On special ideals of non commutative rings

Nico Groenewald

Nelson Mandela University

n-ideals in commutative rings

In 2017 Tekir, Koc and Oral introduced the notion of *n*-ideals for a commutative ring R with identity element: Let $\mathcal{P}(R) = \{a \in R : a^n = 0 \text{ for some } n \in \mathbb{N}\}$ be the prime radical of R.

Definition

A proper ideal I of R is called an n-ideal if whenever $a, b \in R$ with $ab \in I$ and $a \notin \mathcal{P}(R)$, then $b \in I$.

n-ideals in commutative rings

In 2017 Tekir, Koc and Oral introduced the notion of n-ideals for a commutative ring R with identity element: Let $\mathcal{P}(R) = \{a \in R : a^n = 0 \text{ for some } n \in \mathbb{N}\}$ be the prime radical of R.

Definition

A proper ideal I of R is called an n-ideal if whenever $a, b \in R$ with $ab \in I$ and $a \notin \mathcal{P}(R)$, then $b \in I$.

We have:

1. If I is an $n-$ideal of the commutative ring R,then $I \subseteq \mathcal{P}(R)$.
2. $\mathbb{Z}_n$ has an $n-$ideal if and only if $n = p^k$ for some $k \in \mathbb{Z}^+$and p a prime.

n-ideals in commutative rings

In 2017 Tekir, Koc and Oral introduced the notion of n-ideals for a commutative ring R with identity element: Let $\mathcal{P}(R) = \{a \in R : a^n = 0 \text{ for some } n \in \mathbb{N}\}$ be the prime radical of R.

Definition

A proper ideal I of R is called an n-ideal if whenever $a, b \in R$ with $ab \in I$ and $a \notin \mathcal{P}(R)$, then $b \in I$.

We have:

1. If I is an $n-$ideal of the commutative ring R,then $I \subseteq \mathcal{P}(R)$.
2. $\mathbb{Z}_n$ has an $n-$ideal if and only if $n = p^k$ for some $k \in \mathbb{Z}^+$and p a prime.

Compare notion of prime ideals and n-ideals:

1. $3\mathbb{Z}$ is a prime ideal of $\mathbb{Z}$ but not an $n-$ideal since $3\mathbb{Z} \nsubseteq \mathcal{P}(\mathbb{Z}) = \{0\}$.
2. In $\mathbb{Z}_{27}$ $\langle \overline{9} \rangle$ is an $n-$ideal but not a prime ideal

J-ideals in commutative rings

Following this Khashan et al. introduced the notion of a J-ideal for a commutative ring.

Let$\mathcal{J}(R)$ be the Jacobson radical of R.

Definition

A proper ideal I of R is called an $\mathcal{J}$-ideal if whenever $a, b \in R$ with $ab \in I$ and $a \notin \mathcal{J}(R)$, then $b \in I$.

J-ideals in commutative rings

Following this Khashan et al. introduced the notion of a J-ideal for a commutative ring.

Let$\mathcal{J}(R)$ be the Jacobson radical of R.

Definition

A proper ideal I of R is called an $\mathcal{J}$-ideal if whenever $a, b \in R$ with $ab \in I$ and $a \notin \mathcal{J}(R)$, then $b \in I$.

Examples

1. If I is a $\mathcal{J}$-ideal of a ring R, then $I \subseteq \mathcal{J}(R)$.
2. If R is a quasi-local ring, then every proper ideal of R is a $\mathcal{J}$-ideal.
3. In any ring R, every n-ideal I of R is a J-ideal.

Radical ideals in noncommutative rings

In this note we extend these notions to non-commutative rings and show that it is a special case of more a general type of ideal connected to a special radical. The following are some of the well known special radicals, prime radical $\mathcal{P}$, Levitski radical $\mathcal{L}$, Kőthe's nil radical $\mathcal{N}$, Jacobson radical $\mathcal{J}$ and the Brown McCoy radical $\mathcal{G}$.

Definition

Let ρ be a special radical. A proper ideal I of the ring R is called a ρ-ideal if whenever $a, b \in R$ and $aRb \subseteq I$ and $a \notin \rho(R)$, then $b \in I$.

Radical ideals in noncommutative rings

In this note we extend these notions to non-commutative rings and show that it is a special case of more a general type of ideal connected to a special radical. The following are some of the well known special radicals, prime radical $\mathcal{P}$, Levitski radical $\mathcal{L}$, Kőthe's nil radical $\mathcal{N}$, Jacobson radical $\mathcal{J}$ and the Brown McCoy radical $\mathcal{G}$.

Definition

Let ρ be a special radical. A proper ideal I of the ring R is called a ρ-ideal if whenever $a, b \in R$ and $aRb \subseteq I$ and $a \notin \rho(R)$, then $b \in I$.

Remark

If R is an Artinian ring, then since $\mathcal{P}(R) = \mathcal{L}(R) = \mathcal{N}(R) = \mathcal{J}(R) = \mathcal{G}(R)$ the notions of $\mathcal{P}, \mathcal{L}, \mathcal{N}, \mathcal{J}$ and $\mathcal{G}$-ideals are the same. For a commutative ring R, we have $\mathcal{P}(R) = \mathcal{L}(R) = \mathcal{N}(R)$. Hence for commutative rings the notions $\mathcal{P}, \mathcal{L}$ and $\mathcal{N}$-ideals are the same.

If the ring R is commutative

As was mentioned the notions of N-ideals and J-ideals were introduced by Tekir et al. and Khashan et al.for commutative rings.

Definition

If ρ is the prime radical or the Jacobson radical of a commutative ring, then a proper ideal I of R is a ρ-ideal if whenever $a, b \in R$ with $ab \in I$ and $a \notin \rho(R)$, then $b \in I$.

In what follows R is a noncommutative ring and ρ a special radical

Example

If R is a prime ring, then the zero ideal is a ρ -ideal.

In what follows R is a noncommutative ring and ρ a special radical

Example

If R is a prime ring, then the zero ideal is a ρ -ideal.

Proposition

If I is a ρ-ideal ideal of R, then $I \subseteq \rho(R)$.

In what follows R is a noncommutative ring and ρ a special radical

Example

If R is a prime ring, then the zero ideal is a ρ -ideal.

Proposition

If I is a ρ-ideal ideal of R, then $I \subseteq \rho(R)$.

Remark

In general the converse of the above result is not true.

In what follows R is a noncommutative ring and ρ a special radical

Example

If R is a prime ring, then the zero ideal is a ρ -ideal.

Proposition

If I is a ρ-ideal ideal of R, then $I \subseteq \rho(R)$.

Remark

In general the converse of the above result is not true.
Consider the Jacobson radical and the ring $\mathbb{Z}_{36}$. Now $\mathcal{J}(\mathbb{Z}_{36}) = \langle \overline{6} \rangle$ and $I = \langle \overline{12} \rangle \subseteq \mathcal{J}(\mathbb{Z}_{36})$. But I is not a $\mathcal{J}-$ideal since $\overline{3}\mathbb{Z}_{36}\overline{4} \subseteq I$ with $\overline{3} \notin \mathcal{J}(\mathbb{Z}_{36})$ and $\overline{4} \notin I$.

Theorem

Let R and S be rings and $f : R \to S$ be a surjective ring-homomorphism. If ρ is a special radical, then the following statements hold:

1. *If I is a ρ-ideal of R and $\ker(f) \subseteq I$, then $f(I)$ is a ρ-ideal of S.*
2. *If J is a ρ-ideal of S and $\ker(f) \subseteq \rho(R)$, then $f^{-1}(J)$ is a ρ-ideal of R.*

Corollary

Let ρ be a special radical and let R be a ring and let I, K be two ideals of R with $K \subseteq I$. Then the following hold.

1. *If I is a ρ-ideal of R, then I/K is a ρ-ideal of R/K.*
2. *If I/K is a ρ-ideal of R/K and $K \subseteq \rho(R)$, then I is a ρ-ideal of R.*
3. *If I/K is a ρ-ideal of R/K and K is a ρ-ideal of R, then I is a ρ-ideal of R.*

Proposition

Let ρ be a special radical and R a ring. If $I \lhd R$ such that $R/I \in \mathcal{S}_\rho \cap \mathcal{P} = \{R : \rho(R) = 0\} \cap \mathcal{P}$ where , $\mathcal{P}$ is the class of prime rings, then I is a ρ-ideal if and only if $I = \rho(R)$.

Proposition

Let ρ be a special radical and R a ring. If $I \lhd R$ such that $R/I \in \mathcal{S}_\rho \cap \mathcal{P} = \{R : \rho(R) = 0\} \cap \mathcal{P}$ where , $\mathcal{P}$ is the class of prime rings, then I is a ρ-ideal if and only if $I = \rho(R)$.

Corollary

Let ρ be a special radical.

1. *For a ring R we have that $\rho(R)$ is a ρ-ideal if and only if $\rho(R)$ is a prime ideal.*
2. *If R is a ring such that $R \in \mathcal{S}_\rho$ but $R \notin \mathcal{P}$, then R has no ρ-ideals.*
3. *Let $R \in \mathcal{S}_\rho$. Then 0 is a ρ-ideal if and only if R is a prime ring.*

Proposition

Let ρ be a special radical and let R be a ring with identity. If P is a proper ideal of R, then the following are equivalent:

1. *P is a ρ-ideal of R.*
2. *If A, B are ideals of R such that $AB \subseteq P$ and $A \not\subseteq \rho(R)$, then $B \subseteq P$.*

Proposition

Let ρ be a special radical and let R be a ring with identity. If P is a proper ideal of R, then the following are equivalent:

1. *P is a ρ-ideal of R.*
2. *If A, B are ideals of R such that $AB \subseteq P$ and $A \not\subseteq \rho(R)$, then $B \subseteq P$.*

Proposition

Let ρ be a special radical and let R be a ring with identity and S a nonempty subset of R. If I is a ρ-ideal of R and $S \not\subseteq I$, then $(I : \langle S \rangle) = \{r \in R : r \langle S \rangle \subseteq I\}$ is a ρ-ideal of R.

Proposition

Let ρ be a special radical and let R be a ring with identity. If P is a proper ideal of R, then the following are equivalent:

1. *P is a ρ-ideal of R.*
2. *If A, B are ideals of R such that $AB \subseteq P$ and $A \nsubseteq \rho(R)$, then $B \subseteq P$.*

Proposition

Let ρ be a special radical and let R be a ring with identity and S a nonempty subset of R. If I is a ρ-ideal of R and $S \nsubseteq I$, then $(I : \langle S \rangle) = \{r \in R : r \langle S \rangle \subseteq I\}$ is a ρ-ideal of R.

Proposition

If ρ is a special radical and I a maximal ρ-ideal of R, then I is a prime ideal. If in particular $\rho(R) = I$, then the converse is true.

Systems

Definition

Let S be a nonempty subset of R with $R-\rho(R) \subseteq S$. S is called a $\rho-$m-system if $\langle x\rangle \langle y\rangle \cap S \neq \emptyset$ for all $x \in R-\rho(R)$ and $y \in S$.

Systems

Definition

Let S be a nonempty subset of R with $R - \rho(R) \subseteq S$. S is called a $\rho-$m-system if $\langle x\rangle \langle y\rangle \cap S \neq \emptyset$ for all $x \in R - \rho(R)$ and $y \in S$.

Proposition

For a proper ideal I of R, I is a ρ-ideal of R if and only if $R - I$ is a $\rho-$m-system of R.

Systems

Definition

Let S be a nonempty subset of R with $R - \rho(R) \subseteq S$. S is called a $\rho-$m-system if $\langle x \rangle \langle y \rangle \cap S \neq \emptyset$ for all $x \in R - \rho(R)$ and $y \in S$.

Proposition

For a proper ideal I of R, I is a ρ-ideal of R if and only if $R - I$ is a $\rho-$m-system of R.

Recall that if I is an ideal which is disjoint from a m-system S of R, then there exists a prime ideal P of R containing I such that $P \cap S = \emptyset$. The following proposition states a similar result for ρ-ideals.

Systems

Definition

Let S be a nonempty subset of R with $R-\rho(R)\subseteq S$. S is called a $\rho-$m-system if $\langle x\rangle\langle y\rangle\cap S\neq\emptyset$ for all $x\in R-\rho(R)$ and $y\in S$.

Proposition

For a proper ideal I of R, I is a ρ-ideal of R if and only if $R-I$ is a $\rho-$m-system of R.

Recall that if I is an ideal which is disjoint from a m-system S of R, then there exists a prime ideal P of R containing I such that $P\cap S=\emptyset$. The following proposition states a similar result for ρ-ideals.

Proposition

Let I be an ideal of R such that $I\cap S=\emptyset$ where S is a $\rho-$m-system of R. Then there exists a ρ-ideal K of R containing I such that $K\cap S=\emptyset$.

Special submodules

Definition

Let ρ be a special radical and let M be an $R-$module. The proper submodule N of M is a $\rho-$submodule if for all $a \in R$ and $m \in M$, whenever $aRm \subseteq N$ and $a \notin (\rho(R)M : M)$, then $m \in N$.

Special submodules

Definition

Let ρ be a special radical and let M be an $R-$module. The proper submodule N of M is a $\rho-$submodule if for all $a \in R$ and $m \in M$, whenever $aRm \subseteq N$ and $a \notin (\rho(R)M : M)$, then $m \in N$.

Proposition

Let ρ be a special radical and let M be an $R-$module. For N a submodule of M and I an ideal of R. If N is a ρ-submodule of M and $(\rho(R)M : M) = \rho(R)$, then $(N : M) =$ is a ρ-ideal of R.

Special submodules

Definition

Let ρ be a special radical and let M be an $R-$module. The proper submodule N of M is a $\rho-$submodule if for all $a \in R$ and $m \in M$, whenever $aRm \subseteq N$ and $a \notin (\rho(R)M : M)$, then $m \in N$.

Proposition

Let ρ be a special radical and let M be an $R-$module. For N a submodule of M and I an ideal of R. If N is a ρ-submodule of M and $(\rho(R)M : M) = \rho(R)$, then $(N : M) =$ is a ρ-ideal of R.

Remark

If $(\rho(R)M : M) \nsubseteq \rho(R)$, it need not be true. Let $\mathcal{P}$ be the prime radical. For the $\mathbb{Z}$ module $M = \mathbb{Z}_2$ we have $\mathcal{P}(\mathbb{Z}) = (0)$ and $(\mathcal{P}(\mathbb{Z})\mathbb{Z}_2 : \mathbb{Z}_2) = ((0) : \mathbb{Z}_2) = 2\mathbb{Z}$. Now, $N = (0)$ is clearly a $\mathcal{P}$ submodule. $(N : M) = ((0) : \mathbb{Z}_2) = 2\mathbb{Z}$ is not a $\mathcal{P}$ ideal of $\mathbb{Z}$. We have $2\mathbb{Z}3 \subseteq 2\mathbb{Z}$ with $3 \notin 2\mathbb{Z}$.

Characterization of ρ submodules

In the following proposition, we give a characterization of ρ-submodules for a special radical ρ.

Characterization of ρ submodules

In the following proposition, we give a characterization of ρ-submodules for a special radical ρ.

Proposition

Let ρ be a special radical and let M be an $R-$module where R is a ring with identity. Let N be a proper submodule of M. Then N is a ρ-submodule of M if and only if for any ideal I of R and every submodule K of M, we have $IK \subseteq N$ with $I \nsubseteq (\rho(R)M : M)$ implies $K \subseteq N$.

Idealization

We now show how to construct ρ-ideals using the Method of Idealization. In what follows, R is a ring (associative, not necessarily commutative and not necessarily with identity) and M is an $R-R$-bimodule. The idealization of M is the ring $R \boxplus M$ with $(R \boxplus M, +) = (R, +) \oplus (M, +)$ and the multiplication is given by $(r, m)(s, n) = (rs, rn + ms)$

Idealization

We now show how to construct ρ-ideals using the Method of Idealization. In what follows, R is a ring (associative, not necessarily commutative and not necessarily with identity) and M is an $R-R$-bimodule. The idealization of M is the ring $R \boxplus M$ with $(R \boxplus M, +) = (R, +) \oplus (M, +)$ and the multiplication is given by $(r, m)(s, n) = (rs, rn + ms)$

Proposition

Let ρ is a special radical. Let I be a ρ-ideal of R and N an $R-R$-bi-submodule of the $R-R$-bi-module M. Then

1. *$I \boxplus M$ is a ρ-ideal of $R \boxplus M$.*
2. *If $(\rho(R)M : M) = \rho(R)$ and N is a ρ-submodule of M with $IM + MI \subseteq N$, then $I \boxplus N$ is a ρ-ideal of $R \boxplus M$.*

Example

If I is a ρ-ideal of a ring R and N is a $R-R$-bi-submodule of M with $IM+MI \subseteq N$, then $I \boxplus N$ need not be a ρ-ideal of $R \boxplus M$. For example if ρ is the prime radical, $\{0\}$ is a ρ-ideal of the ring of integers $\mathbb{Z}$ and 0 is a submodule of the $\mathbb{Z}$-module $\mathbb{Z}_6$. But $0 \boxplus (0)$ is not a ρ-ideal of $\mathbb{Z} \boxplus \mathbb{Z}_6$ since $(2,0)\mathbb{Z} \boxplus \mathbb{Z}_6(0,3) \subseteq 0 \boxplus (0)$ and $(2,0) \notin \mathcal{P}(\mathbb{Z} \boxplus \mathbb{Z}_6) = \mathcal{P}(\mathbb{Z}) \boxplus \mathbb{Z}_6$ but $(0,3) \notin 0 \boxplus (0)$.

Example

If I is a ρ-ideal of a ring R and N is a $R-R$-bi-submodule of M with $IM + MI \subseteq N$, then $I \boxplus N$ need not be a ρ-ideal of $R \boxplus M$. For example if ρ is the prime radical, $\{0\}$ is a ρ-ideal of the ring of integers $\mathbb{Z}$ and 0 is a submodule of the $\mathbb{Z}$-module $\mathbb{Z}_6$. But $0 \boxplus (0)$ is not a ρ-ideal of $\mathbb{Z} \boxplus \mathbb{Z}_6$ since $(2,0)\mathbb{Z} \boxplus \mathbb{Z}_6(0,3) \subseteq 0 \boxplus (0)$ and $(2,0) \notin \mathcal{P}(\mathbb{Z} \boxplus \mathbb{Z}_6) = \mathcal{P}(\mathbb{Z}) \boxplus \mathbb{Z}_6$ but $(0,3) \notin 0 \boxplus (0)$.

Proposition

Let ρ is a special radical. Let I be an ideal of R and N a proper $R-R$-bi-submodule of the $R-R$-bi-module M. If $I \boxplus N$ is a ρ-ideal of $R \boxplus M$, then I is a ρ-ideal of R and N is a ρ-submodule of M.

Product rings

Suppose that R_1,R_2 are two noncommutative rings with nonzero identities and $R = R_1 \times R_2$. Then R becomes a noncommutative ring with coordinate-wise addition and multiplication. Also, every ideal I of R has the form $I = I_1 \times I_2$, where I_i is an ideal of R_i for $i = 1, 2$. Now, we give the following result.

Proposition

Let R_1 and R_2 be two noncommutative rings and let ρ be a special radical then $R_1 \times R_2$ has no ρ-ideals

P-ideals

In this section the special radical will be the prime radical. Tekir et.al introduced the notion of N-ideals for commutative rings with identity element. They investigate many properties of N-ideals with properties similar to that of prime ideals. We show that for the prime radical many of the results proved by Tekir et.al are also true for non-commutative rings.

P-ideals

In this section the special radical will be the prime radical. Tekir et.al introduced the notion of N-ideals for commutative rings with identity element. They investigate many properties of N-ideals with properties similar to that of prime ideals. We show that for the prime radical many of the results proved by Tekir et.al are also true for non-commutative rings.

In what follows for the non-commutative ring R, $\mathcal{P}(R)$ will denote the prime radical of the ring R.Throughout this section the rings are non-commutative but not necessarily assumed to have a unity unless indicated.

P-ideals

In this section the special radical will be the prime radical. Tekir et.al introduced the notion of N-ideals for commutative rings with identity element. They investigate many properties of N-ideals with properties similar to that of prime ideals. We show that for the prime radical many of the results proved by Tekir et.al are also true for non-commutative rings.

In what follows for the non-commutative ring R, $\mathcal{P}(R)$ will denote the prime radical of the ring R.Throughout this section the rings are non-commutative but not necessarily assumed to have a unity unless indicated.

Definition

A proper ideal I of a ring R is a $\mathcal{P}$-ideal if whenever $a, b \in R$ such that $aRb \subseteq I$ and $a \notin \mathcal{P}(R)$, then $b \in I$.

If R is a commutative ring, then the notion of a $\mathcal{P}$-ideal coincides with an N-ideal as been defined by Tekir et.al

Example

In any prime ring R the zero ideal is a $\mathcal{P}$-ideal. Let $a, b \in R$ such that $aRb = 0$ and $a \notin \mathcal{P}(R) = (0)$. Since R is a prime ring and $a \neq 0$, we have $b = 0$. Hence the zero ideal is a $\mathcal{P}$-ideal.

Results of Tekir et al for non-commutative rings

For the prime radical and a non-commutative ring we now have the following results from which the results of Tekir et al follow as special cases.

1. If a proper ideal I of a ring R is a $\mathcal{P}$-ideal, then $I \subseteq \mathcal{P}(R)$.
2. For a prime ideal I of R, I is a $\mathcal{P}$-ideal of R if and only if $I = \mathcal{P}(R)$.
3. For a ring R we have that $\mathcal{P}(R)$ is a $\mathcal{P}$-ideal if and only if $\mathcal{P}(R)$ is a prime ideal.
4. If R is a semi-prime ring which is not a prime ring, then R has no $\mathcal{P}-$ideals.
5. Let R be a semi-prime ring. Then R is a prime ring if and only if 0 is a $\mathcal{P}$-ideal.
6. If I is a maximal $\mathcal{P}$-ideal of R, then $I = \mathcal{P}(R)$.

Theorem

For any ring the following are equivalent:

1. *R is a prime ring.*
2. *(0) is the only $\mathcal{P}$-ideal of R.*

$\mathcal{P}$-m systems

Definition

Let S be a nonempty subset of R with $R - \mathcal{P}(R) \subseteq S$. S is called a $\mathcal{P}-$m-system if $\langle x \rangle \langle y \rangle \cap S \neq \emptyset$ for all $x \in R - \mathcal{P}(R)$ and $y \in S$.

$\mathcal{P}$-m systems

Definition

Let S be a nonempty subset of R with $R - \mathcal{P}(R) \subseteq S$. S is called a $\mathcal{P}-$m-system if $\langle x \rangle \langle y \rangle \cap S \neq \emptyset$ for all $x \in R - \mathcal{P}(R)$ and $y \in S$.

Proposition

For a proper ideal I of R, I is a $\mathcal{P}$-ideal of R if and only if $R - I$ is a $\mathcal{P}-$m-system of R.

$\mathcal{P}$-m systems

Definition

Let S be a nonempty subset of R with $R - \mathcal{P}(R) \subseteq S$. S is called a $\mathcal{P}-$m-system if $\langle x \rangle \langle y \rangle \cap S \neq \emptyset$ for all $x \in R - \mathcal{P}(R)$ and $y \in S$.

Proposition

For a proper ideal I of R, I is a $\mathcal{P}$-ideal of R if and only if $R - I$ is a $\mathcal{P}-$m-system of R.

Proposition

Let I be an ideal of R such that $I \cap S = \emptyset$ where S is a $\mathcal{P}-$m-system of R. Then there exists a $\mathcal{P}$-ideal K of R containing I such that $K \cap S = \emptyset$.

J-ideals

In this section the special radical will be the Jacobson radical. Khashan et.al introduced the notion of $J-$ideals for commutative rings with identity element. We show that for the Jacobson radical many of the results proved by Khashan et.al are also true for non-commutative rings.In what follows for the non-commutative ring R, $\mathcal{J}(R)$ will denote the Jacobson radical of the ring R.Throughout this section the rings are non-commutative but not necessarily assumed to have a unity unless indicated.

J-ideals

In this section the special radical will be the Jacobson radical. Khashan et.al introduced the notion of $J-$ideals for commutative rings with identity element. We show that for the Jacobson radical many of the results proved by Khashan et.al are also true for non-commutative rings.In what follows for the non-commutative ring R, $\mathcal{J}(R)$ will denote the Jacobson radical of the ring R.Throughout this section the rings are non-commutative but not necessarily assumed to have a unity unless indicated.

Definition

A proper ideal I of a ring R is a $\mathcal{J}$-ideal if whenever $a, b \in R$ such that $aRb \subseteq I$ and $a \notin \mathcal{J}(R)$, then $b \in I$.

If R is a commutative ring, then the notion of a $\mathcal{J}$-ideal coincides with a J-ideal as been defined by Khashan et.al.

Results of Khashan et al for non-commutative rings

Proposition

Let R be a ring.

1. *If R is a semiprimitive ring which is not a prime ring, then R has no $\mathcal{J}$-ideals.*
2. *Let R be a semiprimitive ring. Then R is a prime ring if and only if the zero ideal is a $\mathcal{J}$-ideal of R.*

Results of Khashan et al for non-commutative rings

Proposition

Let R be a ring.

1. *If R is a semiprimitive ring which is not a prime ring, then R has no $\mathcal{J}$-ideals.*
2. *Let R be a semiprimitive ring. Then R is a prime ring if and only if the zero ideal is a $\mathcal{J}$-ideal of R.*

Theorem

Let R be a ring. The following are equivalent:

1. *R is a local ring.*
2. *Every proper ideal of R is a $\mathcal{J}$ ideal.*
3. *Every proper principal ideal of R is a $\mathcal{J}$ ideal.*

Another result of Khashan et al for non-commutative rings

Proposition

Let R be a ring and I be a proper ideal of R. Then $I[|x|]$ is a $\mathcal{J}$-ideal of $R[|x|]$ if and only if I is a $\mathcal{J}$-ideal of R

References

Hani A. Khashan and Amal B. Bani-Ata, J-ideals of commutative rings, *International Electronic Journal of Algebra* Volume **29** (2021) 148-164

U. Tekir, S. Koc and K.H. Oral, n-Ideals of commutative rings, *Filomat*, **31**(10) (2017), 2933-2941.

Thank You