1.2 $a^2 = b^2 \iff a^2 - b^2 = 0 \iff (a-b)(a+b) = 0$

a)

$\Rightarrow a = b \vee a = -b$

Vermerke: $xy = 0 \Rightarrow x = 0 \vee y = 0$

sei $xy = 0$ mit $x \neq 0 \Rightarrow \exists x^{-1}$

Also $0 = x^{-1} 0 = x^{-1}(xy) = (x^{-1}x)y = 1 \cdot y = y$

$\Rightarrow y = 0$

b) $a^2 = b^2 \wedge ab \geq 0 \Rightarrow a = b$

$ab = 0 \Rightarrow a = 0 \vee b = 0 \Rightarrow a^2 = b^2 = 0 \Rightarrow a = b = 0$

sei $ab \neq 0$ also $ab > 0$

$ab > 0 \Rightarrow (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

Also

$a^2 = b^2 \iff (a+b)(a-b) = 0$

$ab > 0 \Rightarrow a + b > 0 \vee a + b < 0 \Rightarrow a + b \neq 0$

$\Rightarrow a - b = 0 \Rightarrow a = b$

Beweis

- $ab > 0 \Rightarrow (a > 0 \wedge b > 0) \vee (a < 0 \wedge b < 0)$

$a > 0 \Rightarrow a^{-1} > 0$

$a > 0 \wedge a^{-1} < 0 \Rightarrow -a^{-1} \overset{2}{=} (-1)a^{-1} > 0$

$\Rightarrow 0 < a(-a^{-1}) = a((-1)a^{-1}) = (-1)(aa^{-1}) = (-1)1 = -1$ *

Beweis

- $-x = (-1)x$

$1 + (-1) = 0 \Rightarrow 0 = x \cdot 0 = x(1 + (-1))$

$= x + (-1)x \Rightarrow -x = (-1)x$

1-3 $|x| = \begin{cases} x & x > 0 \\ 0 & x = 0 \\ -x & x < 0 \end{cases}$

- $|x|^2 = x^2$

$x \geq 0 \Rightarrow |x| = x \rightarrow |x|^2 = x^2$

$x < 0 \Rightarrow |x| = -x$

$|x|^2 = (-x)(-x) = (-1)x(-1)x = \cdots = (-1)(-1)x^2 = x^2$

- $(-1)(-1) = 1 \Leftrightarrow (-1)(-1) - 1 = 0$

$(-1)(-1) - 1 = (-1)(-1) + (-1) = (-1)(-1) + (-1)\cdot 1$

$= (-1)\left((-1) + 1\right) = (-1)\cdot 0 = 0$

- $|ab|^2 = (ab)^2 = (ab)(ab) = a(b(ab)) = a((ba)b)$

$= a((ab)b) = a(ab^2) = a^2 b^2$

$= |a|^2 |b|^2 = (|a||b|)^2$

$\Rightarrow |ab| = |a||b|$ mit $|ab| \geq 0$ und $|a||b| \geq 0$