MODRED 2019

4th WORKSHOP ON MODEL REDUCTION OF COMPLEX DYNAMICAL SYSTEMS

August 28–30, 2019

University of Graz, Institute of Mathematics and Scientific Computing

LOCAL ORGANIZING COMMITTEE

- Tobias Breiten | Karl-Franzens-Universität Graz
  tobias.breiten@uni-graz.at

- Karl Kunisch | Karl-Franzens-Universität Graz
  karl.kunisch@uni-graz.at

SECRETARY | CONTACT

- **Vanessa Peinhart**
  Institute of Mathematics and Scientific Computing
  Karl-Franzens-Universität Graz
  Heinrichstraße 36, 8010 Graz, Austria
  Tel.: +43 316 3805160
  Fax: +43 316 3809815
  vanessa.peinhart@uni-graz.at

LOCATION

- **Institute of Mathematics and Scientific Computing**
  Heinrichstraße 36, 8010 Graz, Austria
  https://mathematik.uni-graz.at/en/

SCIENTIFIC COMMITTEE

- Peter Benner | Max Planck Institute Magdeburg
- Tobias Breiten | University of Graz
- Heike Faßbender | Technical University of Braunschweig
- Michael Hinze | University of Koblenz-Landau
- Tatjana Stykel | University of Augsburg
- Ralf Zimmermann | University of Southern Denmark, Odense

DEAR PARTICIPANT,

this workshop will gather researchers in the field of model order reduction of dynamical systems. Recent theoretical and numerical contributions for large-scale problems are addressed. Scientific topics include, e.g., system-theoretic methods, rational interpolation, POD and generalizations, vector fitting, Loewner matrix and pencil based approaches, dynamic mode decomposition (DMD) and kernel-based methods.

We wish you a fruitful and stimulating time at the workshop and hope you will enjoy your stay in Graz. If you have any questions, please do not hesitate to contact us.

The organizing committee

CONTENTS

VENUE | TRANSPORT

The conference will take place at the Institute of Mathematics and Scientific Computing of the University of Graz. The address of the building is Heinrichstraße 36 (building 11 of the university campus).

All talks will take place in the big lecture halls on the ground floor. The rooms are equipped with projectors and big chalkboards.

- **Arriving by plane:** The closest airport is Graz Thalerhof. From the airport, you can take the train line S5 to the main railway station or a bus to Jakominiplatz (the central hub for public transport in Graz).

  You can also take a train or bus from Vienna International Airport to Graz.

- **Public transport in Graz:** Graz has an efficient public transport system based on buses and trams. Perhaps the easiest way to purchase a ticket is to use one of the machines within the trams or at some of the bigger stations (in particular, at the main railway station and Jakominiplatz). You can also purchase tickets at a Trafik (kiosk). Hour and day tickets can also be purchased from a bus driver (but not week tickets!).

  We recommend that you buy a week ticket. It costs €15.20 and covers all of Graz (zone 101), including the airport, for 7 days.

MEALS

A limited number of meals are provide by **Uni-Café** located on the ground floor of the math building. Several small restaurants offering a lunch menu can be found in **Zinzendorfgasse** which is in walking distance to the university.

WIFI

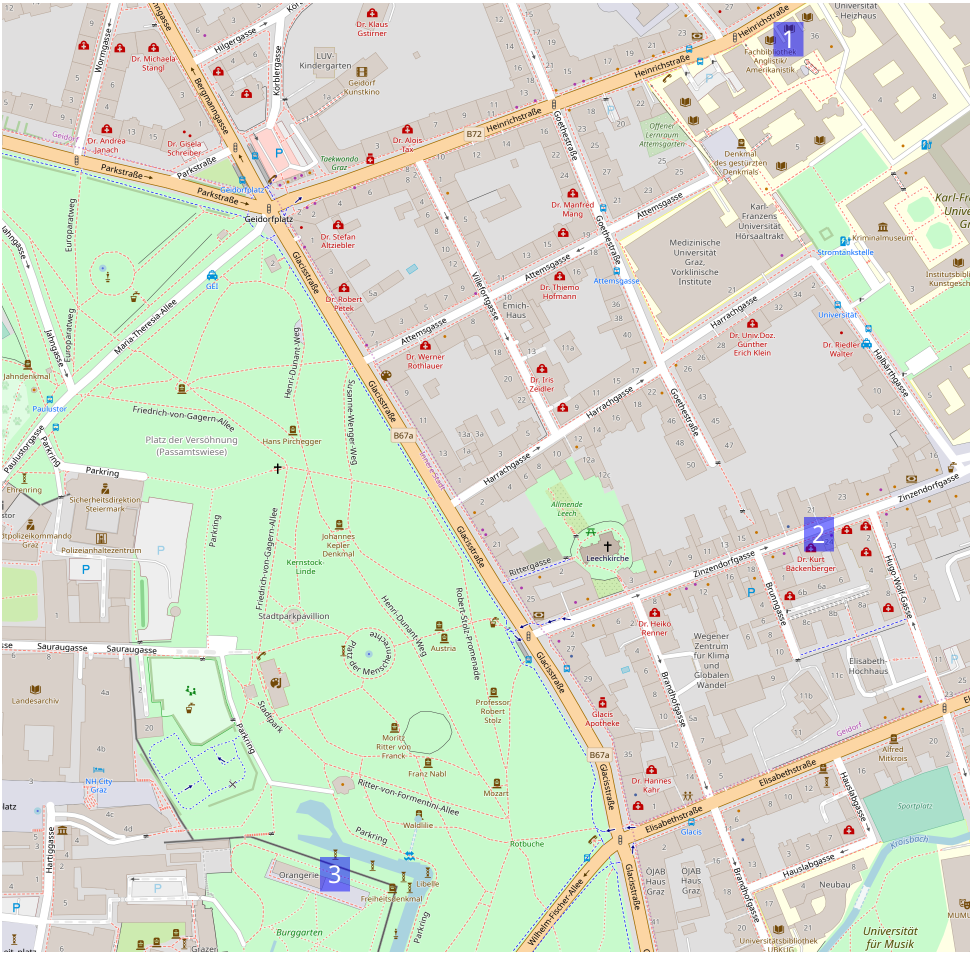
There are two possibilities for free Internet access.

- You can access internet using **eduroam** credentials of your home institution.
- The participants of the workshop can also access **UNIGRAZguest** with the following credentials:

  User: math
  Password: math18

RECEPTION BY THE GOVERNOR

On Wednesday, August 28th, 2019, there will be a reception by the Governor of the Federal State of Styria, in the *Orangerie im Grazer Burggarten*. The reception starts at 19:00 and includes a buffet.



1 Institute of Mathematics   2 Zinzendorfgasse   3 Orangerie

MODRED 2019: Wednesday, August 28

08:50–09:00	**Opening remarks** **HS 11.02**	
09:00–09:50	**N. Kutz:** Machine Learning and Data-Driven Methods for Model Reduction, **Chair:** P. Benner **HS 11.02**	
09:50–10:20	**Coffee break**	
	Chair: C. Himpe **HS 11.01**	**Chair:** J. Saak **HS 11.02**
10:20–10:45	**M. Billaud-Fries:** Geometry based algorithms for dynamical low-rank approximations	**S. Chellappa:** Adaptive parameter sampling using surrogate error model
10:45–11:10	**Z. Drmac:** Numerical aspects of DMD and data driven Koopman spectral analysis	**L. Feng:** Efficient Error Estimation for Model Order Reduction of Linear Non-parametric and Parametric Systems
11:10–11:35	**V. Zambrano:** A Digital-Twin-Building Kernel for Real-Time Computer-Aided Engineering	**M. Hund:** An Optimization Approach for Parametric Model Order Reduction
11:35–12:00	**Z. Tomljanovic:** Sampling-free parametric model reduction of structured systems	**B. Fröhlich:** PMOR in high dimensional parameter spaces and application to structural shape optimization
12:00–14:00	**Lunch break**	
14:00–14:50	**S. Gugercin:** Data-driven modeling of dynamical systems, **Chair:** H. Faßbender **HS 11.02**	
14:50–15:20	**Coffee break**	
	Chair: P. Goyal **HS 11.01**	**Chair:** R. Zimmermann **HS 11.02**
15:20–15:45	**I.V. Gosea:** A data-driven model order reduction approach for linear systems with quadratic output	**P. Schwerdtner:** Structure Preserving and Realization Independent H-infinity Approximation
15:45–16:10	**D. Karachalios:** The Loewner and Volterra frameworks for nonlinear system analysis	**P. Benner:** Localized Balanced Truncation
16:10–16:35	**Y. Yue:** An Adaptive Method for Interpolating Reduced-Order Models Based on Matching and Continuation of Poles	**C. Bertram:** A link between quadrature based approximate balanced truncation and moment matching
16:35–17:00	**D. Quero:** Model reduction of aeroelastic systems in the Loewner framework	**I. Pontes Duff:** Balanced truncation for linear system with quadratic output: theory, error bounds and numerics
18:45–22:00	**Reception** by the Governor of the Federal State of Styria in the *Orangerie im Grazer Burggarten.*	

MODRED 2019: Thursday, August 29

09:00–09:50	**L. Iapichino:** Reduced Basis approaches for efficient solutions of optimal control problems, **Chair:** M. Hinze **HS 11.02**	
09:50–10:20	**Coffee break**	
	Chair: Z. Tomljanovic **HS 11.01**	**Chair:** V. Simoncini **HS 11.02**
10:20–10:45	**C. Lochner:** RB methods in optimal control of electromagnetic wave problems	**H. Sreekumar:** Krylov-Based Model Order Reduction in Vibroacoustics
10:45–11:10	**M. Fossati:** Model-based Adaptive RBM framework for Unsteady Flow Around Lifting Bodies	**D. Palitta:** The projected Newton-Kleinman method for the algebraic Riccati equation
11:10–11:35	**M. Hinze:** Convergence of the reduced basis method for parameter dependent optimal control problems with control constraints	**C. Lee:** Reduced Order Methods in Medical Imaging
11:35–12:00	**D. Korolev:** Reduced basis methods for parabolic problems and applications to permanent magnet synchronous machines	**S. Marques:** Model Order Reduction for Aerodynamic Shape Optimisation
12:00–14:00	**Lunch break**	
14:00–14:50	**U. Wever:** Model Order Reduction as a Key Technology for Realizing Digital Twins, **Chair:** T. Stykel **HS 11.02**	
14:50–15:20	**Coffee break**	
	Chair: W. Schilders **HS 11.01**	**Chair:** S. Gugercin **HS 11.02**
15:20–15:45	**Q. Aumann:** An Adaptive Reduction Method for Poro-acoustic Systems with Frequency Dependent Material Properties	**M. Balmaseda:** Reduced order models for dynamic analysis of rotating structures with geometrical and contact nonlinearities through a POD based forces correction
15:45–16:10	**J. Fehr:** Automatic Model Reduction of a Car Crash Model	**F. Black:** Computation of reduced order models for transport phenomena via shifted proper orthogonal decomposition
16:10–16:35	**W. Arter:** Surrogate models for MHD and Control	**S. Monem:** Parametrized POD-reduction for syngas production
16:35–17:00	**S. Yildiz:** Structure preserving model order reduction of shallow water equation	**P. Goyal:** Automatic Generation of Minimal and Reduced Models for Structured and Nonlinear Parametric Dynamical Systems

MODRED 2019: Friday, August 30

09:00–09:50	**B. Haasdonk:** Kernel-based Surrogate Modelling for Dynamical Systems, **Chair:** T. Breiten **HS 11.02**	
09:50–10:20	**Coffee break**	
	Chair: L. Feng **HS 11.01**	**Chair:** J. Fehr **HS 11.02**
10:20–10:45	**A. Grimm:** Jointly Optimal Modeling in Frequency and Parameter via Kernel-based Approximation	**T. Stykel:** Passivity-preserving model reduction of nonlinear magneto-quasistatic field problems
10:45–11:10	**C. Himpe:** Empirical Dominant Subspaces	**S. Werner:** Frequency- and Time-Limited Balanced Truncation for Second-Order Systems
11:10–11:35	**L. Yu:** Structure preserving model reduction of linear network systems by eigenvalue assignments	**N. T. Son:** Balanced truncation for parametric linear systems using interpolation of gramians: a comparison of linear algebraic and geometric approaches
11:35–12:00	**P. Mlinarić:** Clustering-Based Model Order Reduction for Nonlinear Network Systems	**S. N. Lordejani:** Model order reduction for linear time delay systems based on energy functionals
12:00–12:05	**Closing**	

Parametrized POD-reduction for syngas production

PETER BENNER, SEBASTIAN SAGER, SHAIMAA MONEM

Motivated by the current climate change, we are concerned with CO_2 net consumption, and using it as an alternative source for carbon. CO_2 is particularly interesting as a source for synthesis gas (syngas) production. Syngas is an essential requirement for liquid fuel manufacturing as a clean replacement for fossil fuels, we consider a parameterized nonlinear system governed by a reverse water-gas shift plug flow reactor (rWGS-PFR) model, which is a promising chemical reaction to produce syngas from CO_2. The spatial discretization for the simulated model leads to very high dimensionality that makes the computations hard to tackle, in particular when optimizing the process, which is the ultimate goal. The computational difficulties for the chemical process model can be manipulated by reducing the order of the simulated system. We show a preliminary study of system complexity and the application of parameterized proper orthogonal decomposition (POD) in order to reduce the simulation complexity of the addressed model.

REFERENCES

[1] P. Benner, E. Sachs, S. Volkwein, *Model order reduction for PDE constrained optimization*. In Trends in PDE constrained optimization, 303-326. Birkhäuser, Cham, 2014.

[2] M. Dihlmann, B, Haasdonk, *Certified nonlinear parameter optimization with reduced basis surrogate models*. PAMM. Proc. Appl. Math **13** (2013), 3-6.

[3] F. Bachmann, D. Beermann, J. Lu, S. Volkwein, *POD-based mixed integer optimal control of the heat equation*. Journal of Scientific Computing, 2019, pp 1-28.

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTR. 1
MAGDEBURG
GERMANY

E-mail address: monem@mpi-magdeburg.mpg.de

Convergence of the reduced basis method for parameter dependent optimal control problems with control constraints

AHMAD AHMAD ALI & MICHAEL HINZE

Abstract

Using the variational discretization concept, we establish a reduced model for the underlying parameter dependent optimal control problem. This approach allows us to establish sharp posterior error estimators that do not have residuals for the control variables, which leads to minimal selection of the snapshots in the greedy algorithm. Furthermore, we show theoretically the convergence of the reduced model to the truth one as the number of snapshots increases.

AFFILIATION: UNIVERSITY OF HAMBURG
STREET: BUNDESSTRASSE 55
CITY: HAMBURG
COUNTRY: GERMANY

E-mail address: `ahmad.ali@uni-hamburg.de`

Surrogate models for MHD and Control

WAYNE ARTER

Achieving power generation from magnetically controlled nuclear fusion devices presents a range of challenging problems. The research described here is designed specifically to address transient magnetohydrodynamics (MHD) effects that are capable of reducing energy confinement and hence reducing the efficiency of reactors, particularly those based on the tokamak device concept. Direct simulation of tokamak MHD phenomena is computationally expensive because of the relatively small dissipation coefficients (thermal, viscous and electrical resistivity) of the plasma, their marked anisotropy due to the strong imposed magnetic field, and in some regions, sufficiently long mean free paths to render the fluid approximation itself questionable. Highlights from a lengthy if intermittent campaign of investigation into surrogates will be presented, under two principal headings.

Symmetry The MHD of the tokamak device exhibits approximate rotational and reflectional symmetries, codified as the “Tokamak Group" in ref [1]. Experimental data indicates a fast timescale non-dissipative field behaviour separated by lengthy intervals of relative quiet occupying a resistive timescale. A model derived using equivariant bifurcation theory has been produced [1], with the unusual property of coupling a Hamiltonian and a dissipative model. Interestingly, even the ordinary differential equations (ODEs) of the surrogate model turn out to be difficult to solve accurately by numerical techniques.

Lie-Taylor Another line of investigation [2] involves a generalisation of Dungey’s reduction of MHD to ODEs by considering linear spatial dependence [3]. Ref [2] shows that the linear dependence may be regarded as simply the first term in a Taylor series expansion in spatial variables, leading to a wider family of models. As with the equivariant approach, a simple, physically interesting case is found to lead to a model of mixed type. Work to confront the ODE models with data from PDE solutions and experiment will be described.

This work was funded by the RCUK Energy Programme and the European Communities under the contract of Association between EURATOM and CCFE.

Bibliography

[1] W. Arter. Symmetry constraints on the dynamics of magnetically confined plasma. *Physical Review Letters*, 102(19):195004, 2009.

[2] W. Arter. Beyond Linear Fields: the Lie-Taylor Expansion. *Proc Roy Soc A*, 473:20160525, 2017.

[3] J.W. Dungey. *Cosmic Electrodynamics*. CUP, 1958.

CCFE
D3 Culham Science Centre
Abingdon, Oxon.
UK

E-mail address: wayne.arter@ukaea.uk

An Adaptive Reduction Method for Poro-acoustic Systems with Frequency Dependent Material Properties

QUIRIN AUMANN

Poroelastic materials are made out of an irregular solid skeleton, whose pores are filled with fluid. Such materials are frequently used as noise and vibration control measures due to their dissipative properties. The coupling of the fluid and solid phase is described by the Biot-Allard theory, which models the porous material as an equivalent homogeneous material [1].

If high-frequency dynamic processes are to be simulated accurately, a fine spatial discretization is necessary and computation time grows prohibitively. Due to the material model, each node has sets of degrees of freedom for each the fluid and the solid regime, which further increases the system size. Classic reduction approaches can not be used, as poroelastic materials are highly frequency dependent and the governing differential equations do not have the form of standard second order dynamic systems.

We present a procedure for the efficient computation of poro-acoustic systems based on an Arnoldi method [2]. The terms of the differential equation are transformed into the standard second order structure using a Taylor series [3]. The method adaptively finds reasonable expansion points inside a defined frequency range, where the reduced model is to be considered valid.

The accuracy of the proposed method is shown in numerical examples comparing the results obtained from the reduced models to reference experiments from literature [4].

REFERENCES

[1] R. Rumpler, *Efficient finite element approach for structural-acoustic applications including 3D modelling of sound absorbing porous materials.* PhD Thesis, 2012.

[2] D. Lu, Y. Su, and Z. Bai, "Stability Analysis of the Two-level Orthogonal Arnoldi Procedure," *SIAM J. Matrix Anal. & Appl.*, **37**(1):195–214, 2016.

[3] X. Xie *et al.*, "Adaptive model reduction technique for large-scale dynamical systems with frequency-dependent damping," *Comput. Methods Appl. Mech. Eng.*, **332**:363–381, 2018.

[4] R. Rumpler, P. Göransson, and J.-F. Deü, "A residue-based mode selection and sorting procedure for efficient poroelastic modeling in acoustic finite element applications," *J. Acoust. Soc. Am.*, **134**(6):4730–41, 2013.

CHAIR OF STRUCTURAL MECHANICS, TECHNICAL UNIVERSITY OF MUNICH
ARCISSTR. 21
80333 MUNICH
GERMANY

E-mail address: quirin.aumann@tum.de

Reduced order models for dynamic analysis of rotating structures with geometrical and contact nonlinearities through a POD based forces correction

MIKEL BALMASEDA

Rotating structures are widely used in industrial applications such as turbo-machinery, helicopter blades and wind turbines. The design tendency to create more slender, more flexible and lighter structural components under greater excitations increases the nonlinear behaviour of these components. Thus, the need to accurately predict the dynamic response of geometrically nonlinear structures becomes essential for the designer. The high fidelity finite element models are capable of accurately predict the dynamic response of geometrically nonlinear structures at a high computational cost. Thus, in order to reduce the resolution time of these models, projection based reduced order models provide an acceptable compromise between accuracy and computational time.

In the present work, the geometrically nonlinear reduced order model with POD based forces correction proposed in [1] is adapted to evaluate contact problems. Component mode synthesis is used to keep the physical displacements of the interface in the reduced order model. Thus, the contact is modelled with the penalisation method and the frictional Coulomb law. The vibrations of the rotating structure around the static equilibrium induced by rotation are considered as nonlinear. The geometrically nonlinear generalised forces are represented by a polynomial expansion obtained by the Stiffness Evaluation Procedure (STEP) [2]. During the polynomial coefficient indentification, the nonlinear forces are corrected by means of a Proper Orthogonal Decomposition (POD) of the full order nonlinear forces. The proposed ROM is evaluated for a rotating beam structure.

REFERENCES

[1] M. Balmaseda, G. Jacquet-Richardet, A.Placzek and D.-M. Tran, *Reduced order models for dynamic analysis of nonlinear rotating structures*. Proceedings of the ECCM-ECFD congress, Glasgow 11-15 June 2018, 405-416 pp.

[2] S. A. Rizzi and A. A. Muravyov, *Determination of nonlinear stiffness with application to random vibration of geometrically nonlinear structures*, Computers & Structures, 81(15), pp. 1513-1523, 2003.

DMAS/ONERA, UNIVERSITÉ PARIS-SACLAY // LAMCOS, INSA LYON
29 AVENUE DE LA DIVISION LECLERC // 27BIS, AVENUE JEAN CAPELLE
F-92322 CHÂTILLON // F-69621 VILLEURBANNE
FRANCE

E-mail address: `mikel.balmaseda@onera.fr`

Localized Balanced Truncation

PETER BENNER

Balanced truncation is the most commonly used model order reduction scheme in control engineering. This is due to its favorable properties of automatic stability preservation and the existence of an *a priori* computable error bound, enabling the adaption of the reduced model order to a specified tolerance. It aims at minimizing the worst case error of the frequency response over the whole frequency range. If a good approximation quality in a localized frequency range is required, frequency-weighted or frequency-limited balanced truncation variants can be employed. Nevertheless, these methods often lack stability preservation and/or computable H_∞-type error bounds.

We address both issues with a novel type of localized balanced truncation techniques. First, we construct a family of parameterized frequency-dependent (PFD) mappings which transform a given linear time-invariant (LTI) system to either a discrete-time or continuous-time PFD system. Then, relationships between the maximum singular value of the given LTI system over pre-specified frequency ranges and the maximum singular value of the PFD mapped systems over the entire frequency range are established. By exploiting the properties of the discrete-time PFD mapped systems, a new parameterized frequency-dependent balanced truncation method providing a finite-frequency type error bound with respect to the maximum singular value of the error systems is developed. Numerical examples illustrate the performance of the method.

(This is joint work with Xin Du (Shanghai University).)

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTR. 1
39106 MAGDEBURG
GERMANY

E-mail address: benner@mpi-magdeburg.mpg.de

A link between quadrature based approximate balanced truncation and moment matching

CHRISTIAN BERTRAM

The goal of projection based model order reduction of stable linear time-invariant systems (LTI) is to replace the large and sparse system matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ of the system

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned}$$

by smaller ones, thereby approximately retaining the input-output behaviour of the original system. For projection matrices $V, W \in \mathbb{R}^{n\times r}$ with $W^\mathsf{T} V = I$ the reduced system matrices are given by $\hat{A} = W^\mathsf{T} AV$, $\hat{B} = W^\mathsf{T} B$ and $\hat{C} = CV$.

In the method balanced truncation the controllability and observability gramians

$$\mathcal{P} = \int_0^\infty e^{At} BB^\mathsf{T} e^{A^\mathsf{T} t}\, \mathrm{d}t, \qquad \mathcal{Q} = \int_0^\infty e^{A^\mathsf{T} t} C^\mathsf{T} C e^{At}\, \mathrm{d}t,$$

play a central role. In approximate balanced truncation the projection matrices are derived from approximations to the gramians. In moment matching the power series expansion around $s_0 \in \mathbb{C}$ for the transfer function is considered, $G(s) = \sum_{j=0}^{\infty} m_j(s_0)(s - s_0)^j$. The projection matrices are chosen such that the moments of the original and reduced system coincide, i.e. $m_j(s_0) = \hat{m}_j(s_0)$ for $j = 0, \dots, N$.

We present a quadrature framework based on Runge-Kutta methods, which produces low-rank approximations to the gramians. We show that hereby moment matching is performed around the inverse (conjugated) eigenvalues of the matrices defined in the Butcher tableaus used in the quadrature method, divided by the time step size.

Our work extends the analysis of Opmeer (IEEE TAC 57(2), 2012), where a different quadrature approach was used to approximate the gramians.

This is joint work with Heike Faßbender, TU Braunschweig.

TU BRAUNSCHWEIG
UNIVERSITÄTSPLATZ 2
38106 BRAUNSCHWEIG
GERMANY

E-mail address: ch.bertram@tu-braunschweig.de

Geometry based algorithms for dynamical low-rank approximations

MARIE BILLAUD-FRIES

Solving high-dimensional dynamical systems with classical numerical methods is usually untractable, especially when they are parameter dependent. Dynamical model order reduction (MOR) methods aim at reducing the complexity by projecting dynamical systems onto low-dimensional manifolds. In this work, we particularly focus on dynamical low-rank method that exploits the geometrical structure of the set of fixed rank matrices. The goal of this talk is two folds. First, we present a new geometric description of the sets of fixed rank matrices which relies on a natural parametrization of matrices. More precisely, it is endowed with the structure of analytic principal bundle, with an explicit description of local charts. Second, algorithms working in local coordinates are proposed for MOR.

This is a joint work with Anthony Nouy (Centrale Nantes) and Antonio Falcó (Universidad CEU Cardenal Herrera).

CENTRALE NANTES, LABORATOIRE DE MATHÉMATIQUES JEAN LERAY
1 RUE DE LA NOË, 44321 NANTES, FRANCE

E-mail address: marie.billaud-friess@ec-nantes.fr

Computation of reduced order models for transport phenomena via shifted proper orthogonal decomposition

FELIX BLACK

Many classical model order reduction methods are formulated in a projection framework, building the reduced order model (ROM) by projecting the full order model (FOM) onto a suitable subspace. If the FOM exhibits advective transport, classical methods often fail to produce low-dimensional models with a small approximation error. One strategy to remedy this problem is the shifted proper orthogonal decomposition (shifted POD) [1], which extends the classical POD by introducing additional transformation operators associated with the modes. The transformation operators are parametrized by paths in a suitable parameter space, thus allowing the transformed modes to cope with the convection. In contrast to classical methods that project onto a fixed subspace, the ROM of the shifted POD is thus obtained by projecting onto a time- and/or state-dependent subspace that adapts itself to the problem. On the one hand this approach is very flexible, on the other hand it introduces additional complexity to the online stage, since in addition to the time-dependent coefficients, also the paths need to be computed from the ROM. In this talk, we present the online stage of the shifted POD and discuss how to build a ROM from which we may compute the coefficients and the paths. Additionally, we show that for a certain class of problems, first transforming the system, projecting, and afterwards applying the inverse transformation is equivalent to directly projecting onto the transformed modes, which illustrates the close relation of the shifted POD to the method of freezing [2,3]. This is joint work with Philipp Schulze and Benjamin Unger (both TU Berlin).

REFERENCES

[1] J. Reiss, P. Schulze, J. Sesterhenn, V. Mehrmann, *The Shifted Proper Orthogonal Decomposition: A Mode Decomposition for Multiple Transport Phenomena*, SIAM J. Sci. Comput. **40** (2018), no. 3, A1322–A1344.

[2] W. J. Beyn, V. Thümmler, *Freezing Solutions of Equivariant Evolution Equations*, SIAM J. Appl. Dyn. Syst., **3** (2004), no. 2, 85–116.

[3] M. Ohlberger, S. Rave, *Nonlinear reduced basis approximation of parameterized evolution equations via the method of freezing*, Comptes Rendus Mathematique, **351** (2013), no. 23–24, 901–906.

TECHNISCHE UNIVERSITÄT BERLIN - FAKULTÄT II INSTITUT FÜR MATHEMATIK - AG MODELLIERUNG, NUMERIK, DIFFERENTIALGLEICHUNGEN
STRASSE DES 17. JUNI 136
BERLIN
GERMANY

E-mail address: black@math.tu-berlin.de

Adaptive parameter sampling using surrogate error model

SRIDHAR CHELLAPPA, Lihong Feng, Peter Benner

In the Model Order Reduction (MOR) of complex, parametric systems, the choice of an appropriate training set is often critical. A small training set could lead to the reduced model being valid for only those parameters in the training set and unfit for independent test parameters (overfitting). On the other hand, a large, high-dimensional training set with wide parameter ranges poses computational challenges [1, 2, 3, 4]. In this talk, we propose an approach based on a Radial Basis Function (RBF) interpolation of the *a-posteriori* error estimator. The training set is enriched or depleted by querying the RBF error surrogate, based on some criteria. We present the adaptive sampling framework for both time and frequency domain MOR methods, such as the Reduced basis method and the Multi-moment matching method [5], respectively. Simulations on benchmark examples are provided to show the performance of the proposed approach.

REFERENCES

[1] J. L. Eftang, D. J. Knezevic, A.T. Patera, *An hp certified reduced basis method for parametrized parabolic partial differential equations*, Math. Comput. Model. Dyn. Syst. **17:4** (2011), 395–422.

[2] B. Haasdonk, M. Dihlmann, M. Ohlberger, *A training set and multiple bases generation approach for parameterized model reduction based on adaptive grids in parameter space*, Math. Comput. Model. Dyn. Syst. **17:4** (2011), 423–442.

[3] J. S. Hesthaven, B. Stamm, S. Zhang, *Efficient greedy algorithms for high-dimensional parameter spaces with applications to empirical interpolation and reduced basis methods*, ESAIM: Math. Model. Numer. Anal. **48:1** (2014), 259–283.

[4] A. Paul-Dubois-Taine, D. Amsallem, *An adaptive and efficient greedy procedure for the optimal training of parametric reduced-order models*, Internat. J. Numer. Methods Engrg. **102:5** (2015), 1262–1292.

[5] L. Feng, A. C. Antoulas, P. Benner, *Some a posteriori error bounds for reduced order modelling of (non-)parametrized linear systems*, ESAIM: Math. Model. Numer. Anal. **51:6** (2017), 2127–2158.

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTRASSE 1
MAGDEBURG
GERMANY

E-mail address: chellappa@mpi-magdeburg.mpg.de

Numerical aspects of DMD and data driven Koopman spectral analysis

Zlatko Drmač

In this talk, we present recent development of numerically robust computational tools for dynamic mode decomposition and data driven Koopman spectral analysis.

We consider both the SVD based Schmid's DMD and the natural formulation via the Krylov decomposition with the Frobenius companion matrix. We show how to use the eigenvectors of the companion matrix explicitly – these are the columns of the inverse of the notoriously ill–conditioned Vandermonde matrix. The key step to curb ill–conditioning is the discrete Fourier transform of the snapshots; in the new representation, the Vandermonde matrix is transformed into a generalized Cauchy matrix, which then allows accurate computation by specially tailored algorithms of numerical linear algebra. Numerical experiments show robustness in extremely ill-conditioned cases.

This is a joint work with Igor Mezić (University of California, Santa Barbara) and Ryan Mohr (AIMdyn Inc., Santa Barbara).

The research is supported by the DARPA Contract HR0011-16-C-0116 *"On a Data-Driven, Operator-Theoretic Framework for Space-Time Analysis of Process Dynamics"* and the DARPA Contract HR0011-18-9-0033 *"The Physics of Artificial Intelligence"*

References

[1] Z. Drmač, I. Mezić, R. Mohr, *Data Driven Modal Decompositions: Analysis and Enhancements*, SIAM Journal on Scientific Computing, Vol. 40, (2018) A2253-A2285.

[2] Z. Drmač, I. Mezić, R. Mohr, *On least squares problems with certain Vandermonde–Khatri–Rao structure with applications to DMD*, arXiv:1811.12562.

[3] Z. Drmač, I. Mezić, R. Mohr, *Data driven Koopman spectral analysis in Vandermonde-Cauchy form via the DFT: numerical method and theoretical insights*, arXiv:1808.09557.

University of Zagreb, Department of Mathematics

Bijenička 30 Zagreb Croatia

E-mail address:

drmac@math.hr

Automatic Model Reduction of a Car Crash Model

Jörg Fehr

The crash simulation of finite element car models is still a cumbersome task in the day to day work of an automotive engineer since the simulation takes usually several hours even on high-performance computing clusters due to the high dimension. Model reduction is only applied in rare cases or to certain parts of the model due to its highly nonlinear behavior. We present a new, fully-automated workflow for the reduction of a full car model in the industrial simulation environment LS-DYNA.

In the first step, the model is separated into four classes based on its behavior, which will be treated with different MOR methods later. Clustering methods based on the displacement (see [1]), stress-strain curves or the plastic strain lead to clusters which are then sorted into four classes depending on the degree of nonlinearity of all elements of each part.

The first class of parts that do not experience any or negligible displacement during the crash is approximated with rigid bodies. Parts undergoing linear deformations are approximated with linear MOR methods like Craig-Bampton or Krylov reduction with the help of the software package MatMorembs. The constraints modeled in LS-DYNA are a challenge for the reduction, which is tackled with fully automated process cutting the parts around the constraints. If parts experience nonlinear behavior, they are either not reduced at all or the Energy-Conserving Sampling and Weighting (ECSW) method (see [2,3]) is applied directly in the LS-DYNA Fortran code.

The overall workflow is mainly automated and applied to a model of a 1996 Dodge Neon with around $270\,000$ FE elements.

References

[1] D. Grunert, J. Fehr, *Identification of Nonlinear Behavior with Clustering Techniques in Car Crash Simulations for Better Model Reduction*, Advanced Modeling and Simulation in Engineering Sciences, Vol. 3, No. 1, pp. 1–19, 2016.

[2] C. Farhat, P. Avery, T. Chapman, J. Cortial, *Dimensional Reduction of Nonlinear Finite Element Dynamic Models with Finite Rotations and Energy-based Mesh Sampling and Weighting for Computational Efficiency*, International Journal for Numerical Methods in Engineering, Vol. 98, No. 9, pp. 625–662, 2014.

[3] C. Bach, L. Song, T. Erhart, F. Duddeck: Dimensionality reduction of crash and impact simulations using LS-DYNA. 12th European LS-DYNA Conference 2019, Koblenz, Germany.

Institute of Engineering and Computational Mechanics, University of Stuttgart
Pfaffenwaldring 9
70569 Stuttgart
Germany

E-mail address: `joerg.fehr@itm.uni-stuttgart.de`

Efficient Error Estimation for Model Order Reduction of Linear Non-parametric and Parametric Systems

LIHONG FENG and PETER BENNER

We propose an error estimator for reduced-order modeling of linear non-parametric or parametric dynamical systems. The error estimator estimates the error of the reduced transfer function in frequency domain and can be easily extended to output error estimation for reduced-order models of steady linear parametric systems. It is sharp and cheap to compute. Using the error estimator, the reduced-order model can be adaptively obtained with high reliability. Numerical results show that the error estimator can accurately estimate the true error even for transfer functions with many resonances. Compared with an existing error bound [1], the proposed error estimator can be orders of magnitudes sharper and needs much less computational time. It is also sharper than the output error estimation based on randomized residual proposed in [2].

REFERENCES

[1] L. Feng, A. C. Antoulas, and P. Benner, "Some a posteriori error bounds for reduced order modelling of (non-)parametrized linear systems," ESAIM: M2AN, vol. 51, 2127-2158, 2017.

[2] K. Smetana, O. Zahm and A. T. Pater. Randomized residual-based error estimators for parametrized equations. arXiv e-prints, Cornell University. 2018. https://arxiv.org/abs/1807.10489. math.N.A.

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTR 1
MAGDEBURG
GERMANY

E-mail address: feng@mpi-magdeburg.mpg.de, benner@mpi-magdeburg.mpg.de

Model-based Adaptive RBM framework for Unsteady Flow Around Lifting Bodies

MARCO FOSSATI, GAETANO PASCARELLA, GABRIEL BARRENECHEA

The present work investigates the problem of performing accurate reconstructions of vortex-dominated unsteady flows by means of Reduced Basis Methods (RBM). When faced with the necessity of reconstructing the flow field over a specified time window, the proposed method aims at automatically and adaptively selecting the most accurate reduction technique among a collection of model reduction techniques such as the classical Proper Orthogonal Decomposition (POD), Spectral POD (SPOD) [1] and Recursive Dynamic Mode Decomposition [2]. The rationale behind the development of such an adpative framework is to try to cope with the potential loss of important dynamic information that accompanies classical methods, e.g. POD, where snapshots are treated as statistically independent observation of the dynamical system at study [3].

The adaptive framework will be assessed with respect to two different ways of estimating the reconstruction error by the various methods. One method, referred to as direct error, will employ additional snapshots, not used to build the RBM, and will compare explicitly the reduced solution with the reference data. The second method will instead consider a Finite Volume discretisation of the equations and evaluate the error in terms of the unsteady residual of the reduced solution. A Backward Differencing Formula (BDF) will be used to ensure second order accuracy in the estimation of the residual. Emphasis will be put on the comparative assessment of the two error estimation methods with respect to the identification of the most suitable RBM to be used for the reconstruction at a specific instant of time. Problems of relevance to aircraft aerodynamics will be considered such as the impulsive start of 2D and 3D high-lift configurations and other lifting bodies.

REFERENCES

[1] M. Sieber, C.O. Paschereit, K. Oberleithner, *Spectral Proper Orthogonal Decomposition*. J. Fluid Mech., 792 (2017), pp. 798–828.

[2] R.W. Gilmer, *Recursive Dynamic Mode Decomposition of transient and post-transient wake flows*. J. Fluid Mech., 809 (2016), pp. 843–872.

[3] B.R. Noack, *From snapshots to modal exapnsionsbridging low residuals and pure frequencie*. J. Fluid Mech., 802 (2016), pp. 1–4.

UNIVERSITY OF STRATHCLYDE
75 MONTROSE STREET
GLASGOW, G1 5HE
UNITED KINGDOM

E-mail address: `marco.fossati@strath.ac.uk`

PMOR in high dimensional parameter spaces and application to structural shape optimization

BENJAMIN FRÖHLICH*, PETER EBERHARD

Shape optimization in structural mechanics is one way to realize light weight structures. Usually, such optimization problems are solved numerically using the linear finite element method where the governing equations for the elastic components are given as

$$\mathbf{M}(\mathbf{p})\ddot{\mathbf{x}}(t) + \mathbf{D}(\mathbf{p})\dot{\mathbf{x}}(t) + \mathbf{K}(\mathbf{p})\mathbf{x}(t) = \mathbf{f}(t), \quad \mathbf{x}(t) \in \mathbb{R}^N. \tag{5.1}$$

In the case of shape optimizations, the optimization parameter $\mathbf{p} \in \Omega \subset \mathbb{R}^d$ is used to describe geometric quantities of the structure. As the number of degrees of freedom N can be very high due to fine finite element discretizations, it is advantageous to apply PMOR and to use a parameterized reduced order model to conduct the actual optimization.

It is the goal of PMOR to provide a parameterized reduced order model with a low approximation error over the complete domain Ω. However, structural shape optimization very often requires versatile design parameterizations meaning that the number of parameters d describing geometric quantities can easily become large, e.g. $d > 10$. In such cases, finding a parameterized reduced order model of moderate dimension which captures the entire parameter dependent dynamics can become a challenging task making the solution of shape optimization problems a difficult task, too.

It is the idea of this contribution to exploit the fact that a parameterized reduced order model does not have to be accurate over the entire parameter domain. Instead, one is usually only interested in finding the optimizer $\mathbf{p}_{\text{opt}}$ solving the shape optimization problem. This contribution suggests adaptive PMOR methods which refine the parameterized reduced order model during the optimization until the optimizer $\mathbf{p}_{\text{opt}}$ is found. The methods are for example based on error-control or on the solution of subsequent optimization problems. An example from structural shape optimization with a high dimensional parameter space is used to illustrate and compare the approaches.

REFERENCES

[1] P. Benner, S. Gugercin, and K. Willcox, *A survey of projection-based model reduction methods for parametric dynamical systems*, SIAM Review, 57(4):483–531, 2015.

[2] B. Fröhlich, J. Gade, F. Geiger, M. Bischoff, and P. Eberhard, *Geometric element parameterization and parametric model order reduction in finite element based shape optimization*. Computational Mechanics, doi.org/10.1007/s00466-018-1626-1, 2018.

INSTITUTE OF ENGINEERING AND COMPUTATIONAL MECHANICS, UNIVERSITY OF STUTTGART
PFAFFENWALDRING 9
70569 STUTTGART
GERMANY

E-mail address: {benjamin.froehlich,peter.eberhard}@itm.uni-stuttgart.de

A data-driven model order reduction approach for linear systems with quadratic output

I. V. Gosea and A. C. Antoulas

The goal of most model order reduction (MOR) methods is to construct much smaller systems with the same structure and similar response characteristics as the original one.

The Loewner framework (LF) is a data-driven MOR method that was developed over the last decade. For a recent tutorial, see [1]. LF constructs reduced order models directly from measured data by interpolating the transfer function of the original system.

Nonlinear dynamics is intrinsically present in time-dependent processes that are modeled in many engineering branches. Hence, the study, analysis, and modeling of nonlinear dynamical systems have been the focus of increased research. In some cases, it is suitable to devise reduction methods that can be directly applied to the nonlinear system and not to a linearized version thereof. Considerable progress has been made by extending classical MOR methods for reducing certain classes of mildly nonlinear systems. The later category includes e.g., bilinear or quadratic-bilinear(QB) systems. For such systems, the internal (state) variable enters in the differential state equation in a nonlinear way. LF has recently been extended for reducing bilinear systems in [2] and QB systems in [3].

The main focus of this work is to study MOR of linear systems with quadratic output (LQO). For these systems, the nonlinearity is present in the state-output equation only. This allows to explicitly write the input-output mappings (or kernels) of the system in time-domain. By transforming these kernels, one can identify mappings in the frequency domain (transfer functions): one for the linear part and one for the quadratic part.

We propose a data-driven MOR method that uses measured data as samples of the two transfer functions to construct high fidelity reduced-order LQO models. This methodology can be considered as a natural extension of LF for this class of systems.

We compare the numerical results of our method against other LQO methods (such as balanced truncation or rational Krylov techniques) for certain large-scale benchmarks examples from mechanical engineering.

References

[1] A C. Antoulas, S. Lefteriu and A C. Ionita, A tutorial introduction to the Loewner framework for model reduction, In Model Reduction and Approximation, Chapter 8, pp. 335–376, (2017).

[2] A. C. Antoulas, I. V. Gosea and A. C. Ionita, Model reduction of bilinear systems in the Loewner framework, SIAM J. Sci. Comput., 38(5), pp. B889–B916, (2016).

[3] I. V. Gosea and A. C. Antoulas, Data-driven model order reduction of quadratic-bilinear systems, Numer. Linear Algebra Appl., 25(6), (2018).

DRI Group, Max Planck Institute for Dynamics of Complex Technical Systems, Sandtorstrasse 1, 39106 Magdeburg, Germany, gosea@mpi-magdeburg.mpg.de
ECE Department, Rice University, 6100 Main St, Houston, TX 77005, USA, aca@rice.edu

Automatic Generation of Minimal and Reduced Models for Structured and Nonlinear Parametric Dynamical Systems

PAWAN GOYAL, Peter Benner, Igor Duff Pontes

In this talk, we discuss a novel model order reduction framework for structured and nonlinear parametric dynamical systems with inputs and outputs. Inputs are often control functions, and outputs model measurements or derived quantities. Such dynamical systems include e.g., second-order systems, delay systems, polynomial systems, which may also have parameter dependencies. Our main goal is to construct reduced-order models for the considered systems to reduce numerical complexity, thus easing engineering studies such as optimization and control design. To that aim, we first characterize reachability and observability for such dynamical systems. Thus, we can construct the numerical minimal realization by Petrov-Galerkin projection. Furthermore, we extend the connection between interpolation-based and Loewner approaches, known for standard linear systems, to structured and nonlinear systems, thus allowing us to construct reduced-order models in a totally automatic way. Also, special attention is paid to computational aspects of the approaches, and we discuss their applicability to large-scale problems. We illustrate the efficiency of the proposed approaches with several numerical benchmark examples.

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTRASSE 1
MAGDEBURG GERMANY

E-mail address: `goyal@mpi-magdeburg.mpg.de`

Jointly Optimal Modeling in Frequency and Parameter via Kernel-based Approximation

ALEXANDER GRIMM, GABRIELE SANTIN

We consider the model reduction problem for parametric linear dynamical systems via its input-output mapping in the frequency domain $\hat{y}(\imath\omega, \mathbf{p}) = \mathcal{H}(\imath\omega, \mathbf{p})\hat{u}(\imath\omega)$. Here $\hat{u}(\imath\omega)$ and $\hat{y}(\imath\omega, \mathbf{p})$ denote, respectively, Fourier transforms (with respect to the time variable) of the input and the output quantity of interest; $\mathbf{p} \in \mathbb{C}^{n_p}$ is a parameter vector, n_p the parameter dimension and $\mathcal{H}(s, \mathbf{p})$ is the transfer function. We assume that $\mathcal{H}(s, \mathbf{p})$ is analytic in $(s, \mathbf{p}) \in \{\mathrm{Re}(s) > 0\} \times \mathbb{D}^{n_p}$, the complex polydisc.
We consider the following norm in frequency and parameter space:

$$\|\mathcal{H}\|_{\sharp} := \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{0}^{2\pi} \cdots \int_{0}^{2\pi} \left|\mathcal{H}(i\omega; e^{i\theta_1}, \dots, e^{i\theta_{n_p}})\right|^2 d\theta_1 \cdots d\theta_{n_p} d\omega. \quad (1)$$

Data driven model reduction is a well known tool for parametric systems [4,5], often considering lower dimensional parameter spaces to derive optimality conditions, for example[3]. Our implementation focuses on higher dimensional parameter spaces, utilizing the reproducing kernel(s) of the underlying Hardy space(s):

$$k_C(s,t) := \frac{1}{s + \bar{t}}, \quad k_D(s,t) := \frac{1}{1 - s\bar{t}}, \quad s, t \in \mathbb{C}, \quad (2)$$

where $k_C(\cdot,\cdot)$ corresponds to the time variable and $k_D(\cdot,\cdot)$ to the parameter variables. Note that our approach does not require access to internal dynamics of the transfer function, merely function evaluations of $\mathcal{H}(s; \mathbf{p})$ are required.
In [3], optimal approximations for parametric systems with respect to a similar norm (1) are characterized via a particular selection of interpolation points. We extend the results for several parameters and use the reproducing kernel methods [1,2] for the selection of the interpolation points.
The reduced model is chosen to be of a tensor-like structure:

$$\widetilde{\mathcal{H}}(s, \mathbf{p}) := \sum_{i=1}^{r_s} \sum_{\mathbf{j}=1}^{\mathbf{r_p}} \frac{\phi_{i,\mathbf{j}}}{(s - \lambda_i)(\mathbf{p} - \pi_{\mathbf{j}})}, \quad (3)$$

with separable poles in s and $\mathbf{p}$, for each dimension of $\mathbf{p}$ denoted with the multi-index $\mathbf{j}$. We highlight the approximation quality of the form (3) on several examples.

REFERENCES

[1] B. Haasdonk and G. Santin, *Greedy Kernel Approximation for Sparse Surrogate Modeling*, Reduced-Order Modeling (ROM) for Simulation and Optimization. Springer (2018).

[2] D. Wittwar and G. Santin and B. Haasdonk, *Interpolation with uncoupled separable matrix-valued kernels*, arXiv:1807.09111v1 (2018).

[3] A. Grimm, *Parametric Dynamical Systems: Transient Analysis and Data Driven Modeling*, Dissertation, Virginia Polytechnic Institute and State University (2018).

[4] P. Benner, S. Gugercin, K. Willcox, *A survey of projection-based model reduction methods for parametric dynamical systems*, SIAM review (2015).

[5] U. Baur, C. Beattie, P. Benner and S. Gugercin, *Interpolatory Projection Methods for Parameterized Model Reduction*, SIAM J. Sci. Comput. vol. 33, no. 3 pp 2489-2518 (2011).

IANS, STUTTGART UNIVERSITY
ALLMANDRING 5B
70563 STUTTGART GERMANY

E-mail address: grimmar@mathematik.uni-stuttgart.de

Data-driven modeling of dynamical systems

SERKAN GUGERCIN

Projection-based methods are a common approach to model reduction in which reduced-order quantities are obtained via explicit use of full-order quantities. However, these full-order quantities are not always accessible and instead a large-set of input/output data, e.g., in the form of transfer function evaluations, are available. In this talk, we will discuss how one can employ this available data to construct high-fidelity (in some cases optimal) data-driven dynamical models. We will investigate both non-parametric and parametric systems, and present applications to estimating dispersion curves in structural materials. Interpolation and least-squares based modeling will form the basics of the underlying theory.

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
BLACKSBURG, VA 24061, USA

E-mail address: `gugercin@vt.edu`

Kernel-based Surrogate Modelling for Dynamical Systems

Bernard Haasdonk

Data-based approaches are gaining increasing attention for generating or improving simulation models in CSE. Application settings comprise modelling from data, i.e. measurements are given, and we aim to find a model, that can be used for simulation, or approximative surrogate modelling, where a model is given and a cheap surrogate model is constructed based on simulation data of the former.

In this presentation I focus on kernel methods for generating such models. These powerful techniques have proven to be successful in various applications in data-science such as pattern recognition, machine learning, bioinformatics, etc. In addition to relevant applicability, they also enable elegant mathematical analysis in so called reproducing kernel Hilbert spaces (RKHS).

In the context of dynamical systems, kernel methods can be used for sparse vectorial function approximation, for example by vectorial support vector regression or the vectorial kernel orthogonal greedy algorithm (VKOGA). For the VKOGA theoretical analysis can be given in terms of local optimality and convergence rates [2,3]. Extensions including regularization can cope with noise in the observed target values. The resulting approximants allow efficient computational acceleration in multiscale problems [4]. Also they enable higher-level simulation tasks such as efficient sampling for uncertainty quantification of stochastic PDEs, where they outperform other state-of-the art methods [5]. A recent application demonstrates the use for inverse problems by parameter and state estimation [6].

References

[1] Wirtz, D. & Haasdonk, B.: Efficient a-posteriori error estimation for nonlinear kernel-based reduced systems, Systems and Control Letters, 2012, 61, 203 - 211.

[2] Wirtz, D. & Haasdonk, B.: An Improved Vectorial Kernel Orthogonal Greedy Algorithm,

Dolomites Research Notes on Approximation, 2013, 6, 83-100.

[3] Santin, G.; Haasdonk, B.: Convergence rate of the data-independent P-greedy algorithm in

kernel-based approximation. Dolomites Res. Notes Approx., Vol. 10, pp. 68.78, 2017.

[4] Wirtz, D.; Karajan, N. & Haasdonk, B.: Surrogate Modelling of multiscale models using kernel methods, International Journal of Numerical Methods in Engineering, 2015, 101, 1-28.

[5] Köppel, M.; Franzelin, F.; Kröker, I.; Oladyshkin, S.; Santin, G.; Wittwar, D.; Barth, A.; Haasdonk, B.; Nowak, W.; Pflšuger, D.; Rohde, C.: Comparison of data-driven uncertainty quantification methods for a carbon dioxide storage benchmark scenario. Computational Geosciences, Vol. 23, No. 2, pp. 339.354, 2019.

[6] Köppl, T.; Santin, G.; Haasdonk, B.; Helmig, R.: Numerical modelling of a peripheral arterial stenosis using dimensionally reduced models and kernel methods. International Journal for Numerical Methods in Biomedical Engineering, Vol. 34, No. 8, p. e3095, 2018. E3095 cnm.3095.

UNIVERSITY OF STUTTGART
INSTITUTE OF APPLIED ANALYSIS AND NUMERICAL SIMULATION
PFAFFENWALDRING 57
STUTTGART
GERMANY

E-mail address: `bernard.haasdonk@ians.uni-stuttgart.de`

Empirical Dominant Subspaces

Christian Himpe

Balanced truncation is the gold standard for system-theoretic projection-based model reduction methods, because it preserves stability of the reduced order model and provides a bound for the model reduction error. The associated truncated balancing transformations result from equilibrating controllability and observability subspaces of the full order model, and discarding the least controllable, and at the same time least observable, subspaces.

Twenty years ago, T. Penzl proposed a related, but more straight-forward approach: the *dominant subspace projection model reduction* (DSPMR) method [1]. Instead of balancing controllability and observability, the dominant controllable and observable subspaces are just conjoined. We revisit this dominant subspace idea from a cross Gramian point of view, summarize conditions for stability preservation and derive an error indicator [2].

Curiously, the computation of the dominant subspaces using the cross Gramian matrix, can be accomplished in low-rank, as well as error-driven, and formulated as a single *hierarchical approximate proper orthogonal decomposition* (HAPOD) [3]. Furthermore, due to relation of balanced truncation and dominant subspaces methods, generalizations, such as empirical balanced truncation, directly extend to empirical dominant subspaces.

We numerically demonstrate this *empirical dominant subspace* method combining the DSPMR, with the empirical cross Gramian matrix [4] and the HAPOD for linear and nonlinear input-output systems.

References

[1] T. Penzl. Algorithms for Model Reduction of Large Dynamical Systems. Linear Algebra and its Application, 415(2–3): 322–343, 2006. (Reprint of SFB393/99-40, TU Chemnitz, 1999.)

[2] P. Benner, C. Himpe. Cross-Gramian-Based Dominant Subspaces. arXiv, math.OC: 1809.08066, 2018.

[3] C. Himpe and T. Leibner and S. Rave. Hierarchical Approximate Proper Orthogonal Decomposition. SIAM Journal on Scientific Computing, 40(5): A3267–A3292, 2018.

[4] C. Himpe. `emgr` – The Empirical Gramian Framework. Algorithms, 11(7): 91, 2018.

Max Planck Institute for Dynamics of Complex Technical Systems Magdeburg
Sandtorstr. 1
Magdeburg
Germany

E-mail address: `himpe@mpi-magdeburg.mpg.de`

An Optimization Approach for Parametric Model Order Reduction

MANUELA HUND

We consider model order reduction of parametric linear time-invariant systems

$$\begin{aligned} E(\mu)\dot{x}(t,\mu) &= A(\mu)x(t,\mu) + B(\mu)u(t), \\ y(t,\mu) &= C(\mu)x(t,\mu). \end{aligned} \tag{1}$$

We assume (1) to be asymptotically stable and $E(\mu)$ invertible for all parameters $\mu \in M \subset \mathbb{R}^d$, where M is a given bounded set. The only assumption we make on the reduced-order model (ROM) is an affine decomposition of its matrices. In case the matrices of the full-order model (FOM) are affine decomposable, this approach preserves the given structure.

Our aim is to minimize the error of the input-output behavior between the FOM and the ROM. In order to measure this error, we will make use of Hardy norms in the frequency domain and Lebesgue norms in the parameter domain.

In the non-parametric setting, Wilson [1], e.g., used the $\mathcal{H}_2$-norm and derived necessary optimality conditions, which are given as a set of coupled matrix equations. Extending this idea to the parametric case, we also have to take the optimality with respect to the parameters into account. One possibility to handle this is using the $\mathcal{L}_2$-norm, that measures the average error of the transfer functions of the error system over the parameter domain M. For the case of $\mathcal{H}_2 \otimes \mathcal{L}_2$-optimal MOR, we have derived first-order necessary optimality conditions [2] that are given as integral matrix equations. Another possibility in this setting would be to consider the maximum error of the input-output behavior of the error system resulting in the $\mathcal{H}_2 \otimes \mathcal{L}_\infty$-norm. While the first case can be tackled using derivative-based methods, due to the non-smooth functional, numerical treatment in the second case is far more involved. In our contribution, we will present latest results in this framework.

REFERENCES

[1] D. A. Wilson. *Optimum solution of model-reduction problem*, Proceedings of the Institution of Electrical Engineers, **117** (1970), No. 6, 1161–1165.

[2] M. Hund, P. Mlinarić, and J. Saak. *An $\mathcal{H}_2 \otimes \mathcal{L}_2$-optimal model order reduction approach for parametric linear time-invariant systems.*, Proc. Appl. Math. Mech. **18** (2018), No. 1, e201800084.

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTR. 1
D-39106 MAGDEBURG
GERMANY

E-mail address: hund@mpi-magdeburg.mpg.de

Reduced Basis approaches for efficient solutions of optimal control problems

Laura Iapichino

Reduced order methods are discussed in the framework of optimal control problems involving parametric partial differential equations (PDEs).Classical numerical techniques (e.g. Finite Element method) are usually computationally expensive when solutions need to be computed in a many query context, for many different values of the parameters as it happens in the optimization problems or multiobjective optimization. The reduced basis method is proposed to achieve the accuracy and reliability of a high fidelity approximation by drastically decreasing the problem complexity and the required computational time. First, a reduced order technique is presented for the numerical solution of PDE-constrained multiobjective optimization with the aim of solving problems where several objective functions have to be simultaneously optimized. The idea is to identify a possibly infinite set of optimal solutions (Pareto points), which do not penalize the optimization of any objective function and which represent good compromises for all the individual ones. Then, a model reduction approach is proposed to control dynamical systems characterized by one or more parameters describing physical features of the problem or geometrical configurations of the computational domain. As a consequence, by assuming that the system is controllable, a range of optimal controls exists corresponding to different parameter values. The goal of the proposed approach is to avoid the computation of a control function for any instance of the parameters. The presented reduced greedy controllability approach consists in the selection of the most representative values of the parameter set that allows a rapid approximation of the control function for any desired new parameter value, ensuring that the system is steered to the target within a certain accuracy. Numerical tests demonstrate as the use of the reduced basis method in these frameworks allows handling the computational complexity and resolution times of the problems without compromising the solution accuracy.

Scientific Computing Group
Department of Mathematics and Computer Science
TU Eindhoven
Eindhoven
The Netherlands

E-mail address: `l.iapichino@tue.nl`

The Loewner and Volterra frameworks for nonlinear system analysis

D. S. Karachalios, I. V. Gosea and A. C. Antoulas

One common approach for projection-based model reduction, both for linear and non-linear dynamical systems, is by means of interpolation. Projection-based methods require access to the internal dynamics of the system which is not always available. The aim is to seek reduced order models (ROMs) whose transfer function matches that of the original system at selected interpolation points. In this work, this particular class of realization independent methods is represented by introducing a data-driven approach. The proposed method, which will be referred to as the Loewner framework, was initially introduced in [1]. It constructs state-space models from given data in a direct way. A significant attribute of the Loewner framework is that it provides a trade-off between the accuracy of fit and complexity of the reduced model. In the linear case, the Loewner framework offers a complete identification tool which constructs reduced-order models from frequency-domain data [1] as well as from time-domain data [2]. Moreover, we present results on approximating non-rational functions and constructing ROMs by compression as well as by adaptive/optimal selection. The test cases include engineering examples and problems typically encountered in approximation theory [3]. In the broader class of nonlinear systems, the Loewner framework has been extended to classes of systems with special structure such as bilinear and quadratic-bilinear systems. In this direction, the current work includes a study that incorporates in the Loewner framework the concept of interpolation of the generalized higher order transfer function (GFRFs) as they appear in the classical Volterra framework. The scope of the proposed extension is to address procedures that experimentally reveal high fidelity linear and nonlinear models from time domain measurements.

References

[1] A.J. Mayo and A.C. Antoulas, *A framework for the solution of the generalized realization problem*. Linear Algebra and its Applications, 425(2), pp. 634 – 662, (2007).

[2] B. Peherstorfer, S. Gugercin, and K. Willcox, *Data-driven reduced model construction with time-domain Loewner models*, SIAM Journal on Scientific Computing 39(5), pp. A2152–A2178, (2017).

[3] D.S. Karachalios, I. V. Gosea and A. C. Antoulas, *Data-driven approximation methods applied to non-rational functions*, PAMM, 18(1), (2018).

Data-Driven System Reduction and Identification Group, Max Planck Institute for Dynamics of Complex Technical Systems
Sandtorstrasse 1
Magdeburg
Germany
karachalios,gosea,antoulas@mpi-magdeburg.mpg.de

Krylov-Based Model Order Reduction in Vibroacoustics

HARIKRISHNAN K. SREEKUMAR[1], RUPERT ULLMANN[2], STEFAN SICKLINGER[2], SABINE C. LANGER[1]

Vibroacoustic simulations at an early stage of product development are crucial in anticipating a model's behavior in its real application. Structure-borne vibration problems can be numerically modeled with the Finite Element Method (FEM). However, the complexity of models in practical applications often leads to large systems of linear equations that demands huge computational efforts for solving. Therefore, it is advisable to perform a reduction in model order by using Model Order Reduction (MOR) techniques, thereby yielding faster computations. However, conventional reduction techniques like the Component Mode Synthesis (CMS) method faces challenges in reducing a model in the presence of localized damping. The contribution investigates another technique of efficient Krylov-based MOR (KMOR) for accurately reducing the models, which are subjected to structure-borne vibration and highly-localized damping. For vibroacoustic applications with different damping models, projection matrices yielding the second order Krylov subspaces are rather in the complex domain. Therefore, maximum moment matching with the multi-point Páde approximation is re-established for applicability to symmetric Multi-Input-Multi-Output (MIMO) system matrices belonging to complex domain with respect to non-hermitian transpose. In the context of numerical stability, the proposed block-wise algorithm implements the rank reveiling QR deflation strategy and iterative Gram-Schmidt with reorthogonalization. Hence the final goal of the contribution is to provide a suitable KMOR algorithm which is able to efficiently compute accurate reduced order models for large scale vibroacoutic problems in complex domain.

In addition, one suitable way to approach coupled system response of complex model assemblies is by evaluating the transfer functions of each FE model representing a single substructure using KMOR techniques and coupling them within a Frequency Based Substructuring (FBS) framework. The procedure thereby yields flexibility and faster computations without compromising on accuracy. Consequently, the vibration energy flowing through the structure is evaluated and compared with other conventional methods.

[1] TECHNISCHE UNIVERSITÄT BRAUNSCHWEIG, INSTITUT FÜR AKUSTIK
LANGER KAMP 8
38106 BRAUNSCHWEIG
GERMANY

E-mail address: hk.sreekumar@tu-braunschweig.de

[2] BMW GROUP RESEARCH, NEW TECHNOLOGIES, INNOVATIONS
PARKRING 19
85748 GARCHING
GERMANY

Reduced basis methods for parabolic problems and applications to permanent magnet synchronous machines

DENIS KOROLEV

A crucial task in the design of electric motors is the creation of proper magnetic circuits. In permanent magnet synchronous motors, the latter is created by electromagnets with alternating current and permanent magnets. The corresponding mathematical model governs a quasilinear parabolic PDE (eddy current problem) which describes the evolution of the magnetic field. One of the engineering design goals consists in improving the performance of the motor through modifying the size and/or location of the permanent magnets. This problem can be viewed as a parameter optimization problem, where the parameters determine the geometry of the computational domain. The underlying optimization problem then requires multiple solutions of the parabolic problem on the parametrized domain. Therefore, there is an increasing demand for the efficient reduced models as surrogates in the optimization problem. The talk discusses the permanent magnets optimal design setting and the application of reduced basis techniques to the arising parametrized eddy current problem.

KOBLENZ-LANDAU UNIVERSITY
UNIVERSITATSSTRASSE 1
KOBLENZ
GERMANY

E-mail address: korolev@uni-koblenz.de

Machine Learning and Data-Driven Methods for Model Reduction

J. NATHAN KUTZ

A major challenge in the study of spatio-temporal dynamical systems is that of model reduction and low-rank model discovery: turning data into models that are not just predictive, but provide insight into the nature of the underlying dynamical system that generated the data. This problem is made more difficult by the fact that many systems of interest exhibit parametric dependencies and diverse behaviors across multiple time scales. We introduce a number of data-driven strategies for discovering nonlinear dynamical systems, their coordinates and their control laws from data. We consider two canonical cases: (i) systems for which we have full measurements of the governing variables, and (ii) systems for which we have incomplete measurements. For systems with full state measurements, we show that the recent sparse identification of nonlinear dynamical systems (SINDy) method can discover governing equations with relatively little data and introduce a sampling method that allows SINDy to scale efficiently to problems with multiple time scales and parametric dependencies. We can also regress to data-driven control laws that are capable of learning how to control a given system. Together, our approaches provide a suite of mathematical strategies for reducing the data required to discover, model and control nonlinear systems.

DEPARTMENT OF APPLIED MATHEMATICS
UNIVERSITY OF WASHINGTON
SEATTLE, WA 90195-3925

E-mail address: `kutz@uw.edu`

Reduced Order Methods in Medical Imaging

Charles H. Lee, Thomas Freeze,
Elias Salomão Helou,
Saifon Chaturantabut

Tomographic images encountered in Medicine, Biology and Petroleum engineering are often reconstructed via the inverse Radon transform. This process has traditionally been complex and computationally expensive which might be critically slow for some applications, especially for large-scale real-time 3D assessment. On the other hand, the inverse Radon transform is a linear transform and therefore is preserved under linear operations. We take advantage of this property and combine it with various Principal Orthogonal Decomposition (POD) approaches to greatly decrease the time required to convert large amounts of tomographic data to images with minimal loss. Mathematical formulations and numerical schemes for the POD method and the Radon transform will be discussed along with actual medical tomographic data.

Department of Mathematics, California State University, Fullerton, USA
Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, Brazil
Department of Mathematics and Statistics, Thammasat University, Thailand

Correspondent E-mail address: `charleshlee@fullerton.edu`

RB methods in optimal control of electromagnetic wave problems

CHRISTIAN LOCHNER

The content of the talk is the optimal control of reduced basis models of 3D electromagnetic wave problems in a closed cavity with coaxial excitation. To generate high fidelity finite element models the software tool COMSOL Multiphysics is used. To handle the special structure of FEM problems within COMSOL the reduced basis method is extended to RB-Lagrange methods and the divided systems. Because of the highly dimensional discretization of the considered 3D problems some further extensions are made for the approximation of the reduced basis error estimator [1] and its calculation. After generating the reduced models, these models are used for optimal control of a heating process with electromagnetic waves.

REFERENCES

[1] M. Heß. *Reduced Basis Approximations for Electromagnetic Applications*. PhD Thesis (2016), Otto-von-Guericke-Universität Magdeburg.

UNIVERSITÄT AUGSBURG
UNIVERSITÄTSSTR. 14
86159 AUGSBURG
GERMANY

E-mail address: Lochner.Christian@gmx.net

Model Order Reduction for Aerodynamic Shape Optimisation

WEIGANG YAO[1,*], SIMÃO MARQUES[2], LIANG SUN[1], TREVOR ROBINSON[1]

This work presents a reduced order model for gradient based aerodynamic shape optimization. The fluid Euler equations are converted to a reduced Newton iteration by using the Least Squares Petrov-Galerkin projection. The reduced order basis are extracted by Proper Orthogonal Decompostion from snapshots of the fluid state. The formulation distinguishes itself by adopting a linear sampling procedure to construct the subspace, along the optimization trajectory, which results in solving one linear system to obtain the snapshots for all design parameters; this is advantageous when optimising in high-dimensional paramter spaces. Similarly, the reduced gradient formulation is derived by projecting the full-order model state onto the subspace spanned by the reduced basis. Throughout the optimisation trajectory, the residual of the reduced Newton iterations is used as an indicator to update the snapshots and enrich the reduced order basis. Auto-Differentiation is used to evaluate the required reduced Jacobian without forming the full fluid Jacobian explicitly during the redcued Newton iteration.

The resulting multi-fidelity optimisation problem is managed by a trust-region algorithm. The ROM is demonstrated for using two-dimensional design problems, including transonic aerofoil drag minimization and progress torwards three-dimensional problems will be discussed.

[1] SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING,
QUEEN'S UNIVERSITY BELFAST
BELFAST
U.K.
[2] DEPARTMENT OF MECHANICAL ENGINEERING SCIENCES,
UNIVERSITY OF SURREY
SURREY
U.K.

E-mail address: s.marques@surrey.ac.uk

Clustering-Based Model Order Reduction for Nonlinear Network Systems

Petar Mlinarić

Nonlinear network systems appear in various application areas, including energy distribution networks, water networks, multi-robot networks, and chemical reaction networks. Model order reduction enables faster simulation, optimization, and control of large-scale network systems. However, standard methods generally do not preserve the network structure.

Preserving the network structure is necessary, e.g., if an optimization method assumes this structure. Furthermore, structure preservation could also give more accurate reduced-order models since the physical interpretation is preserved.

One approach to preserving the network structure proposed in the literature is clustering. For linear network systems, it was shown that clustering can be achieved by a particular Galerkin projection. We consider a general class of nonlinear network systems for which we show that such clustering preserves the structure. We discuss preservation of the synchronization property relevant for networks of oscillators.

Finally, we present a method which restores network structure in an arbitrary reduced-order model obtained by projection. We demonstrate this method on a number of examples.

Max Planck Institute for Dynamics of Complex Technical Systems
Sandtorstr. 1
D-39106 Magdeburg
Germany

E-mail address: `mlinaric@mpi-magdeburg.mpg.de`

Model order reduction for linear time delay systems based on energy functionals*

Sajad Naderi Lordejani*,[1], Bart Besselink[2], Nathan van de Wouw[3]

This abstract presents a model order reduction approach for asymptotically stable linear time delay systems with point-wise delays. The presented approach, which can be regarded as an extension of existing balanced model order reduction techniques for linear delay-free systems, is based on energy functionals that characterize observability and controllability properties of the time delay system. This type of approach was first introduced in [1], and it provides an a priori bound on the reduction error. Moreover, the resulting reduced model is an asymptotically stable time delay system with the same delay-structure as the original model. In the current work, we investigate two ways to enhance the performance of this model order reduction approach. Firstly, we explain how to improve the method in [1] by making the corresponding model order reduction criteria delay-dependent. This extension has advantages such as enabling more accurate model approximations and smaller error bounds for small delays. Secondly, inspired by the stability analysis of time delay systems using descriptor transformations [2], we introduce an extended model order reduction technique for time delay systems. This extension is beneficial when the preservation of physical interconnection structures or uncertainties is desired. Numerical simulation case studies illustrate these advantages of the presented model order reduction approaches.

References

[1] B. Besselink, A. Chaillet, and N. van de Wouw, *Model reduction for linear delay systems using a delay-independent balanced truncation approach*, Proceedings of the 56th Annual Conference on Decision and Control (CDC), Melbourne, Australia, 3793-3798, Dec 2017.

[2] E. Fridman, *Introduction to time-delay systems: analysis and control*, Birkhauser Boston, 2014.

[1,3] Mechanical Engineering Department, Eindhoven University of Technology, Eindhoven, The Netherlands. [2] Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, The Netherlands. [3]Department of Civil, Environmental & Geo-Engineering, University of Minnesota, Minneapolis, USA

E-mail address: s.naderilordejani@tue.nl

*This research has been carried out in the HYDRA project, which has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement No 675731.

Balanced truncation for parametric linear systems using interpolation of gramians: a comparison of linear algebraic and geometric approaches

NGUYEN THANH SON[1,*], PIERRE-YVES GOUSENBOURGER[1], ESTELLE MASSART[1], TATJANA STYKEL[2], P.-A. ABSIL[1]

In balanced truncation model order reduction, one has to solve a pair of Lyapunov equations for the two gramians and uses them for constructing a reduced-order model. Although advances in solving such equations have been made, it is still the most expensive step in this reduction method. For systems that depend on parameters, parametric model order reduction has to deal with the dependence on parameters simultaneously with approximation of the input-output behavior of the full-order system. The use of interpolation in parametric model order reduction has become popular. Nevertheless, interpolation of gramians is rarely mentioned, most probably due to the restriction to symmetric positive semi-definite matrices. In this talk, we will present two approaches for interpolating these structured matrices which are based on linear algebra and a recently developed Riemannian geometry. The result is then utilized in constructing parametric reduced-order systems. Their numerical performances are compared on different models.

[1]INMA, ICTEAM, UNIVERSITÉ CATHOLIQUE DE LOUVAIN
AVENUE GEORGES LEMAÎTRE 4-6/L4.05.01
1348 LOUVAIN-LA-NEUVE
BELGIUM
[2]INSTITUT FÜR MATHEMATIK, UNIVERSITÄT AUGSBURG
UNIVERSITÄTSSTR. 4
86179 AUGSBURG
GERMANY

*E-mail address**: thanh.son.nguyen@uclouvain.be

The projected Newton-Kleinman method for the algebraic Riccati equation

DAVIDE PALITTA

The numerical solution of the algebraic Riccati matrix equation

$$AX + XA^T - XBB^TX + C^TC = 0, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{p\times n}, p + m \ll n$, is an interesting and still challenging task especially when the problem dimension is very large, say $n > 10^4$, as the dense solution X cannot be store and a memory-saving approximation has to be sought. In general, equation (1) has many solutions but, under certain conditions on the coefficient matrices A, B and C, there exists a unique stabilizing solution, that is all the eigenvalues of the matrix $A - XBB^T$ have negative real part. One of the most classical iterative methods for the computation of such an X is the so-called Newton-Kleinman method that, given a stabilizing initial guess X_0 , computes an approximation X_{k+1} to X by sequentially solving Lyapunov equations of the form

$$(A - X_kBB^T)X_{k+1} + X_{k+1}(A - X_kBB^T)^T + C^TC + X_kBB^TX_k = 0. \tag{2}$$

Due to the problem dimension, equations (2) must be iteratively solved, leading to the inexact Newton-Kleinman method which provides a low-rank approximation to X. Other very efficient methods for the solution of (1) have been developed in the last years. In particular, it has been shown that projection methods are very effective in the numerical treatment of (1) even though they straightforwardly generalize the approach used for linear matrix equations like (2). However, to the best of our knowledge, it is not guaranteed that the solution computed by projection methods is the stabilizing solution. In this talk we present a novel approach that combines the inexact Newton-Kleinman scheme with projection methods for Lyapunov equations. In particular, we show that all the iterates X_{k+1} in (2) belong to the same space used to compute X_1, so that only one approximation space has to be constructed. This leads to remarkable reductions in the computational efforts and the cost per iteration of the resulting scheme is comparable to the one of the pure projection procedures. Moreover, the well-established convergence properties of the inexact Newton-Kleinman method are preserved. Several numerical results are reported to illustrate the potential of the discussed method.

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTR. 1, 39106
MAGDEBURG
GERMANY

E-mail address: palitta@mpi-magdeburg.mpg.de

Balanced truncation for linear system with quadratic output: theory, error bounds and numerics

IGOR PONTES DUFF, Pawan Goyal, Peter Benner

Balanced truncation is one of the most common model order reduction techniques. This method mainly relies on reachability and observability energy functionals. For linear systems, these functionals are encoded by the reachability and observability Gramians [3]. In this talk, we propose an extension of balanced truncation for model reduction of linear systems with quadratic output, whose dynamics are governed by

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t),\ \mathbf{x}(0) = 0, \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{x}(t)^T\mathbf{M}\mathbf{x}(t), \end{aligned}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$, $\mathbf{A}, \mathbf{M} \in \mathbb{R}^{n\times n}$, $\mathbf{B} \in \mathbb{R}^{n\times m}$ and $\mathbf{C} \in \mathbb{R}^{p\times n}$. Hence, this system represents a nonlinear input-output mapping. This class of systems has a variety of applications in science and engineering, e.g., linear-quadratic optimal control, random vibration problems, and stock market models. By following [2], we propose new algebraic Gramians for these systems based on Hilbert space adjoint theory [3]. We then show that they satisfy a certain type of generalized Lyapunov equations and investigate their connection to energy functionals. This allows us to find those states that are hard to control and hard to observe via an appropriate balancing transformation. Truncating such states yields reduced-order systems. Finally, based on $\mathcal{H}_2$ energy considerations, we are able to derive a priori error bounds depending on the neglected singular values. The efficiency of the proposed method is demonstrated by means of various semi-discretized nonlinear partial differential equations and compared with the other model reduction techniques from the literature.

REFERENCES

[1] P. Benner and P. Goyal. Balanced truncation model order reduction for quadratic-bilinear control systems. arXiv preprint arXiv:1705.00160, 2017.

[2] K. Fujimoto, J. M. A. Scherpen, and W. S. Gray. Hamiltonian realizations of nonlinear adjoint operators. Automatica, 38(10):1769–1775, 2002.

[3] B. Moore. Principal component analysis in linear systems: Controllability, observability, and model reduction. IEEE transactions on automatic control, 26(1):17–32, 1981.

[4] R. Pulch and A. Narayan. Balanced truncation for model order reduction of linear dynamical systems with quadratic outputs. arXiv preprint arXiv:1709.06677, 2017.

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTR. 1, 39106
MAGDEBURG
GERMANY

E-mail address: `pontes@mpi-magdeburg.mpg.de`

Model reduction of aeroelastic systems in the Loewner framework

David Quero

Model order reduction (MOR) techniques have found many applications in the field of aeroelasticity, which considers the interaction between the motion of a structure and the aerodynamic flow around it. After spatial discretization of the governing nonlinear partial differential equations (PDEs) together with appropriate boundary conditions a set of nonlinear ordinary differential equations (ODEs) containing typically millions of degrees of freedom is obtained.

The specific models available to solve the set of nonlinear ODEs describing the unsteady aerodynamic flow are typically formulated in the frequency domain. The Loewner framework is applied here in order to obtain a complete reduced aeroelastic model for a transport aircraft flying in the transonic regime. The model includes the effect of control surfaces in order to alleviate the loads experienced by the airframe when encountering atmospheric disturbances. A residualization of the non-proper part of the aerodynamic transfer function results in the appearance of the time derivative of the control surface command. The minimal order model size produced by the Loewner approach allows for the design of an optimal controller, for which a specific method that explicitly considers the time derivative of the control signal has been developed. Applications showing the combination of the Loewner framework together with the optimal design controller for loads alleviation are presented.

In order to analyze the applicability of recent extensions of the Loewner framework for nonlinear systems to weak nonlinear aerodynamic phenomena, the quasi one-dimensional Euler equations describing the unsteady compressible flow in a nozzle are considered next. Due to the specific formulation of the boundary conditions the system cannot be formulated as having a bilinear dependency on the input. The analytical expression for the second order Volterra kernel is derived, showing that the functional series expansion of second order is able to capture weak nonlinear behaviour exhibited by the unsteady compressible flow, motivating the application of the Loewner framework for the description of weak nonlinear aerodynamic phenomena.

DLR (German Aerospace Center), Institute of Aeroelasticity
Bunsenstrasse 10
Göttingen
Germany

E-mail address: David.QueroMartin@dlr.de

Structure Preserving and Realization Independent H-infinity Approximation

PAUL SCHWERDTNER

We present a greedy interpolation algorithm to address the H-infinity approximation problem. Using interpolation allows us to either preserve the given model structure or compute surrogate models with a realization having a structure defined by the user to approximate the given model. Starting from an initial reduced order model, we compute the H-infinity norm of the error between the given model and the reduced order model to place a new interpolation point where this H-infinity norm is attained. In this way, a sequence of reduced order models that minimize the distance to the given model with respect to the H-infinity norm is constructed.

The computational bottleneck for such an approach is the repeated computation of the H-infinity norm of the large scale error model. For this, we adapt recently developed interpolation based algorithms [1,2] for the computation of the H-infinity norm to perform these repeated computations efficiently.

The method is applicable to all linear dynamical systems with rational and irrational transfer functions, integrates well with other model order reduction techniques, and the free choice between structure preservation and realization independence allows to construct models that provide high-fidelity simulation results or can be used with general purpose controller design algorithms.

REFERENCES

[1] N. Aliyev, P. Benner, E. Mengi, P. Schwerdtner, and M. Voigt. Large-Scale Computation of L-Infinity-norms by a Greedy Subspace Method. SIAM Journal on Matrix Analysis and Applications, 38(4):1496–1516, 2017.

[2] P. Schwerdtner and M. Voigt. Computation of the L-Infinity-norm Using Rational Interpolation. IFAC-PapersOnLine, 51(25):84–89, January 2018.

TECHNICAL UNIVERSITY OF BERLIN
STRASSE DES 17. JUNI 135
BERLIN
GERMANY

E-mail address: schwerdt@math.tu-berlin.de

Passivity-preserving model reduction of nonlinear magneto-quasistatic field problems

JOHANNA KERLER-BACK, TATJANA STYKEL

We consider nonlinear magneto-quasistatic (MQS) field equations which arise in simulation of low-frequency electromagnetic devices coupled to electrical circuits. A finite element discretization of such equations on 3D domains leads to a singular system of differential-algebraic equations (DAEs). First, we study the structural properties of this system and present a new regularization approach based on projecting out the singular state components. Furthermore, we investigate the passivity of the variational MQS problem and semidiscretized system by defining appropriate storage functions. For model reduction of the nonlinear MQS system, we use the proper orthogonal decomposition (POD) method combined with the discrete empirical interpolation technique (DEIM) for fast evaluation of the nonlinearity. Our model reduction approach is based on transforming the regularized DAE into a system of ordinary differential equations (ODEs) by exploiting a special block structure of the underlying problem and applying standard model reduction methods to the resulting ODE system. For the POD reduced model, we prove the preservation of passivity, while for the POD-DEIM reduced model, we present a passivity enforcement method based on a perturbation of the output which depends on DEIM errors. Numerical experiments demonstrate the performance of the presented model reduction methods and the passivity enforcement technique.

INSTITUT FÜR MATHEMATIK
UNIVERSITÄT AUGSBURG
UNIVERSITÄTSSTRASSE 14
86159 AUGSBURG
GERMANY

E-mail address: stykel@math.uni-augsburg.de

Sampling-free parametric model reduction of structured systems

CHRISTOPHER BEATTIE[1], SERKAN GUGERCIN[1], AND ZORAN TOMLJANOVIĆ[2]

We consider a parametric linear time invariant dynamical systems represented as

$$\begin{aligned} E\dot{x}(t) &= A(p)x(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned}$$

where $E, A(p) \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{l\times n}$. Here $x(t) \in \mathbb{R}^n$ denotes the state variable, while $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^l$ represent, respectively, the inputs and outputs of the system. Transfer function of the full-order dynamical system is given by $H(s;p) = C(sE - A(p))^{-1}B$.

We assume that the matrix matrix $A(p)$ depends on $k \ll n$ parameters $p = (p_1, p_2, \ldots, p_k)$ such that

$$A(p) = A_0 + U\,\mathrm{diag}(p_1, p_2, \ldots, p_k)V^T,$$

where $U, V \in \mathbb{R}^{n\times k}$ are given fixed matrices. Exploiting on this structure, we propose an approach for approximating the full-order transfer function $H(s;p)$ with a reduced-order version that retains the structure of parametric dependence and (typically) offers uniformly high fidelity across the full parameter range. Remarkably, the proposed reduction process removes the need for parameter sampling and thus does not depend on identifying particular parameter values of interest. In our approach the Sherman-Morrison-Woodbury formula allows us to construct a parameterized reduced order model from transfer functions of four subsystems that do not depend on parameters. In this form one can apply well-established model reduction techniques for non-parametric systems. The overall process is well suited for computationally efficient parameter optimization and the study of important system properties.

One of the main applications of our approach is for damping optimization: we consider a vibrational system described by

$$\begin{aligned} M\ddot{q}(t) + (C_{int} + C_{ext})\dot{q}(t) + Kq(t) &= Ew(t), \\ z(t) &= Hq(t), \end{aligned}$$

where the mass matrix, M, and stiffness matrix, K, are real, symmetric positive-definite matrices of order n. Here, $q(t)$ is a vector of displacements and rotations, while $w(t)$ and $z(t)$ represent, respectively, the inputs (typically viewed as potentially disruptive) and outputs of the system. Damping in the structure is modeled as viscous damping determined by $C_{int} + C_{ext}$ where C_{int} and C_{ext} represent contributions from internal and external damping, respectively. Information regarding damper geometry and positioning as well as the corresponding damping viscosities are encoded in $C_{ext} = U\mathrm{diag}(p_1, p_2, \ldots, p_k)U^T$ where $U \in \mathbb{R}^{n\times k}$ determines the placement and geometry of the external dampers.

The main problem is to determine the best damping matrix that is able to minimize influence of the disturbances, w, on the output of the system z. One can consider different optimality measures. In the input-output dynamical systems settings, the optimization criteria are usually based on system norms such as $\mathcal{H}_2$ or $\mathcal{H}_\infty$ system norm.

We propose a new algorithms for optimization of system parameters that is based on efficient calculation of relative error in the objective function. This approach offers a new tool with significant advantages for the efficient optimization of damping in such problems.

[1]DEPARTMENT OF MATHEMATICS, VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY
225 STANGER ST.
BLACKSBURG
USA
[2]DEPARTMENT OF MATHEMATICS, UNIVERSITY OF OSIJEK
TRG LJ. GAJA 6
OSIJEK
CROATIA

E-mail address: [1]{beattie, gugercin}@vt.edu, [2]ztomljan@mathos.hr

Frequency- and Time-Limited Balanced Truncation for Second-Order Systems

STEFFEN W. R. WERNER

The modeling of mechanical systems often leads to linear dynamical systems containing second derivative terms

$$\begin{aligned} M\ddot{p}(t) + E\dot{p}(t) + Kp(t) &= B_u u(t), \\ y(t) &= C_p p(t) + C_v \dot{p}(t), \end{aligned} \tag{1}$$

with $M, E, K \in \mathbb{R}^{n\times n}$, $B_u \in \mathbb{R}^{n\times m}$ and $C_p, C_v \in \mathbb{R}^{q\times n}$. These systems (1) can become very large in practice and therefore, expensive to be used for simulations and controller design. A solution to this problem is provided by model reduction, which aims for an approximation of the original system (1) that is much easier to evaluate.

In applications, often a global approximation of the system is less needed than local ones, approximating the system behavior in certain frequency or time intervals. In case of first-order systems, this problem has been solved by the frequency- and time-limited balanced truncation methods [3]. We present a structure-preserving extension of the limited balanced truncation methods for second-order type systems (1), maintaining the second-order structure in the reduced-order model. Also, we provide efficient numerical implementations of the presented methods for large-scale sparse systems [2] as well as for medium-scale dense ones [1].

REFERENCES

[1] P. Benner, S. W. R. Werner, *MORLAB – Model Order Reduction LABoratory (version 4.0)*, 2018. see also: `http://www.mpi-magdeburg.mpg.de/projects/morlab`. doi:`10.5281/zenodo.1574083`

[2] P. Benner, S. W. R. Werner, *Limited balanced truncation for large-scale sparse second-order systems (version 1.0)*, 2019. doi:`10.5281/zenodo.2553926`

[3] W. Gawronski, J.-N. Juang, *Model reduction in limited time and frequency intervals*. Int. J. Syst. Sci., 21(2):349–376, 1990. doi:`10.1080/00207729008910366`

MAX PLANCK INSTITUTE FOR DYNAMICS OF COMPLEX TECHNICAL SYSTEMS
SANDTORSTR. 1
39106 MAGDEBURG
GERMANY

E-mail address: `werner@mpi-magdeburg.mpg.de`

Model Order Reduction as a Key Technology for Realizing Digital Twins

Utz Wever

An increasing number of disruptive innovations with high economic and social impact shape our digitalizing world. Simulation technologies are key enablers of digitalization, since they facilitate digital twins that mirror physical products and systems into the digital world. However, digital twins require a paradigm shift in computational engineering: Instead of expert centric tools, such as common CAx software, engineering and operation require largely autonomous digital assist systems that continuously interact with the physical environment through background simulation, optimization and control. This new type of digital engineering tools must efficiently integrate models and data from different product life cycle phases and master the resulting exploding computational complexities.

Within this talk we concentrate on technologies supporting the operation phase of a component. We point out the benefit of applying real time capable models generated with MOR-technologies instead of using pure data driven methods and address the importance of integrating these technologies into modern CAE-tools [5]. Furthermore, we discuss applications of the digital twin, where real time capable models running in parallel to operation play an important rule [6]. E.g. using MOR-technologies, online life time monitors may be realized in a very natural way [7].

In the second part of the talk we want to discuss some techniques for the reduction of nonlin- ear mechanical problems. The concept of modal derivatives [1, 3] seems to be a very powerful technique for reducing nonlinear mechanical systems, which are driven not to far from the equilibrium. Applications of this technique could be nonlinear frequency analysis [2] and hyper- reduction for structural mechanics [4, 8]. However, for the general case some knowledge of the design space in terms of snapshots seems to be beneficial. Polynomial hyper reduction based on proper orthogonal decomposition seems to be a practical alternative [9].

References

[1] Idelsohn, S. R., Cardona, A.: *A load-dependent basis for reduced nonlinear structural dynamics*, Computers and Structures, Vol. 29, No. 1, pp 203 - 210 (1985)

[2] Weeger, O., Wever, U., Simeon, B.: *Nonlinear frequency response analysis of structural vibrations* Journal of Computational Mechanics, Vol. 54, Issue 6, pp 1477-1495, DOI 10.1007/s00466-014-1070-9 (2014)

[3] Jain, S., Tiso P., Rixen, D. J., Rutzmoser, J. B.: *A Quadratic Manifold for Model Order Reduction of Nonlinear Structural Dynamics* Computers and Structures, 10.1016/j.compstruc.2017.04.005 (2016)

[4] Weeger, O., Wever, U., Simeon, B.: *On the use of modal derivatives for nonlinear model order reduction*, International Journal for Numerical Methods in Engineering, DOI: 10.1002/nme.5267 (2016)

[5] Hartmann, U., Herz, M., Wever, U. : *Model Order Reduction enabling Simulation beyond classical Applications*. KoMSO Challenge Worksho, Springer (2017)

[6] Brandtstaedter, H., Huebner, L., Jungiewicz, A., Ludwig, C., Tsouchnika, E., Wever, U. : *Digital Twins for large Electric Power Trains*. PCIC Europe Antwerpen (2018)

[7] Heinrich, C., Khalil, M., Martynov, K., Wever, U.: *Online Remaining Lifetime Estimation for Structures*. Mechanical Systems and Signal Processing, Volume 119, Pages 312-327, https://doi.org/10.1016/j.ymssp.2018.09.028 (2018)

[8] Rutzmoser, J.: *Model Order Reduction for Nonlinear Structural Dynamics*, Dissertation, Technical University Of Munich, (2018)

[9] Martynov, K., Wever, U.: *On Polynomial Hyperreduction for Nonlinear Structural Mechanics*. International Journal for Numerical Methods in Engineering, DOI:10.1002/nme.6033 (2019)

SIEMENS AG
CORPORATE TECHNOLOGY
MUNICH
GERMANY

E-mail address: `utz.wever@siemens.com`

Structure preserving model order reduction of shallow water equation

SÜLEYMAN YILDIZ

Abstract:

The two dimensional shallow water equation in Hamiltonion form [1] integrated in time by fully implicit average vector method [2] and as a PDE with quadratic polynomial nonlinearities by linear implicit two step Kahan method [3] which preserve the energy and esentrophy over long time. Based these full order models structure preserving reduced order models (ROMs) using POD, DEIM [4] and tensorial POD [5] are constructed. The ROM are compared with respect to accuracy, integral preserving properties and computational efficiency.

REFERENCES

[1] R. Salmon, *Poisson-Bracket Approach to the Construction of Energy- and Potential-Enstrophy-Conserving Algorithms for the Shallow-Water Equations*, Journal of the Atmospheric Sciences **61** (2004), no. 16, 2016-2036.

[2] D. Cohen, E. Hairer, *Linear energy-preserving integrators for Poisson systems*, BIT Numerical Mathematics **51**, (2011), no. 1, 91–101.

[3] S. Eidnes, L. Li, S. Sato , *Linearly implicit structure-preserving schemes for Hamiltonian systems*, arXiv e-prints **1901.03573**, (2019).

[4] R. Ştefănescu, A.Sandu, I.M. Navon *Comparison of POD reduced order strategies for the nonlinear 2D shallow water equation*, International Journal for Numerical Methods in Fluids **76**, (2014), no. 8, 497–521 .

[5] P. Benner, T. Breiten,*Two-Sided Projection Methods for Nonlinear Model Order Reduction*, SIAM Journal on Scientific Computing , 37 **37** (2015), no. 2 , B239-B260 .

INSTITUTE OF APPLIED MATHEMATICS, MIDDLE EAST TECHNICAL UNIVERSITY
DUMLUPINAR BULVARI
ANKARA
TURKEY

email: yildiz.suleyman@metu.edu.tr

Structure preserving model reduction of linear network systems by eigenvalue assignments

LANLIN YU XIAODONG CHENG JACQUELIEN M.A. SCHERPEN

This abstract studies structure preserving model reduction problem for linear multi-agent network systems consisting of diffusively coupled agents. The network dynamics is given by

$$\dot{x} = (I_N \otimes A - L \otimes BC)x + (F \otimes B)u, \tag{1}$$

where $F \in \mathbb{R}^{N\times p}$, whose (i,j)-th entry is the amplification of the j-th input acting on agent i, which is zero when the external input has no effect on agent i, $x := \begin{bmatrix} x_1^T, & \cdots, & x_n^T \end{bmatrix}^T \in \mathbb{R}^{Nn}$, $u := \begin{bmatrix} u_1^T, & \cdots, & u_p^T \end{bmatrix}^T \in \mathbb{R}^{pm}$, $y := \begin{bmatrix} y_1^T, & \cdots, & y_q^T \end{bmatrix}^T \in \mathbb{R}^{qm}$. $L \in \mathbb{R}^{N\times N}$ is the *Laplacian matrix* associated with an undirected weighted graph $\mathcal{G}$. (A, B, C) denotes the dynamics of each agent. Synchronization is an important property of complex network systems, which occurs when certain agreements are reached via exchanging the information among the agents. The goal is to find the following reduced model

$$\dot{x}_r = (I_r \otimes A - L_r \otimes BC)x_r + (F_r \otimes B)u, \tag{2}$$

where $L_r = (V_r^T V_r)^{-1} V_r^T L V_r,\ F_r = (V_r^T V_r)^{-1} V_r^T F$. L_r is the *Laplacian matrix* associated with a directed weighted graph. Moreover, $\lambda(L_r) = \{\lambda_1, \lambda_2, \cdots, \lambda_r\}$, $\lambda_1 = 0$, $\lambda_i \in \lambda(L)\backslash\{0\}$, $\lambda_1 < \lambda_2 \leq \cdots \leq \lambda_r$, $i \in \{2, \cdots, r\}$. We construct the projection matrix V_r by using the corresponding eigenvalue vectors of L. The resulting reduced model (2) preserves the synchronization property of the original network (1). Moreover, an explicit formula for the $\mathcal{H}_2$ approximation error is obtained. Some comparisons with Almost Equitable Partition [1] are also discussed. Compared with [2], the agent dynamics (A, B, C) in (1) is not restricted to be passive.

REFERENCES

[1] Monshizadeh, Nima, Harry L. Trentelman and M. Kanat Camlibel. *Projection-based model reduction of multi-agent systems using graph partitions*. IEEE Trans. on Control of Network Systems. **2**(2014), no. 1, 145-154.

[2] Xiaodong Cheng, Jacquelien M.A. Scherpen and Bart Besselink. *Balanced truncation of networked linear passive systems*. Automatica. **104**(2019), 17-25.

LANLIN YU AND JACQUELIEN M.A. SCHERPEN ARE WITH JAN C. WILLEMS CENTER FOR SYSTEMS AND CONTROL, ENGINEERING AND TECHNOLOGY INSTITUTE GRONINGEN, FACULTY OF SCIENCE AND ENGINEERING, UNIVERSITY OF GRONINGEN; XIAODONG CHENG IS WITH DEPARTMENT OF ELECTRICAL ENGINEERING EINDHOVEN UNIVERSITY OF TECHNOLOGY, THE NETHERLANDS.

E-mail address: {lanlin.yu, j.m.a.scherpen}@rug.nl, x.cheng@tue.nl

An Adaptive Method for Interpolating Reduced-Order Models Based on Matching and Continuation of Poles

Yao Yue, Lihong Feng, Peter Benner

In parametric analysis of large-scale state-space systems, parametric model order reduction (PMOR) has become a popular tool to drastically reduce the computational cost in the last decades. Among the many methods proposed, projection-based PMOR methods have received the most research attention. However, these methods have several disadvantages. First, they assume that a parametric full-order model is available in the state-space form, which is not always possible in industrial applications. In addition, they tend to lose efficiency when the number of parameters increases. In [1], we proposed a pole-matching PMOR method, which builds a parametric reduced-order model (pROM) by interpolating given nonparametric reduced-order models (ROMs). This method starts from several accurate given ROMs, regardless of how they are built. In [1], we showed that this method can even interpolate ROMs built by different MOR methods, e.g., one by a projection-based method and the other by a data-driven method. In this work, we introduce two new techniques, namely the branch and bound technique and the continuation technique. We show that if all poles change slightly and the nonparametric model reduction method exhibits "algorithmic continuity", the proposed branch and bound method will be very efficient in solving the pole-matching optimization problem. To favor the branch and bound method, a continuation technique is introduced: the branch and bound algorithm is conducted to match the poles of a truly generated ROM with a "predicted" ROM, rather than with another truly generated ROM for a different parameter value. The continuation method also introduces adaptivity: it accepts the new ROM only when some criterion of the pole matching is met; otherwise, a parameter value closer to p_i is chosen as a new p_{i+1} and the procedure above is done again. The proposed method can be implemented in an offline-online manner: building ROMs and conducting pole matching in the offline phase, while executing pole interpolation in the online phase.

References

[1] Y. Yue, L. Feng, P. Benner, *Interpolation of reduced-order models based on modal analysis*, 2018 IEEE MTT-S International Conference on Numerical Electromagnetic and Metaphysics Modeling and Optimization (NEMO) (2018), 1–4.

Max Planck Institute for Dynamics of Complex Technical Systems
Sandtorstrasse 1
Magdeburg
Germany

email: {yue,feng,benner}@mpi-magdeburg.mpg.de

A Digital-Twin-Building Kernel for Real-Time Computer-Aided Engineering

Rafael Rodríguez-Barrachina, Valentina Zambrano, Susana Calvo, Salvador Izquierdo

TWINKLE is a library for performing Reduced Order Modelling (ROM) via Canonical Polyadic Decomposition (CPD) on tensors. ROM strategies are based on numerical methods for multivariable problems simplification through mathematical approximation techniques. These strategies are especially needed when a reduction in terms of system's variables should be taken into consideration, e.g. big data, uncertainties in the physics behind the system, system's equations' solution not trivial and/or computationally cumbersome.
TWINKLE library is written in C++, which enhances the achievement of high computational performance. Moreover, TWINKLE allows processing and recovering highly sparse and unstructured data, which is not commonly achieved in CPD methods, such as Singular Value Decomposition (SVD), Principal Components Analysis (PCA), PARAllel FACtor analysis (PARAFAC) or CANonical DECOMPosition (CANDECOMP). The library uses Armadillo, an open source library for linear algebra and scientific computing, based on LAPACK (Linear Algebra PACKage) and BLAS (Basic Linear Algebra Subprograms) routines in turn.
We aim to present the software's basic usage, potentiality and possible extensions through some numerical examples.

Instituto Tecnológico de Aragón - ITAINNOVA
C/ María de Luna 7-8, 50018
Zaragoza
Spain

E-mail address: `sizquierdo@itainnova.es`

WE GRATEFULLY ACKNOWLEDGE THE FINANCIAL SUPPORT OF

Stadt Graz

European Research Council (ERC)

OCLOC
From Open to Closed Loop Optimal Control of PDEs
Karl-Franzens-Universität Graz

European Commission

Horizon 2020
European Union funding
for Research & Innovation

Karl-Franzens-Universität Graz

Land Steiermark