DCE-MRI tumor registration by using TV-L1 optical flow - IMPOSSIBLE?

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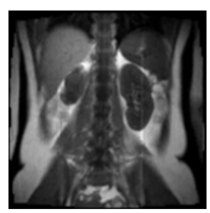
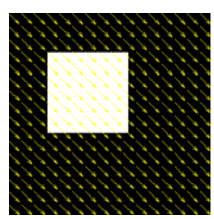




Introduction: DCE-MRI

DCE-MRI is an important method for tumor diagnosis and treatment monitoring

Objective:

Better quantification of dynamic MRI data

Problems:

1.) Physiological and unintentional motion

2.) Contrast media uptake changes the image contrast of the dynamic times series

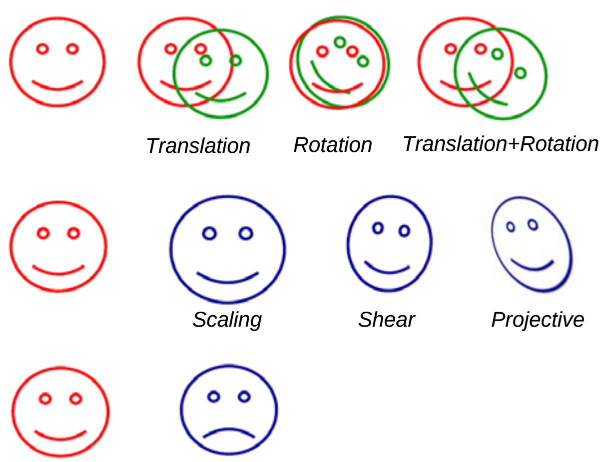
Solution: Image registration

Introduction: Registration transformation

Rigid

Affine & Projective

Non-rigid (Elastic)



Methods: Total Variation

ROF-Model: used for edge preserving image denoising

$$\min_u \left\{ \int_\Omega |\nabla u| \, d\Omega \right\} \quad s.t. \quad \int_\Omega (u-f)^2 \, d\Omega \overset{\le}{\cancel{=}} \sigma^2$$

non-convex problem! [2]
→ replacing constraint
→ dual formulation

f … is the observed image function, assumed to be corrupted by Gaussian noise of variance σ^2
u … is the unknown denoised image

$$\min_u \left\{ \int_\Omega |\nabla u| \, d\Omega + \frac{1}{2\lambda} \int_\Omega (u-f)^2 \, d\Omega \right\}$$

TV-L2 → Anisotropic diffusion

[2] Chambolle, A. and Lions, P.-L. (1997). Image recovery via total variation minimization and related problems

Methods: Optical Flow Registration

- Recovery of motion from images
- ***Main objective:*** compute a flow field (displacement field) estimating the motion in consecutive image frames

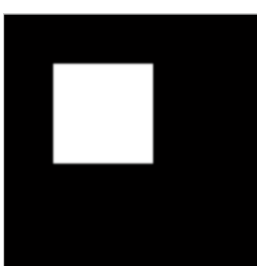


Image 1: Template image

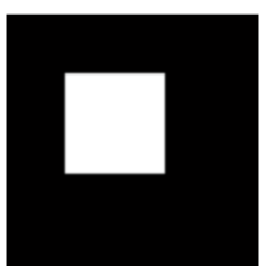


Image 2: Moved image

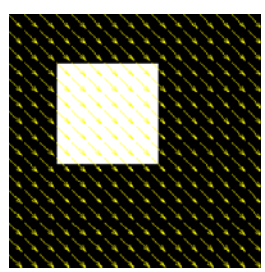


Image 3: Displacement field

Registration using TV-L1 Optical Flow

Computation of the displacement field by solving the optical flow functional:

$$\min_u \left\{ \int_\Omega \underbrace{|\nabla \boldsymbol{u}| d\Omega}_{\text{Total Variation}} + \lambda \int_\Omega \underbrace{|I_0(\boldsymbol{x}) - I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}))| d\Omega}_{\text{Optical Flow constraint}} \right\}$$

I_0, I_1 … image pair

$\boldsymbol{u}$ … two-dimensional displacement field, $\boldsymbol{u} = (u_1(x), u_2(x))^T$

Ω ... image domain

λ … weights between the regularization force and the data fidelity

Solution of the TV-L1 Optical Flow model [2]

Duality based projected gradient descend algorithm:

$$\min_u \left\{ \int_\Omega |\nabla \boldsymbol{u}| d\Omega + \lambda \int_\Omega \underbrace{|I_0(\boldsymbol{x}) - I_1(\boldsymbol{x} + \boldsymbol{u}(\boldsymbol{x}))| d\Omega}_{p(v)} \right\}$$

1. Introduction of an auxilary variable v

$$\min_{u,v} \left\{ \int_\Omega |\nabla \boldsymbol{u}| d\Omega + \frac{1}{2\Theta} \int_\Omega (u-v)^2 d\Omega + \lambda \int_\Omega |p(v)| d\Omega \right\}$$

Θ … small constant, such that v is a close approximation of u

[2] Chambolle, A. and Lions, P.-L. (1997). Image recovery via total variation minimization and related problems

Solution of the TV-L1 Optical Flow model [2]

Alternating steps updating either u or v in every iteration:

$$\min_{u,v}\left\{\int_\Omega |\nabla \boldsymbol{u}|\,d\Omega + \frac{1}{2\Theta}\int_\Omega (u-v)^2\,d\Omega + \lambda\int_\Omega |p(v)|\,d\Omega\right\}$$

2. For v being fixed, solve

$$\min_{u}\left\{\int_\Omega |\nabla \boldsymbol{u}|\,d\Omega + \frac{1}{2\Theta}\int_\Omega (u-v)^2\,d\Omega\right\}$$

3. For u being fixed, solve

$$\min_{v}\left\{\int_\Omega \frac{1}{2\Theta}(u-v)^2\,d\Omega + \lambda\int_\Omega |p(v)|\,d\Omega\right\}$$

[2] Chambolle, A. and Lions, P.-L. (1997). Image recovery via total variation minimization and related problems

DCE-MRI tumor registration by using TV-L1 optical flow

Solution of the TV-L1 Optical Flow model [2]

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2. For v being fixed, solve

$$\min_{u}\left\{\int_\Omega |\nabla \boldsymbol{u}|\,d\Omega + \frac{1}{2\Theta}\int_\Omega (u-v)^2\,d\Omega\right\}$$

ROF-Model: Projected gradient descent, Chambolle

3. For u being fixed, solve

$$\min_{v}\left\{\int_\Omega \frac{1}{2\Theta}(u-v)^2\,d\Omega + \lambda\int_\Omega |p(v)|\,d\Omega\right\}$$

Thresholding scheme

Michael Hofer

[2] Chambolle, A. and Lions, P.-L. (1997). Image recovery via total variation minimization and related problems

Solution of the TV-L1 Optical Flow model [2]

Alternating steps updating either u or v in every iteration:

$$\min_{u,v}\left\{\int_{\Omega}|\nabla \boldsymbol{u}|\,d\Omega+\frac{1}{2\Theta}\int_{\Omega}(u-v)^2\,d\Omega+\lambda\int_{\Omega}|p(v)|\,d\Omega\right\}$$

2. For v being fixed, solve

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Projected gradient descent

3. For u being fixed, solve

$$\min_{v}\left\{\int_{\Omega}\frac{1}{2\Theta}(u-v)^2\,d\Omega+\lambda\int_{\Omega}|p(v)|\,d\Omega\right\}$$

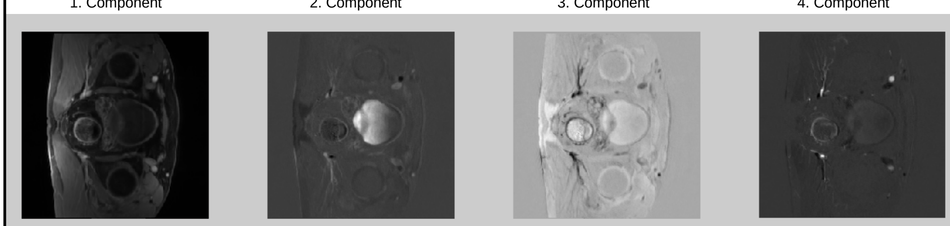
Thresholding sheme

[2] Chambolle, A. and Lions, P.-L. (1997). Image recovery via total variation minimization and related problems

Determining a template image

- Principal component analysis (PCA)
- ***First component:*** including structure information of the DCE-MRI sequence

1. Component 2. Component 3. Component 4. Component

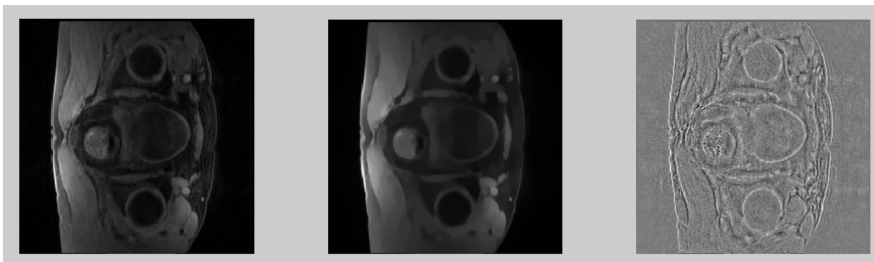


Template image: registration of each frame to the template image

Calculating structure images

- Optical Flow is intensity based → need of intensity invariant structure images
- Subtracting homogenous background information

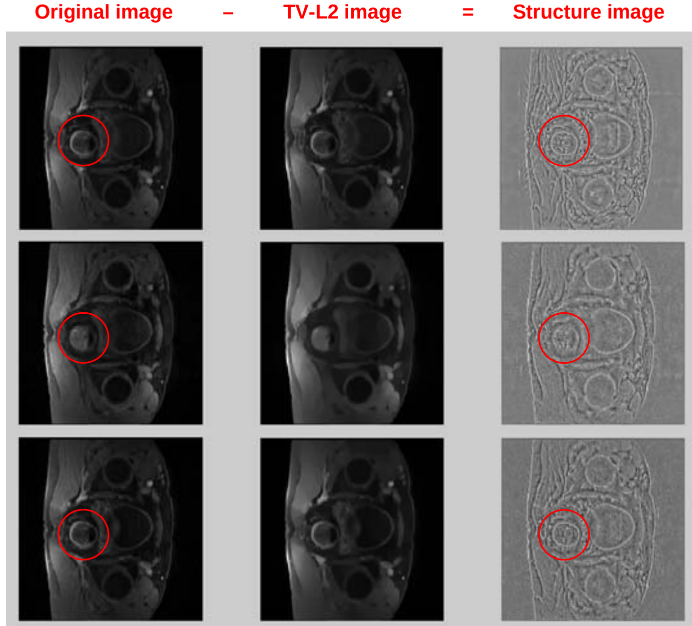
  → ***TV-L2 (Anisotropic diffusion)***



Original image – TV-L2 image = Structure image

DCE-MRI tumor registration by using TV-L1 optical flow

Original image – TV-L2 image = Structure image



Template image
1. Component (PCA)

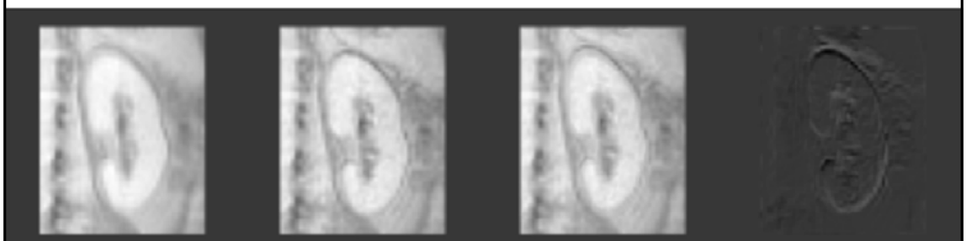
1. Frame
(DCE-MRI sequence)

25. Frame
(DCE-MRI sequence)

Results

Registration: Kidney without intensity changes

Reference Image | Original Image | Registered Image | Difference Image

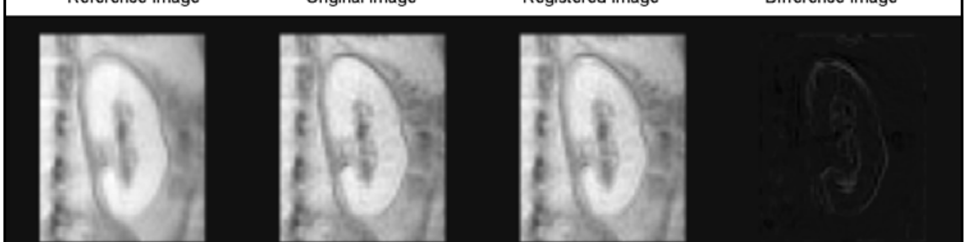


Frame 1/5

Results

Registration: Kidney without intensity changes

Reference Image | Original Image | Registered Image | Difference Image

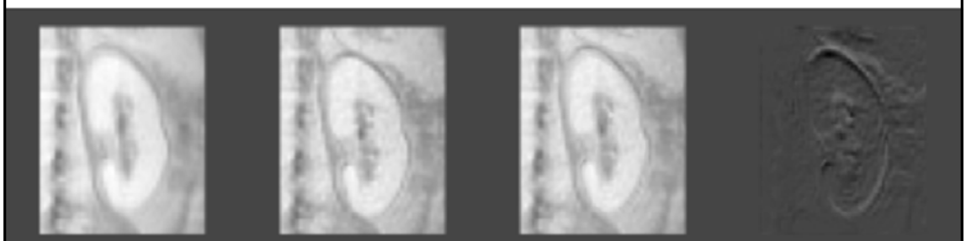


Frame 2/5

Results

Registration: Kidney without intensity changes

Reference Image | Original Image | Registered Image | Difference Image

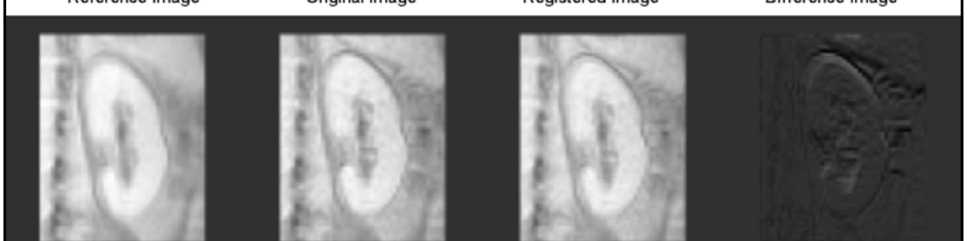


Frame 3/5

Results

Registration: Kidney without intensity changes

Reference Image | Original Image | Registered Image | Difference Image

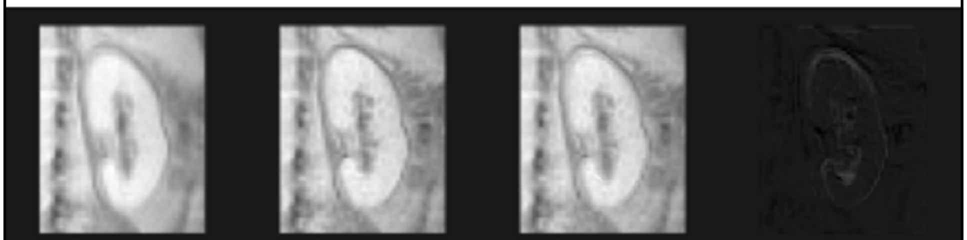


Frame 4/5

Results

Registration: Kidney without intensity changes

Reference Image | Original Image | Registered Image | Difference Image

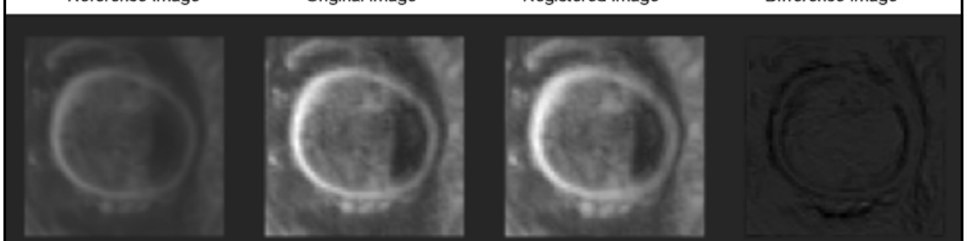


Frame 5/5

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

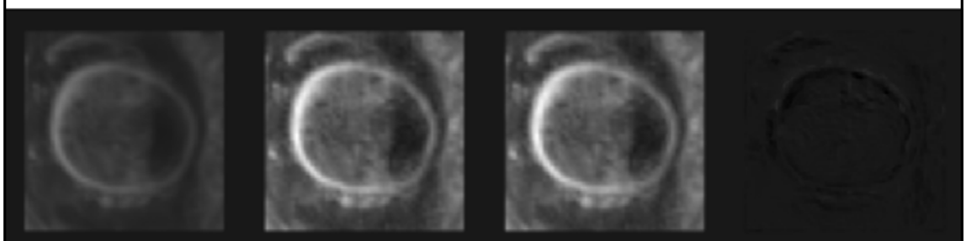


Frame 1/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

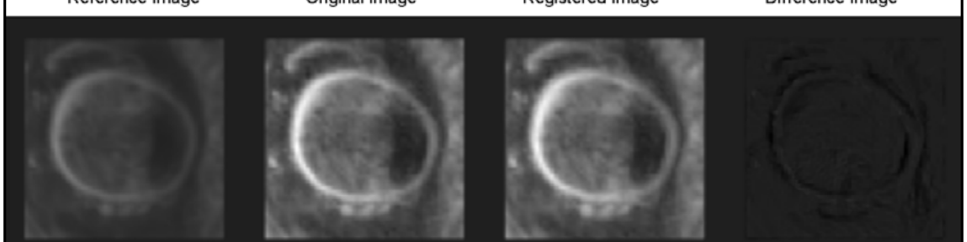


Frame 2/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

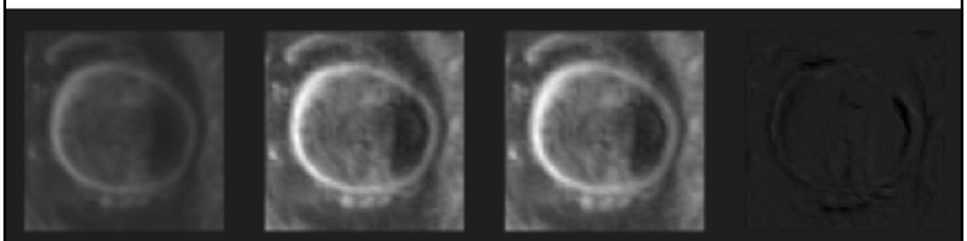


Frame 3/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image



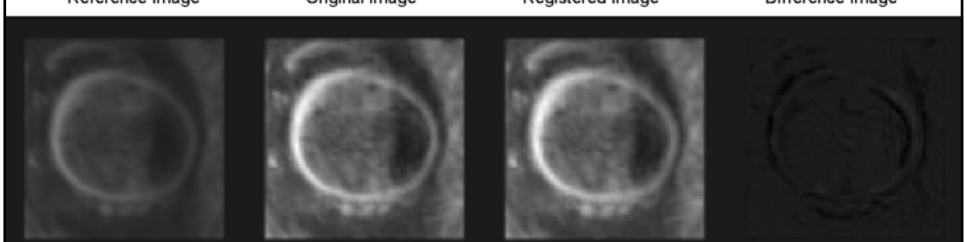
Frame 4/15

DCE-MRI tumor registration by using TV-L1 optical flow

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

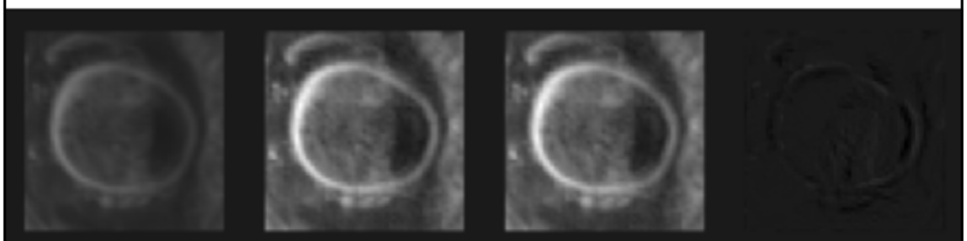


Frame 5/15

Results

Registration: Rectum tumor

Reference Image Original Image Registered Image Difference Image

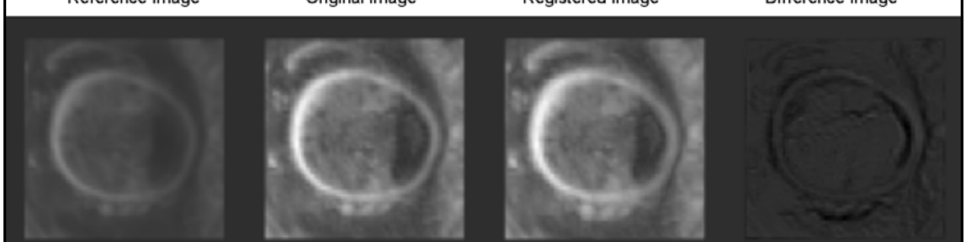


Frame 6/15

Results

Registration: Rectum tumor

Reference Image Original Image Registered Image Difference Image

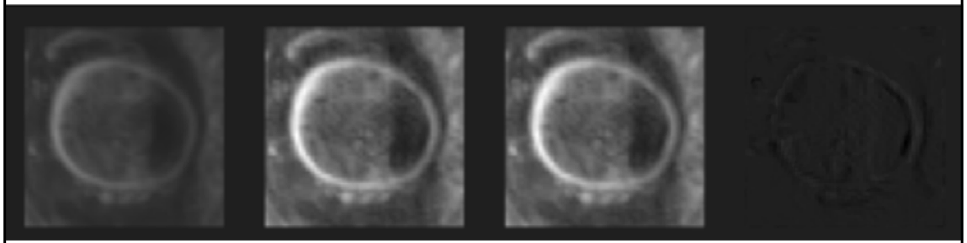


Frame 7/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

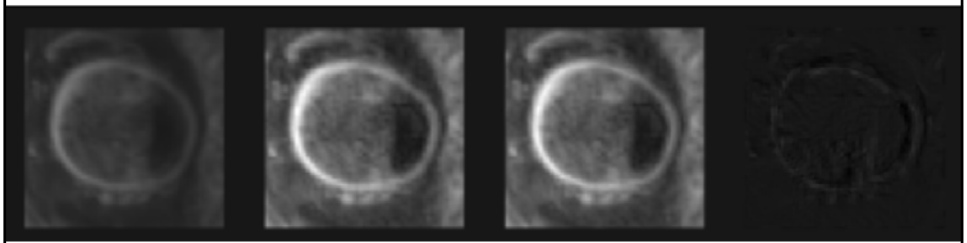


Frame 8/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

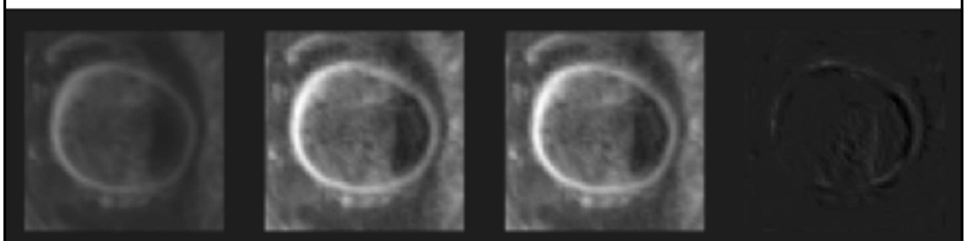


Frame 9/15

Results

Registration: Rectum tumor

Reference Image Original Image Registered Image Difference Image

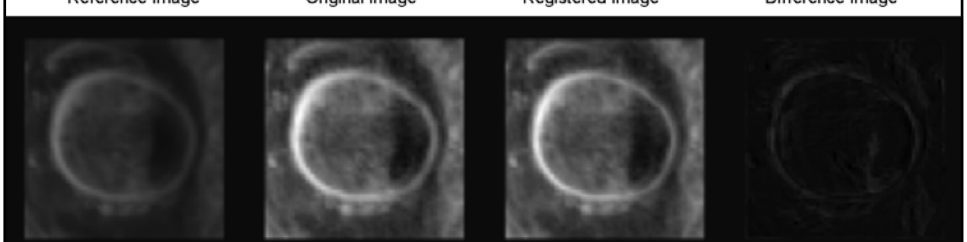


Frame 10/15

Results

Registration: Rectum tumor

Reference Image Original Image Registered Image Difference Image

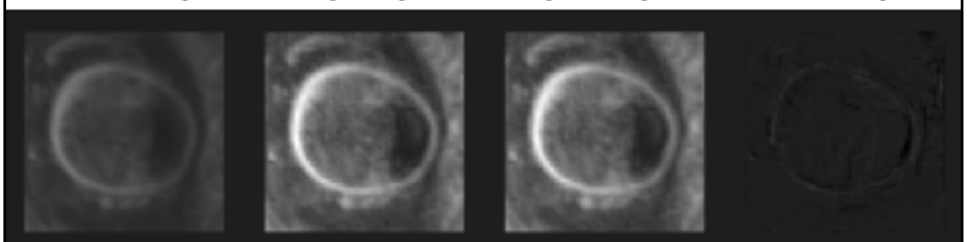


Frame 11/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

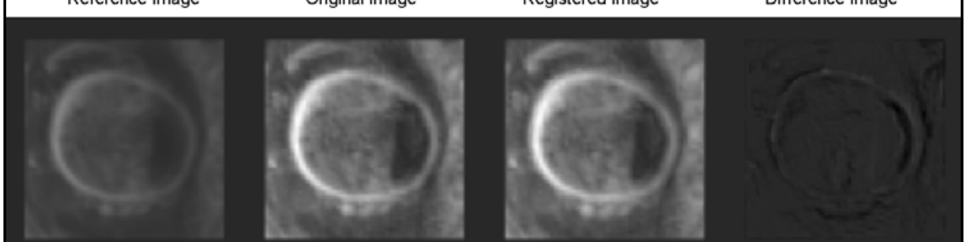


Frame 12/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

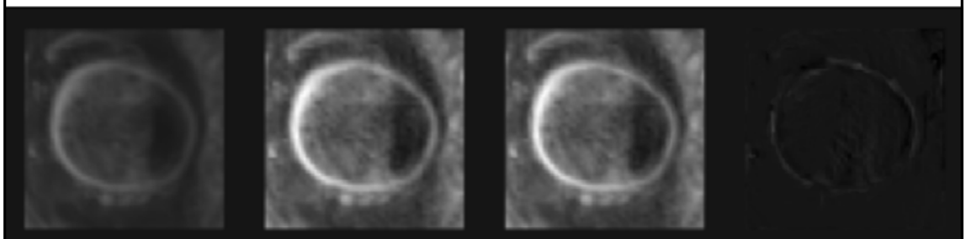


Frame 13/15

Results

Registration: Rectum tumor

Reference Image | Original Image | Registered Image | Difference Image

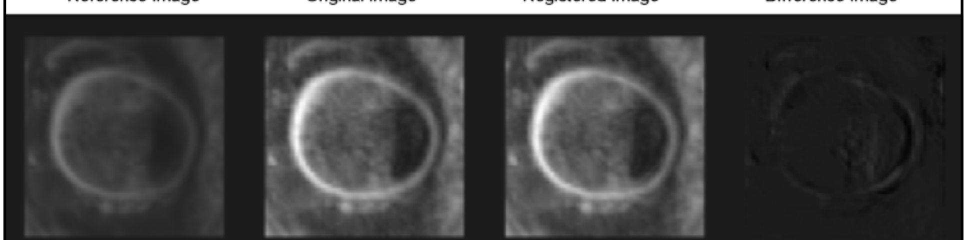


Frame 14/15

Results

Registration: Rectum tumor

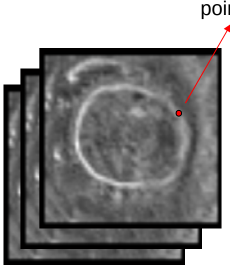
Reference Image | Original Image | Registered Image | Difference Image



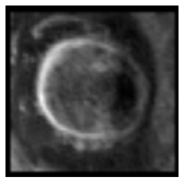
Frame 15/15

Results

point x

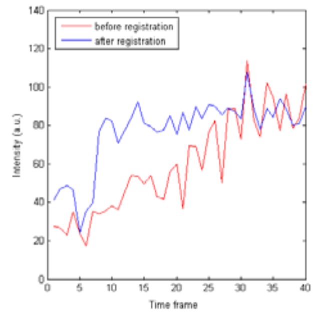


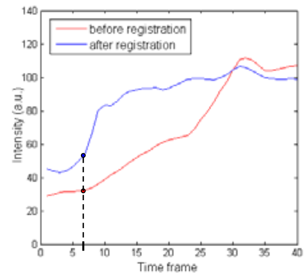
DCE-MRI sequence



Template image

Time dependence of
signal intensity at point x





DCE-MRI tumor registration by using TV-L1 optical flow

Discussion

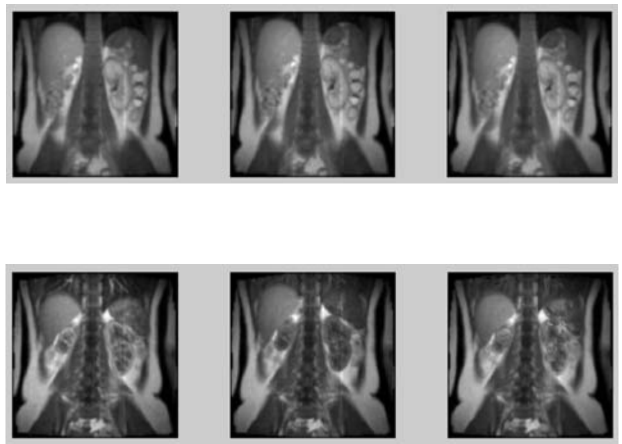
- Good registration performance for images with no intensity changes
- Good performance dealing with simple objects (e.g. rectum tumor)
- Violation of the **optical flow constraint** → intensity changes

Open question(s) ...

Open questions: Intensity changing OF - IMPOSSIBLE?

Problem: intensity changes for the OF algorithm over time

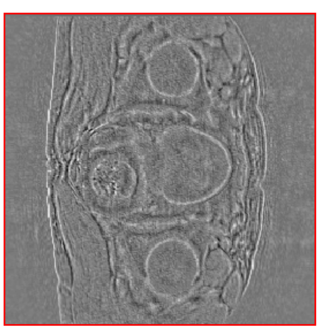
without intensity changes →

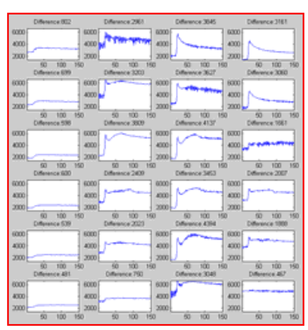


including intensity changes →

Possible solutions

reduce intensity information → structure images	add additional information to the OF functional	segmentation using prior-shape models





?

Thank you!

Dual formulation of the ROF-Model

ROF Model (TV-L2):

$$\min_u \left\{ \int_\Omega |\nabla u| \, d\Omega + \frac{1}{2\lambda} \int_\Omega (u - f)^2 \, d\Omega \right\}$$

Dual formulation of the TV norm:

$$|\nabla u| = \max \{ \boldsymbol{p} \cdot \nabla u \; : \; \|\boldsymbol{p}\| \leq 1 \}$$

Primal-dual formulation of the ROF Model:

$$\min_u \max_{\|\boldsymbol{p}\| \leq 1} \left\{ \int_\Omega \boldsymbol{p} \cdot \nabla u \, d\Omega + \frac{1}{2\lambda} \int_\Omega (u - f)^2 \, d\Omega \right\}$$

Optimality condition with respect to u:

$$u = f + \lambda \nabla \cdot \boldsymbol{p}$$

Dual ROF model:

$$\min_{\|\boldsymbol{p}\| \leq 1} \left\{ - \int_\Omega \boldsymbol{p} \cdot \nabla f \, d\Omega + \frac{\lambda}{2} \int_\Omega (\nabla \cdot \boldsymbol{p})^2 \, d\Omega \right\}$$

Euler-Lagrange equation:

$$- \nabla \left(f + \lambda \nabla \cdot \boldsymbol{p} \right) = 0 \, , \quad \|\boldsymbol{p}\| \leq 1$$