Optimization based estimation of activation sites in the heart

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October 16, 2017

This talk is concerned with an inverse problem in cardiac electrophysiology. In particular, the locations of the activation sites in the heart are estimated from the arrival times of the excitation wave on the epicardium of the heart. The electrophysiologic activity of the heart is often modeled using the Bidomain equations, whose numerical solution is very expensive. If one is only interested in the activation times T of the tissue, the Bidomain model can be reduced to the simpler viscous Eikonal equation

$$\begin{cases} -\varepsilon\,\mathrm{div}(M\nabla T) + M\nabla T\cdot\nabla T = 1 & \text{in } \Omega, \\ T = g_a & \text{on } \Gamma, \\ \varepsilon\nabla T\cdot n = 0 & \text{on } \Gamma_N. \end{cases} \tag{1}$$

The domain Ω models the geometry of the heart. The epicardium of the heart is denoted by Γ_N and the boundaries of the activation region (activation sites) by Γ. The matrix M describes the fiber orientation of the heart tissue and the function g_a the activation times in the activation regions. On the basis of this model we formulate the inverse problem in the following form

$$\min_{\Gamma} J(\Gamma) := \frac{1}{2}\int_{\Gamma_N} (T(\Gamma)-z)^2\,\mathrm{d}x \quad \text{subject to (1)}, \tag{2}$$

where z is the measured data on the epicardium. Problem (2) constitutes a shape optimization problem. We approach problem (2) using a gradient descent method. Thus we calculate the shape derivative DJ of J with respect to Γ on the continuous level. The numerical calculation of a perturbation field for Γ based on DF involves the numerical solution of (1), the adjoint equation of the linearized version of (1) and a vector-valued elliptic equation. These equations are discretized using linear finite elements and the non-linearity is treated using a quasi Newton method. The talk is concluded with the presentation of numerical experiments on a three-dimensional heart geometry with synthetic data.