Efficient Solvers in Biomedical applications
Graz 2012

Efficient solution for the Laplace equation in mesh generation

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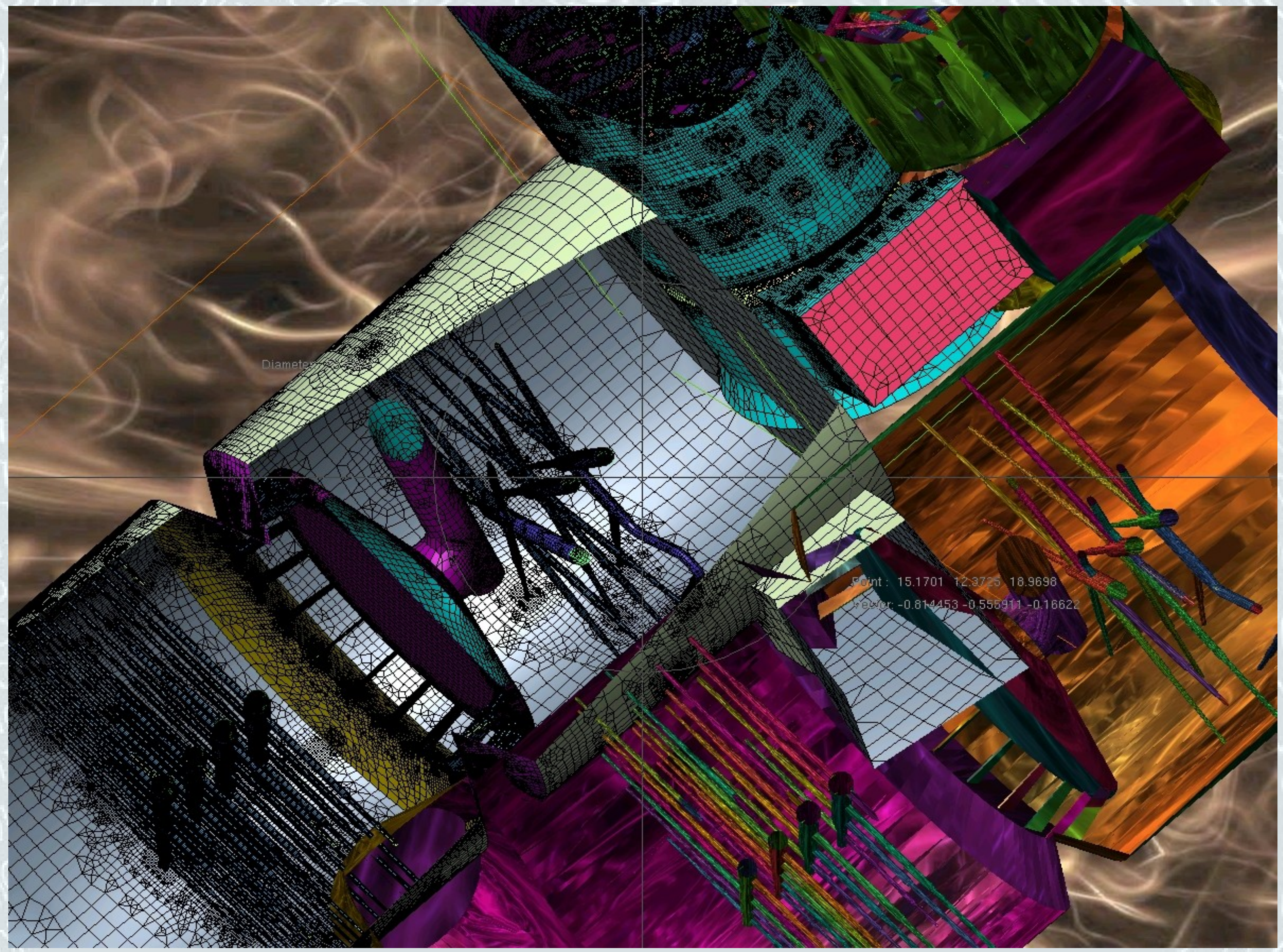
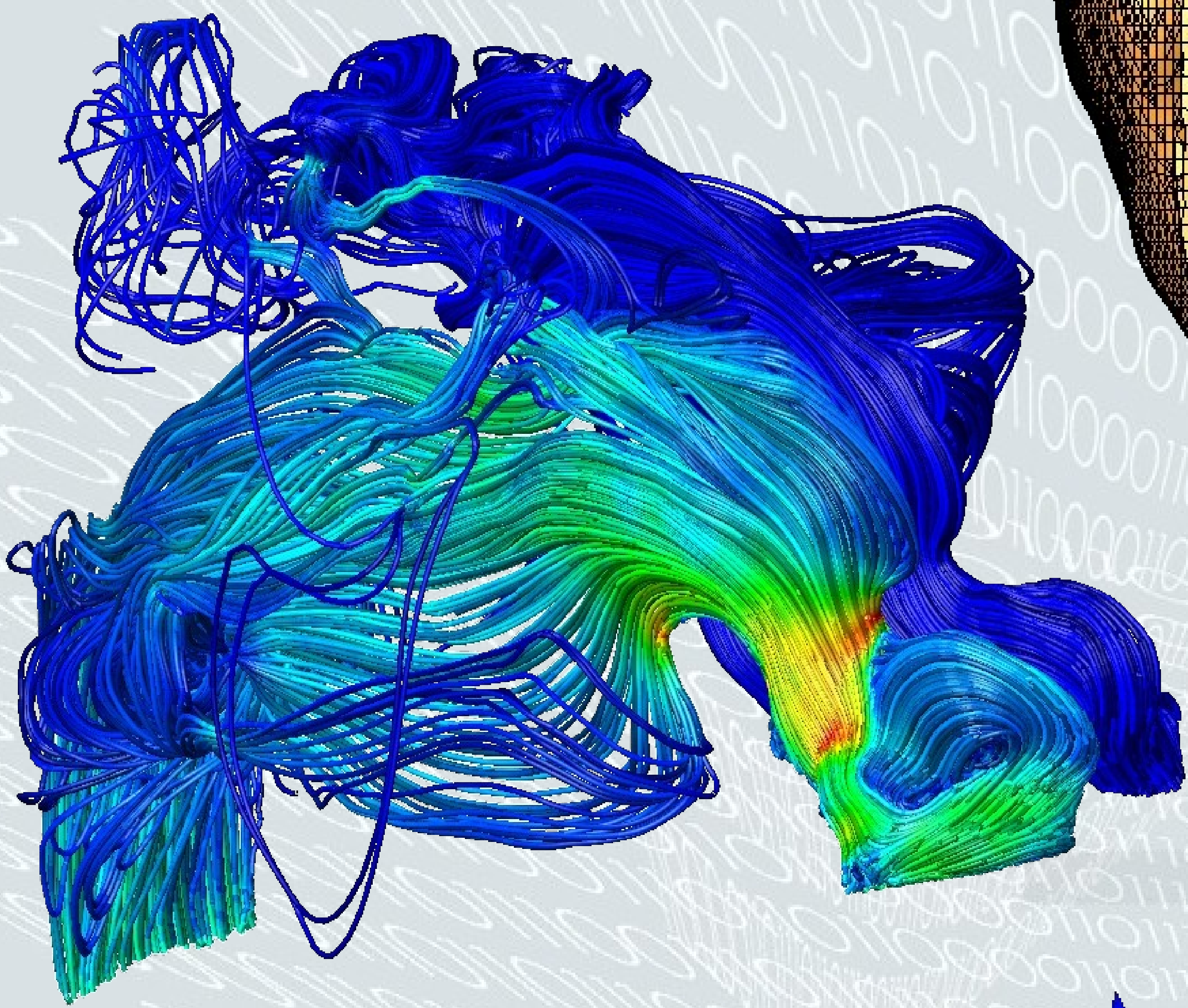
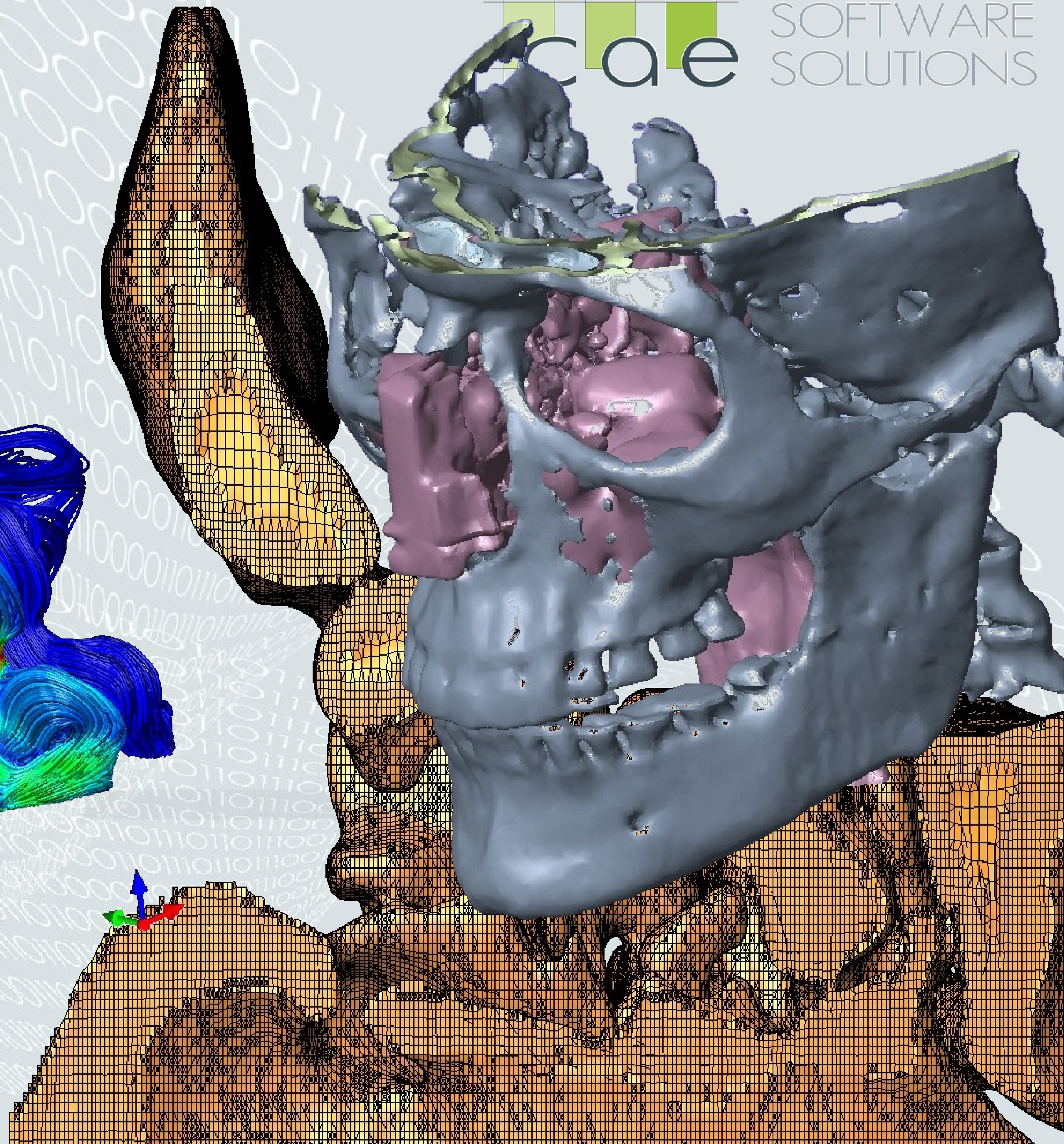
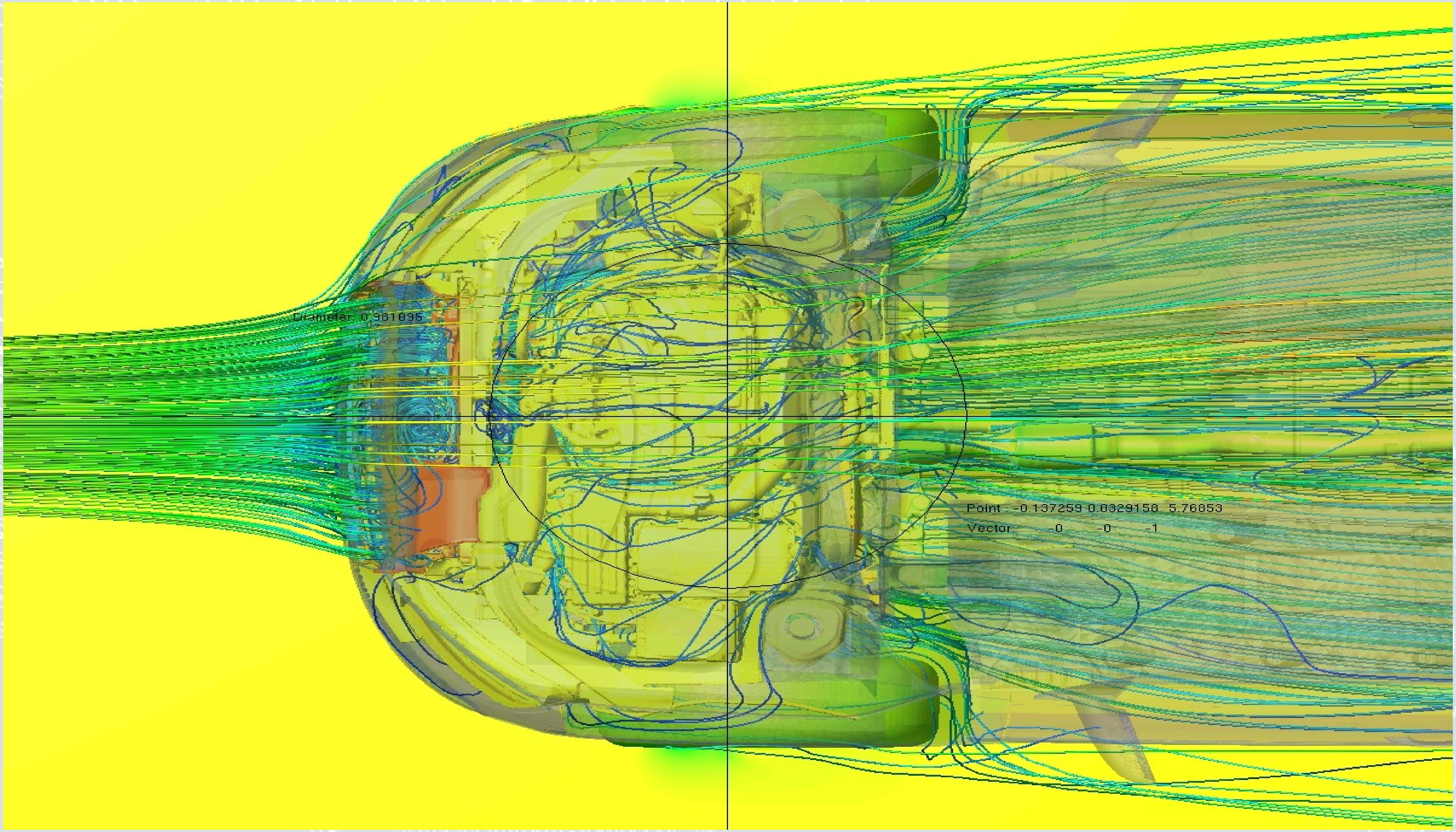


Motivation



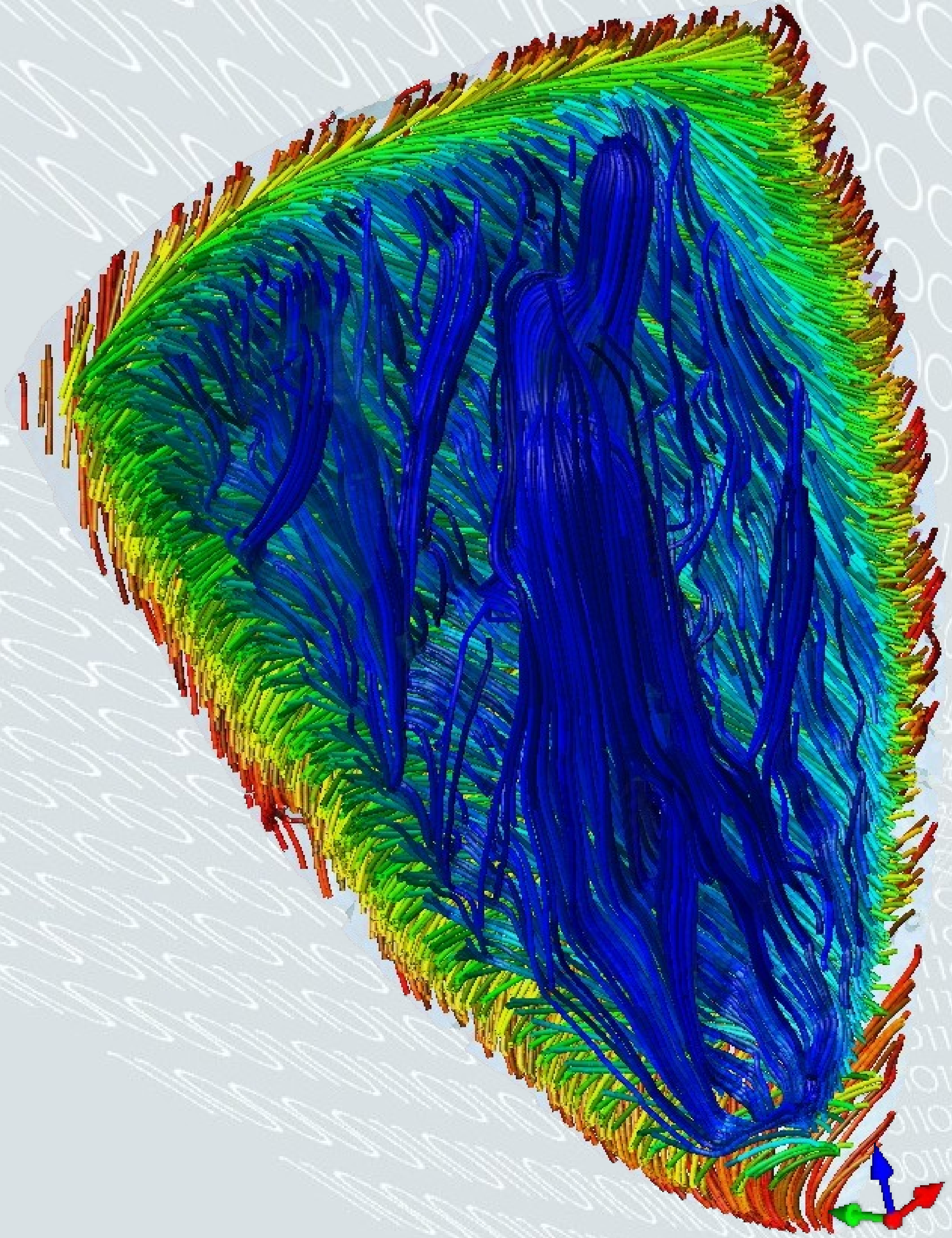


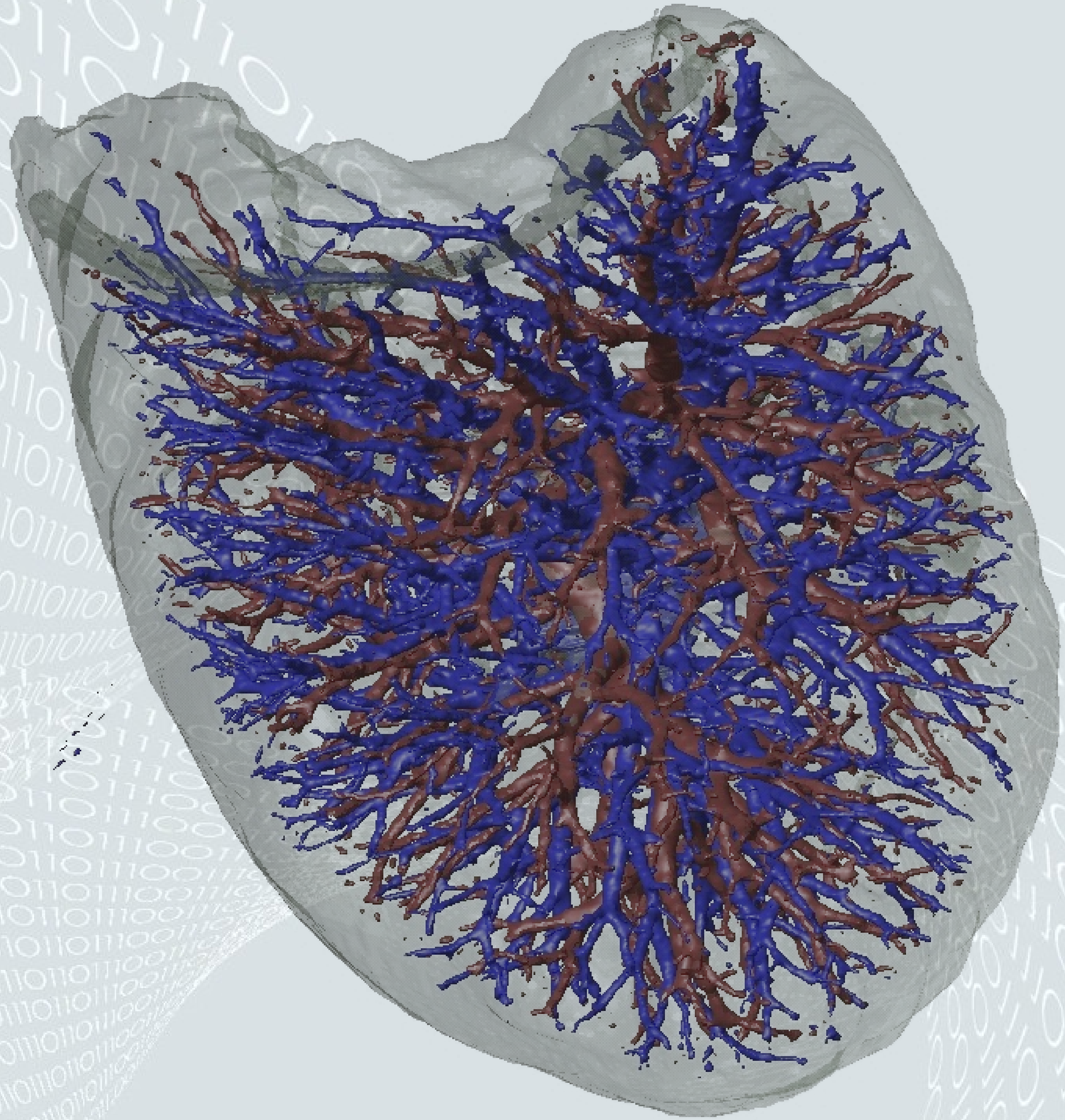
Motivation



Diameter: 0.981095
Point : -0.137259 0.0329158 5.76853
Vector: -0 -0 -1

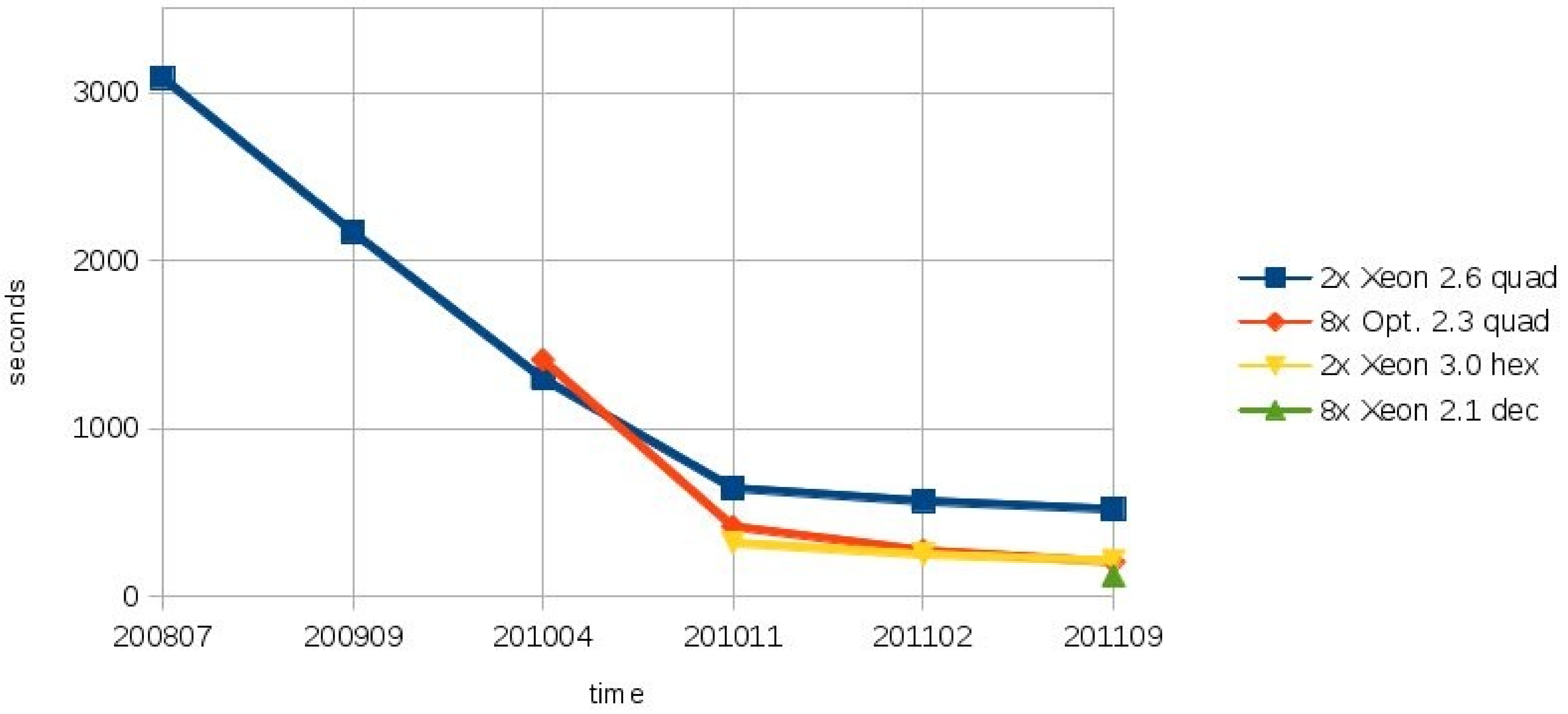
Motivation





Motivation

Example 10MCells		200807	200909	201004	201011	201102	201109
	cores						
2x Xeon 2.6 quad	8	3087,23	2169,60	1295,36	642,73	566,09	517,95
8x Opt. 2.3 quad	32			1407,22	414,43	271,58	204,87
2x Xeon 3.0 hex	12				316,67	248,36	210,98
8x Xeon 2.1 dec	80						121,01



Mesh-sizes in the past years:

Typical application size (u-hood)

- 2005 8 Mcells (with luck)
- 2007 20 Mcells
- 2011 100 Mcells
- 2012 200Mcells

Max-sized mesh created

- 2005 8 Mcells
- 2007 100   Mcells
- 2008 1000 Mcells (oxford rabbit hear
- 2012 3000 Mcells

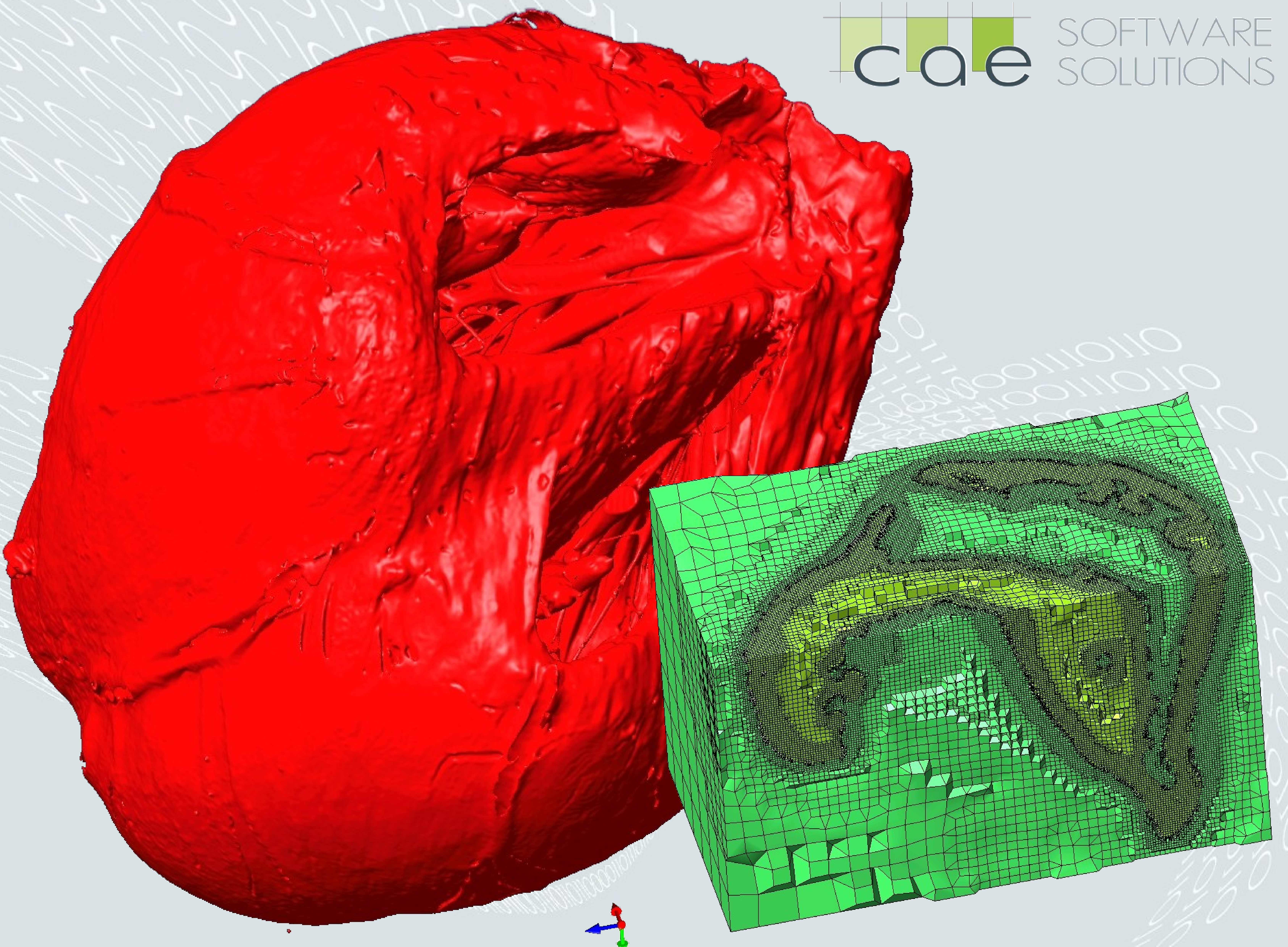
2012 numbers

- 1h->360MCells
- 250Byte a cell memory
- 30Byte a cell on file (hex dominant)
- 1e7 cells/s on file

2012 target numbers

- 1h-> 1000 MCells
- 100Byte a cell memory
- 30Byte a cell on file (hex dominant)
- 1e7 cells/s on file

cae SOFTWARE SOLUTIONS



Solving $-\Delta u=f$ in Ω,
$u=g_1$ on Γ_1
$\partial u/\partial n=g_2$ on Γ_2
with AMG / MG (efficient? parallel?)

but where could that help?

Restrictions:

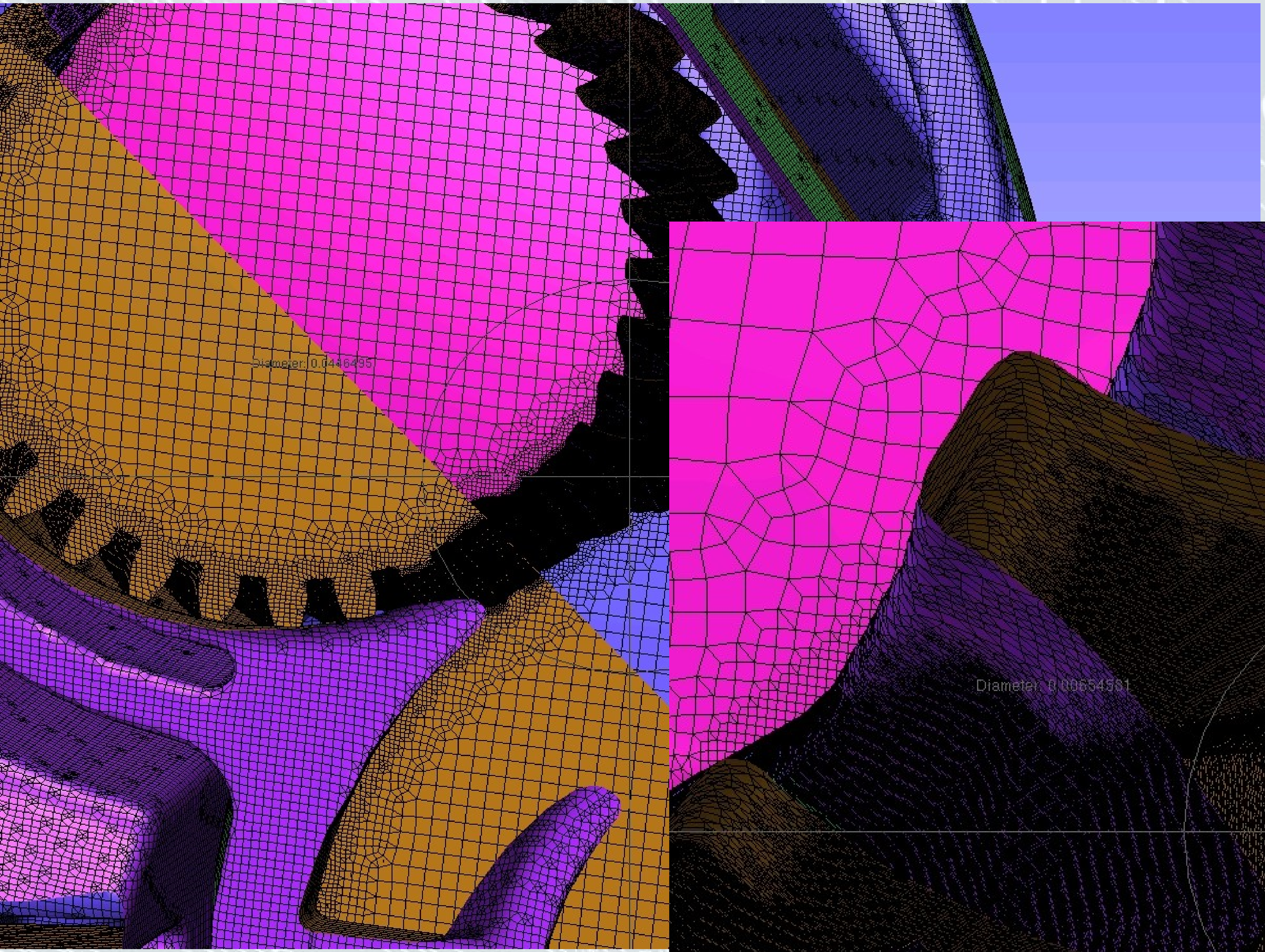
- Solving pde during meshing process is difficult, the mesh is not ready!
- Alternative 1: mesh movement
- Alternative 2: mesh smoothing

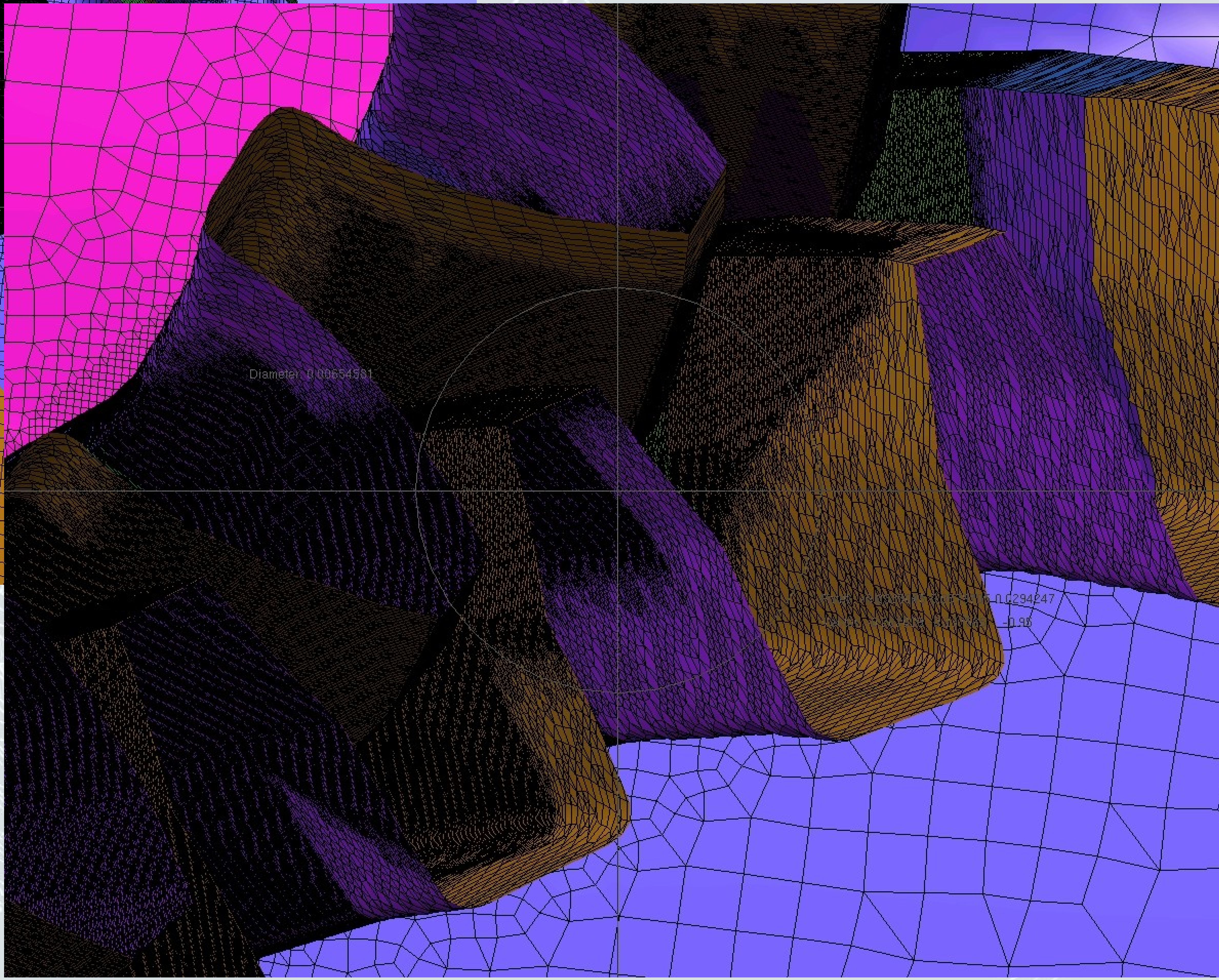
Mesh movement

- Solving linear elastic equations + nonlinear local smoothing is possible
- Solving laplace equation for each component + nonlinear local smoothing is sufficient

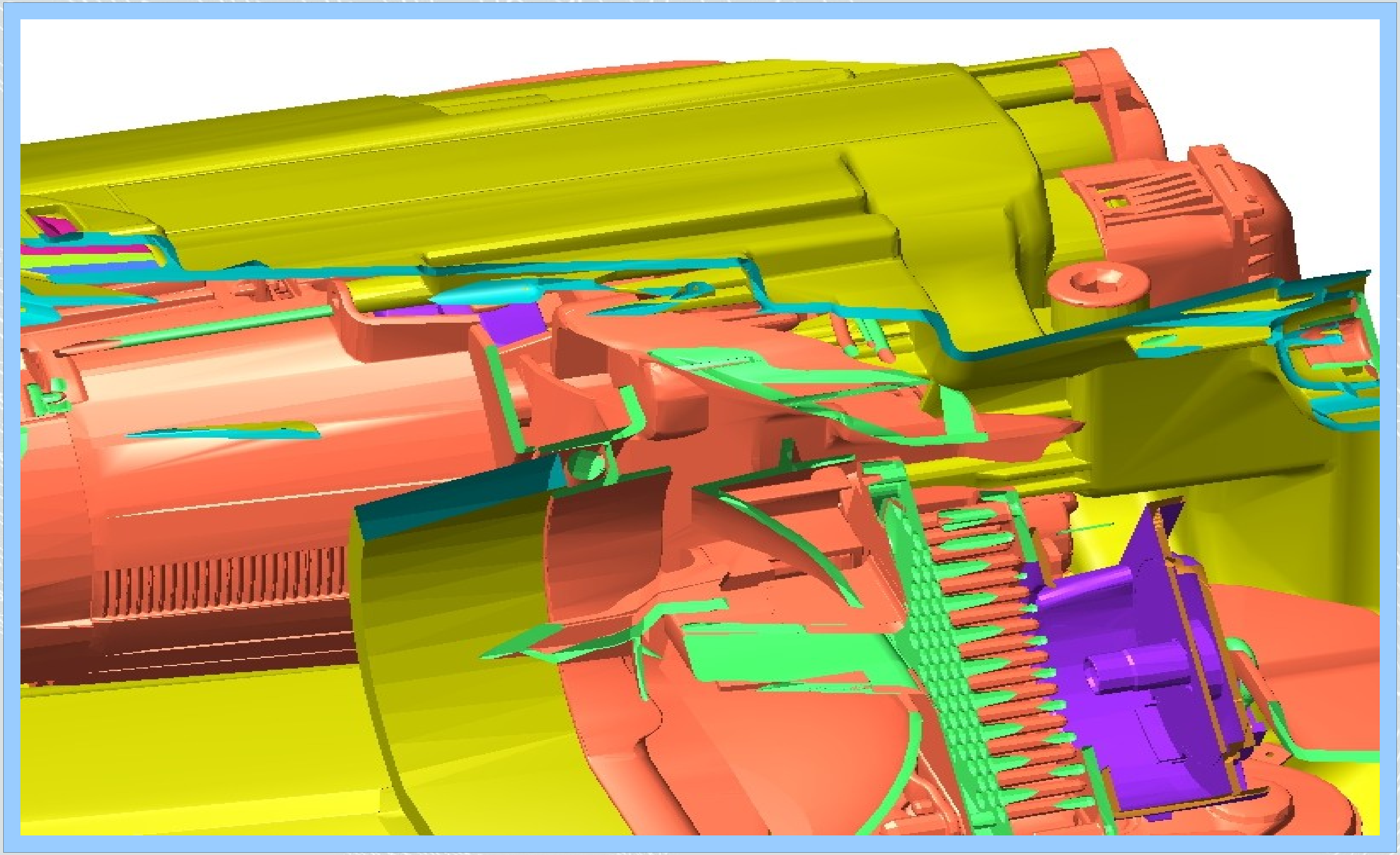
Video mesh movement

•There should be a video in a few moments ...





Mission: create mesh for fluid(air) domain



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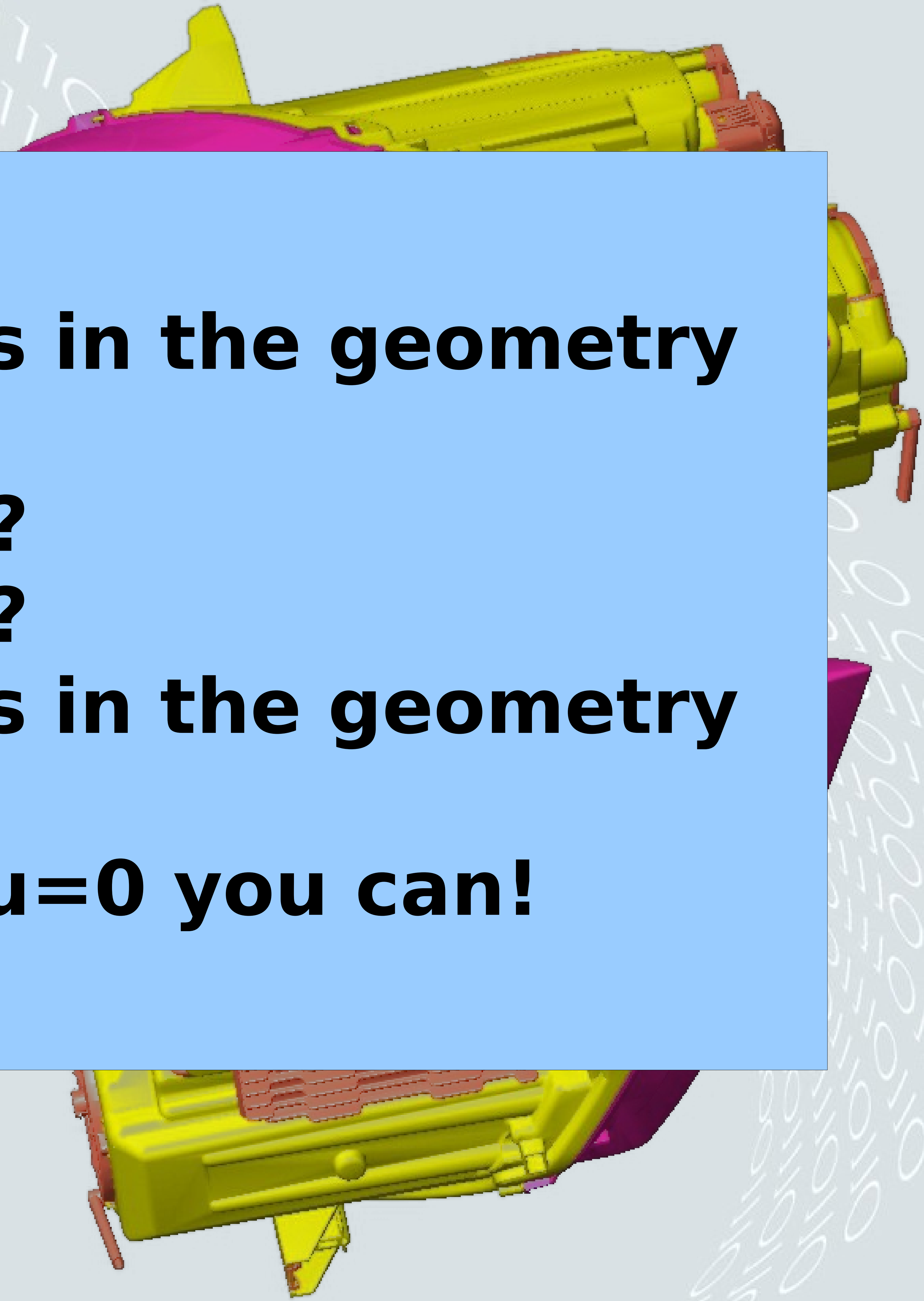
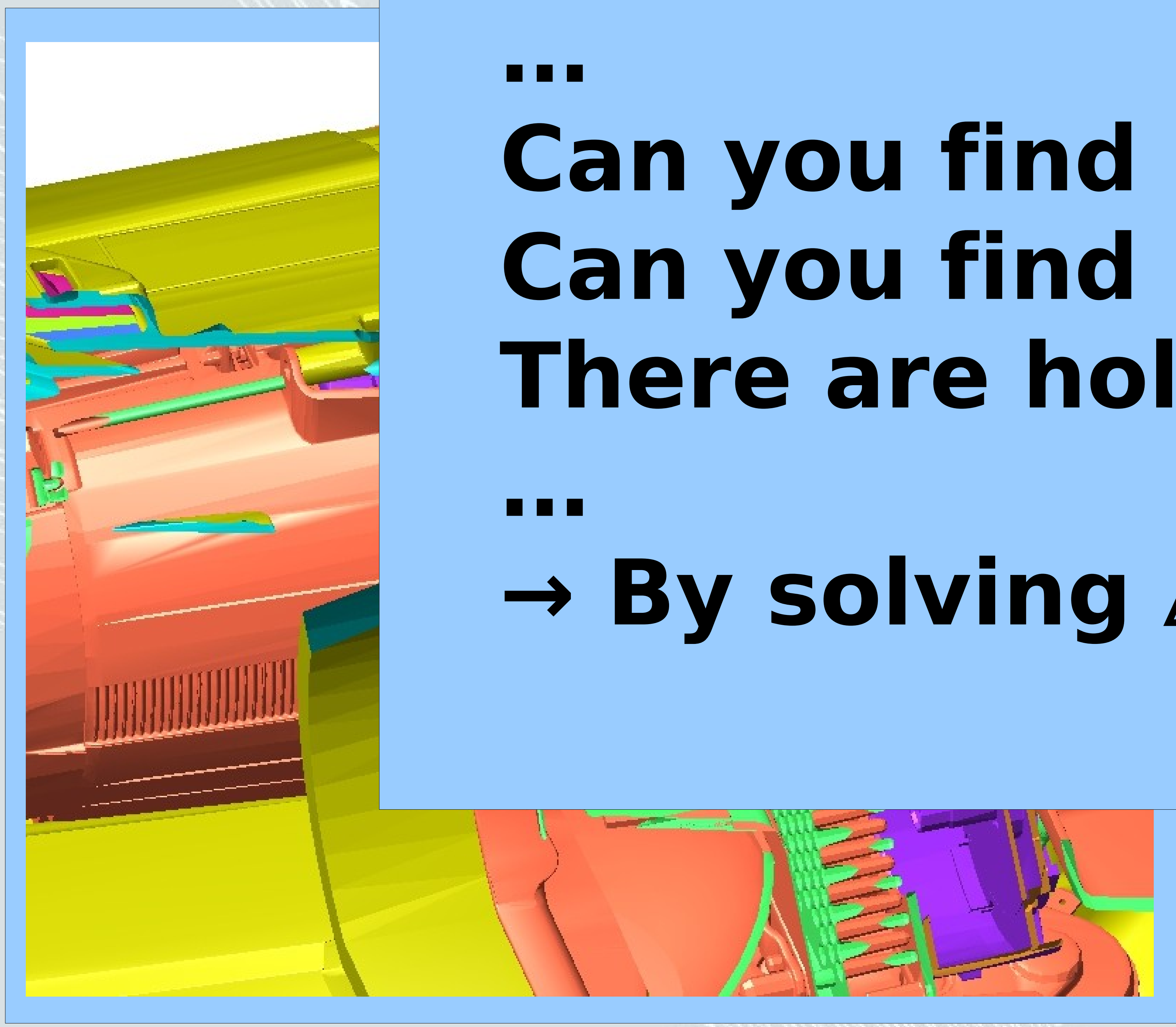




♬ ♬ ♬ ♬

There are holes in the geometry ...

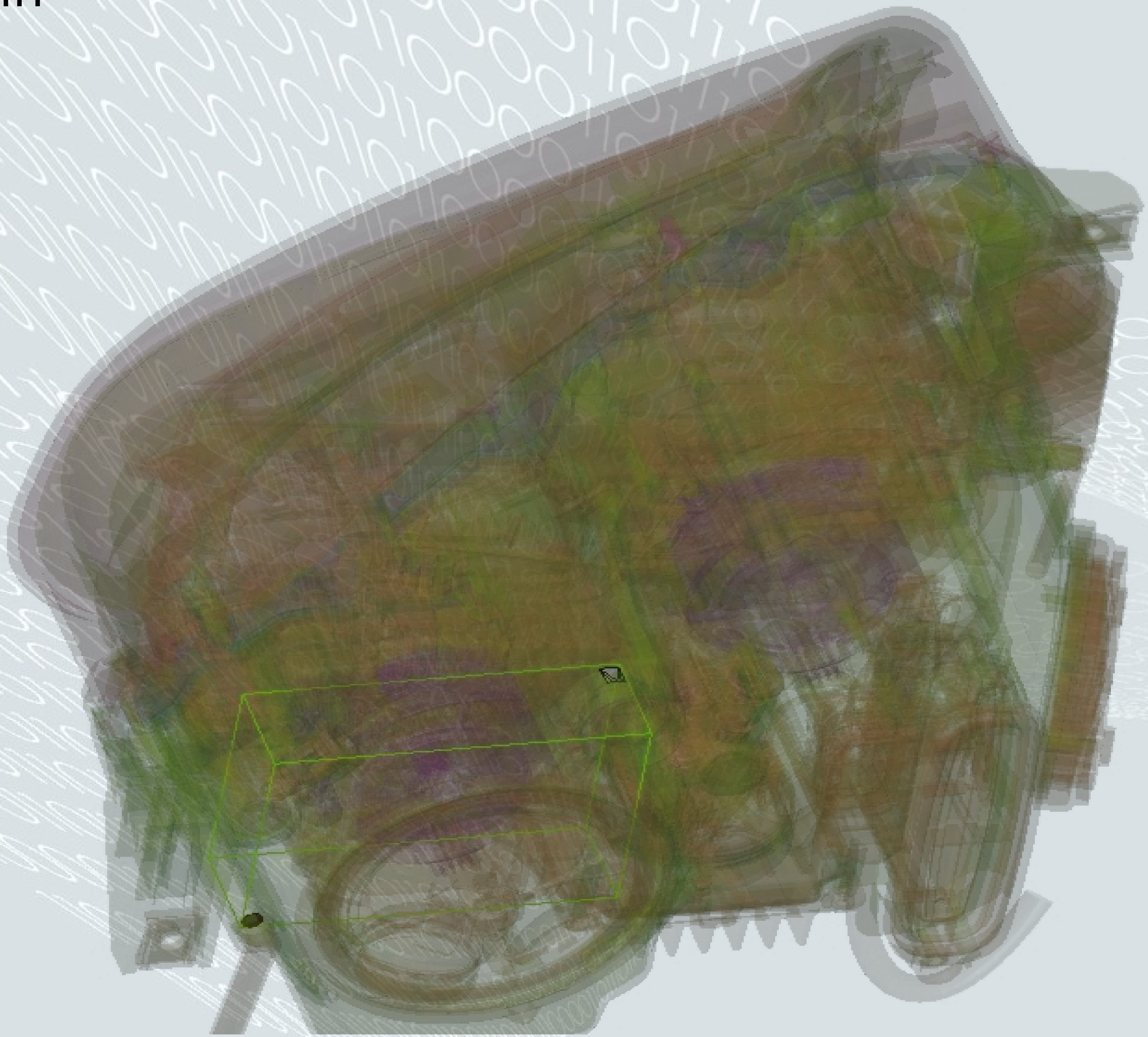
Can you find it?

Can you find it?

There are holes in the geometry ...

→ By solving $\Delta u=0$ you can!

Mission: create mesh for fluid(air) domain



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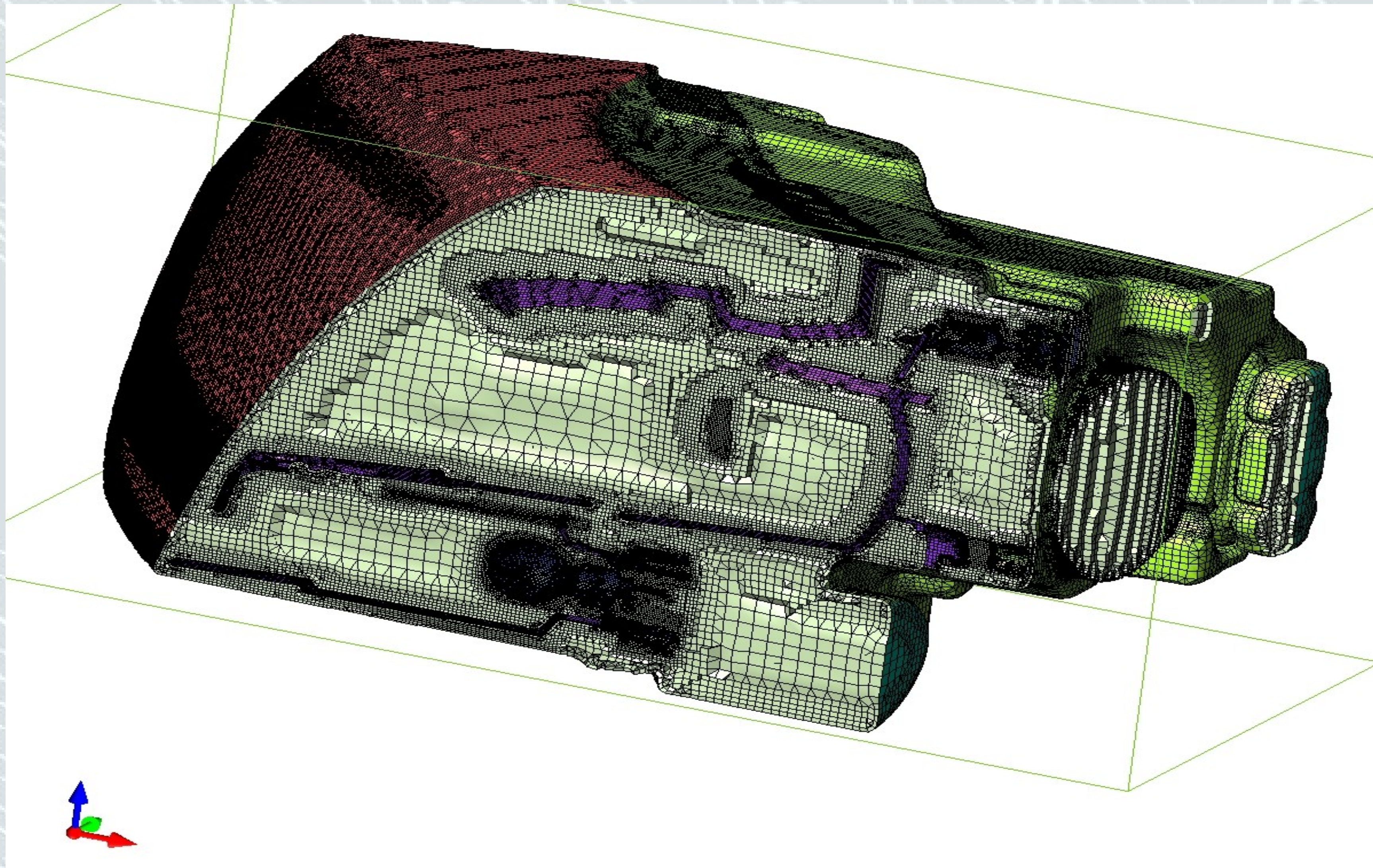

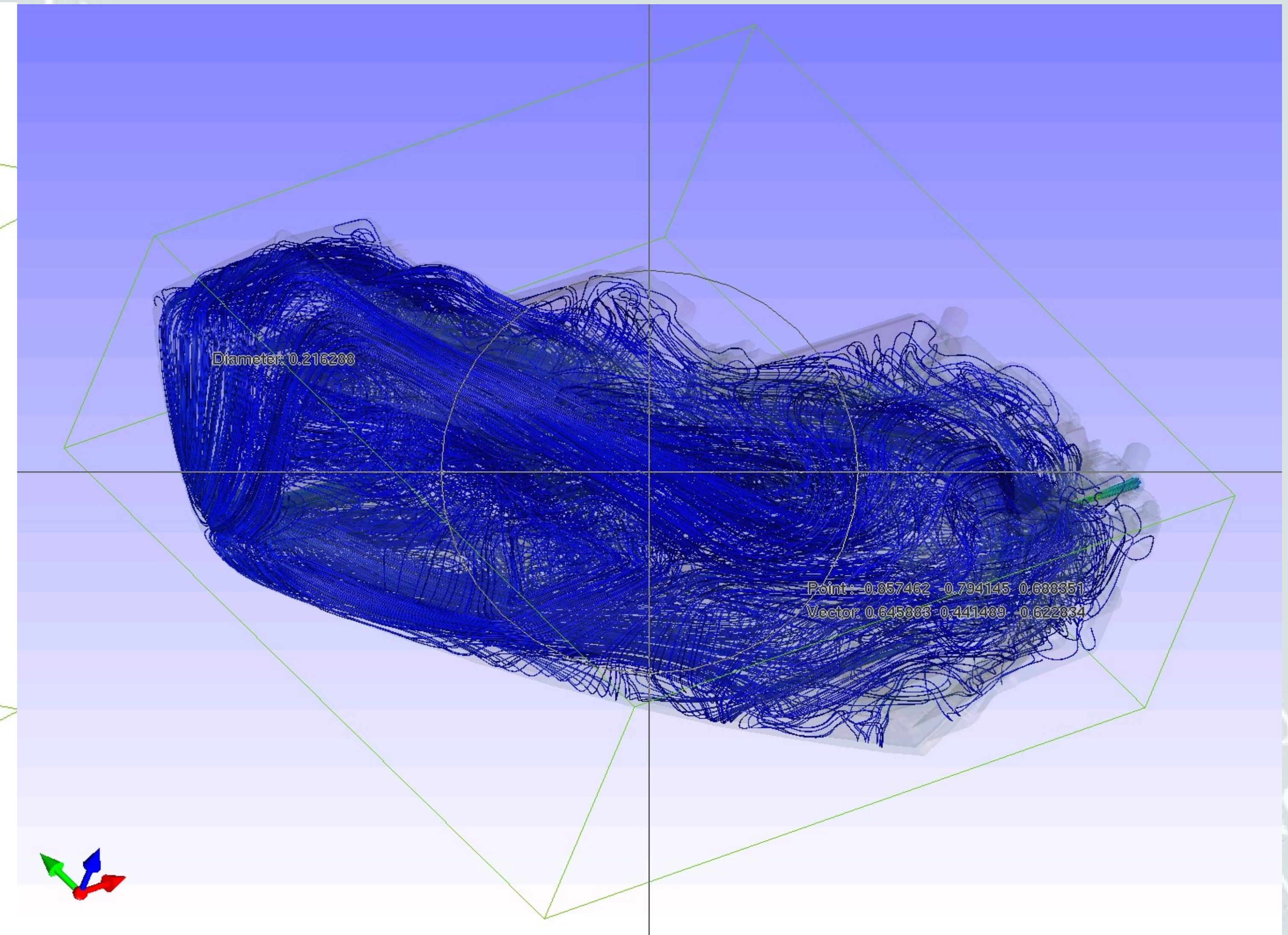

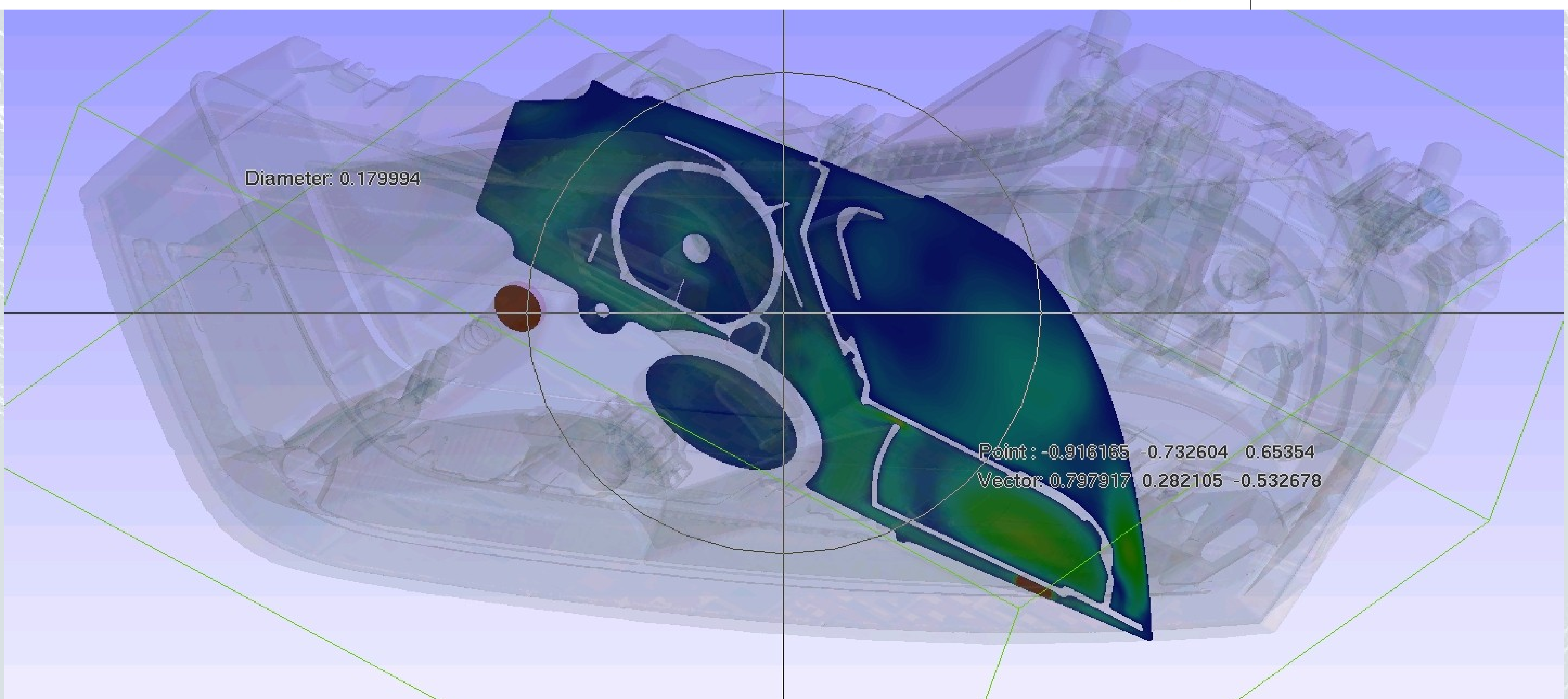

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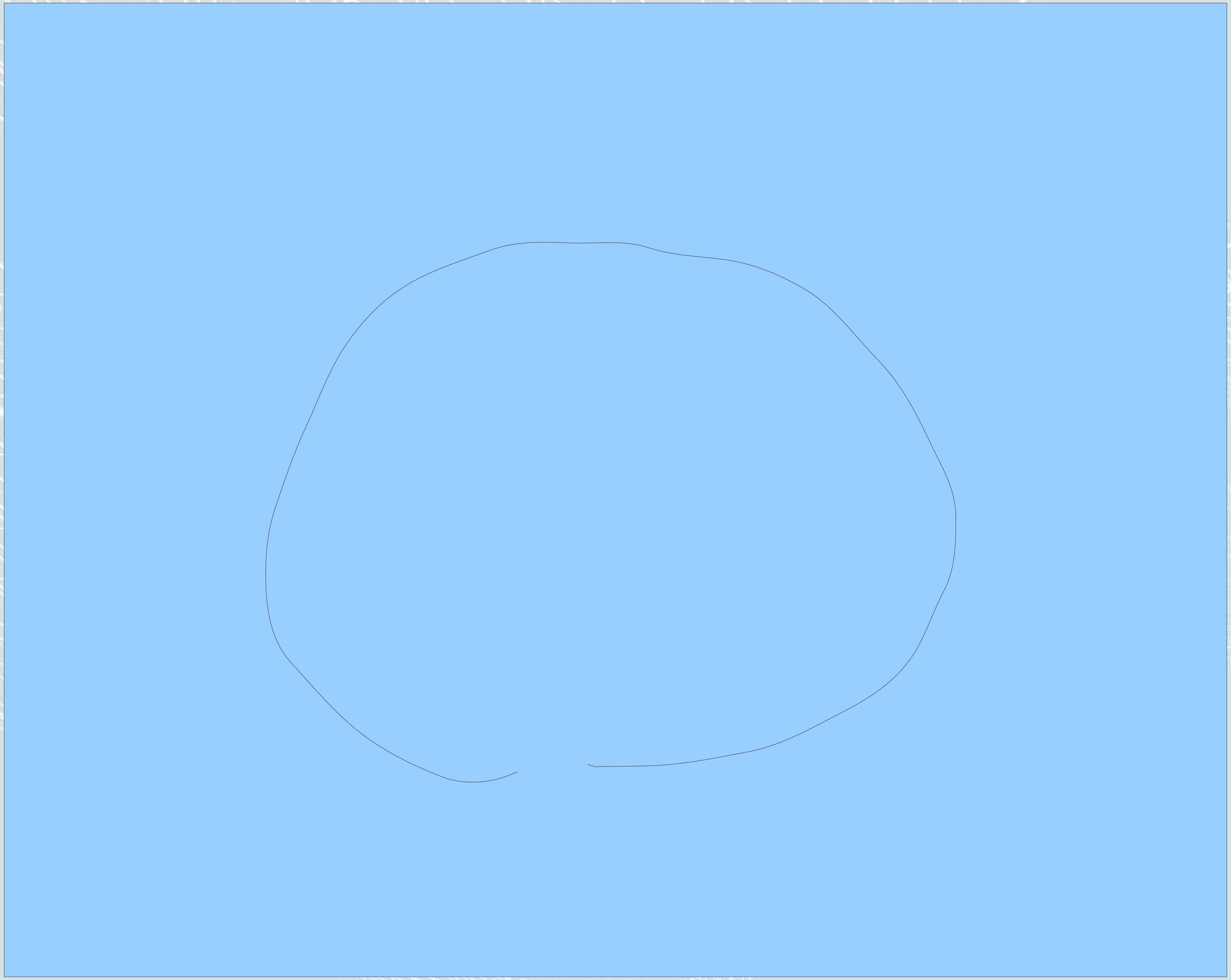
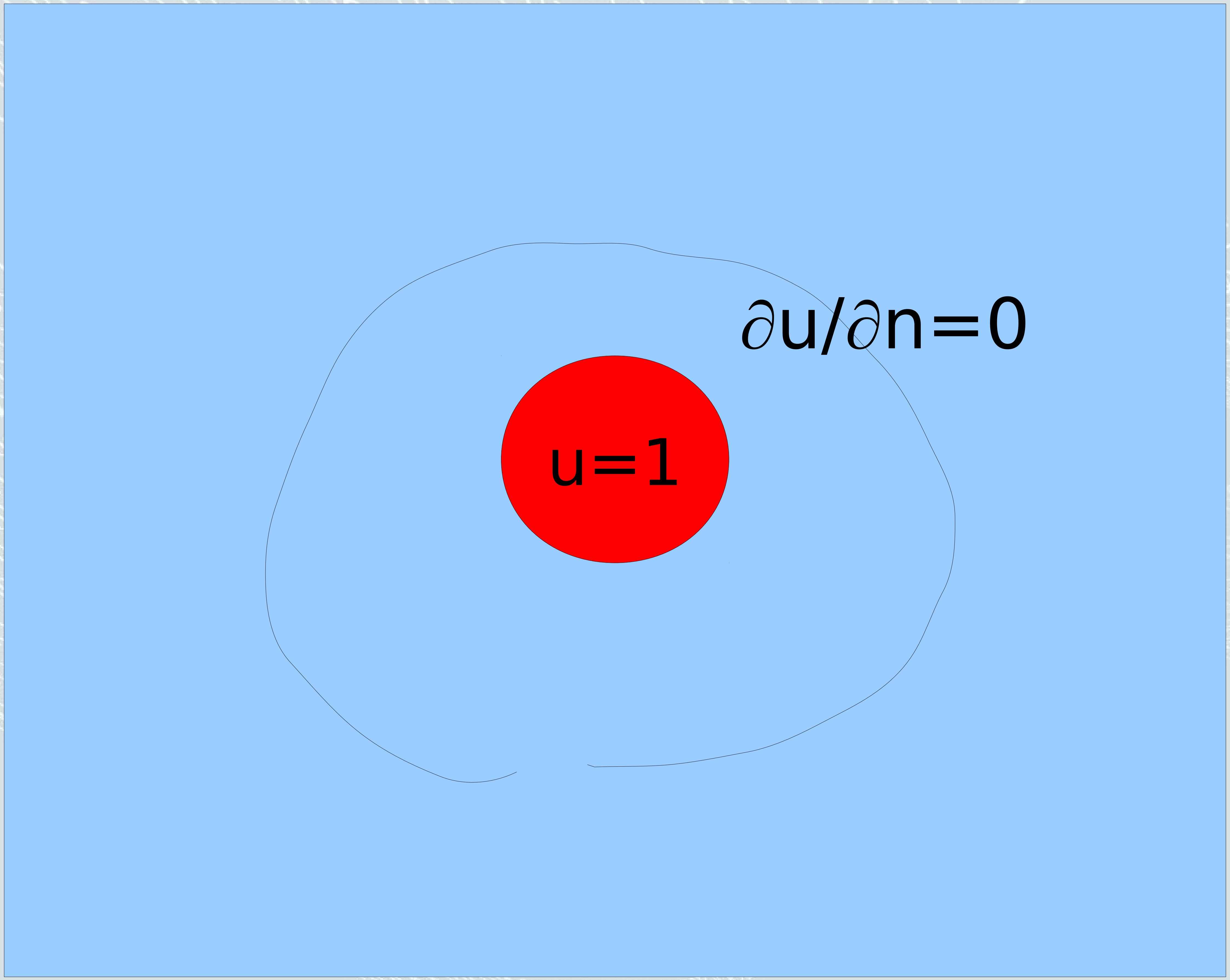


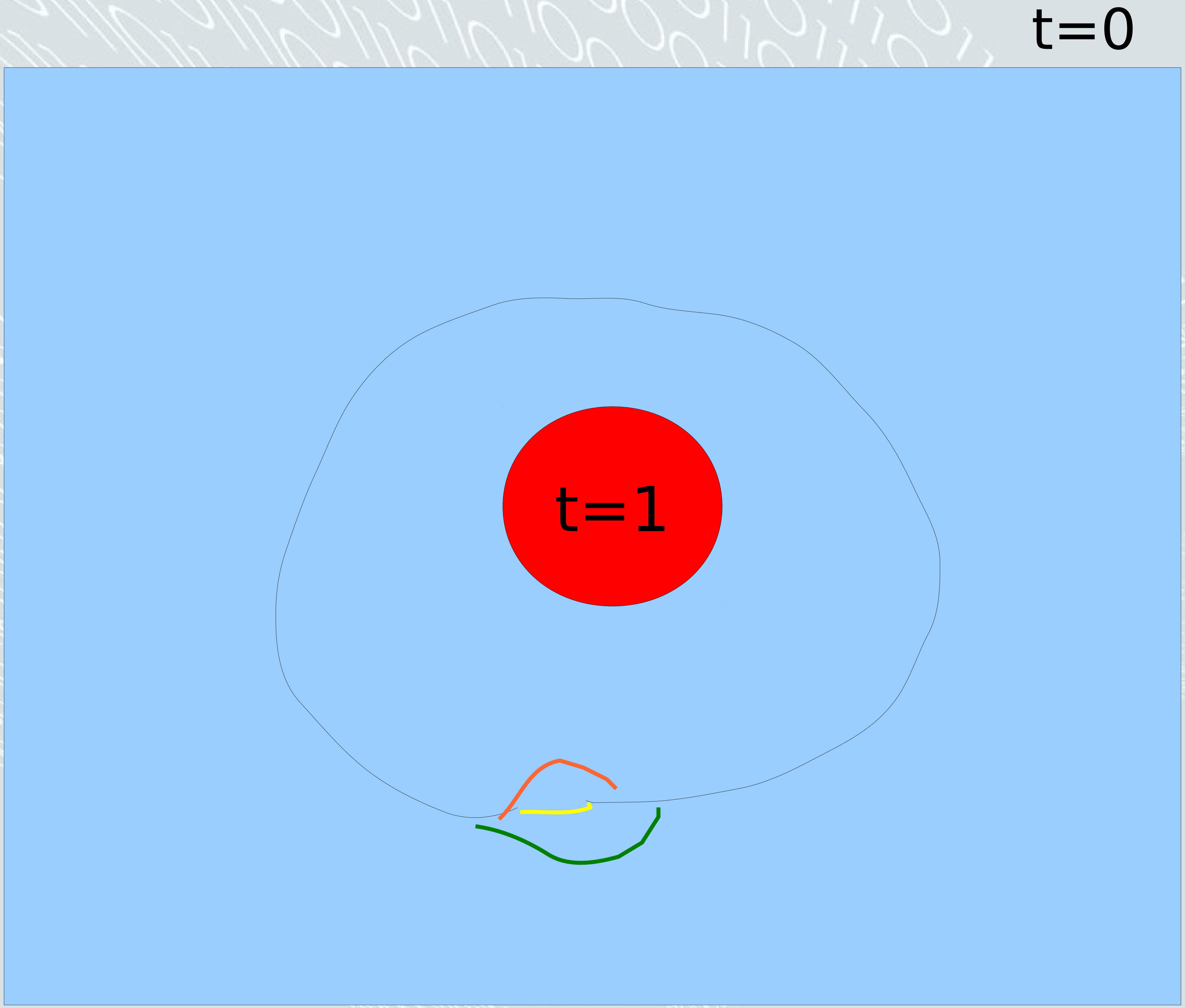
Mission: create mesh for fluid(air) domain

$$-\Delta u=0$$



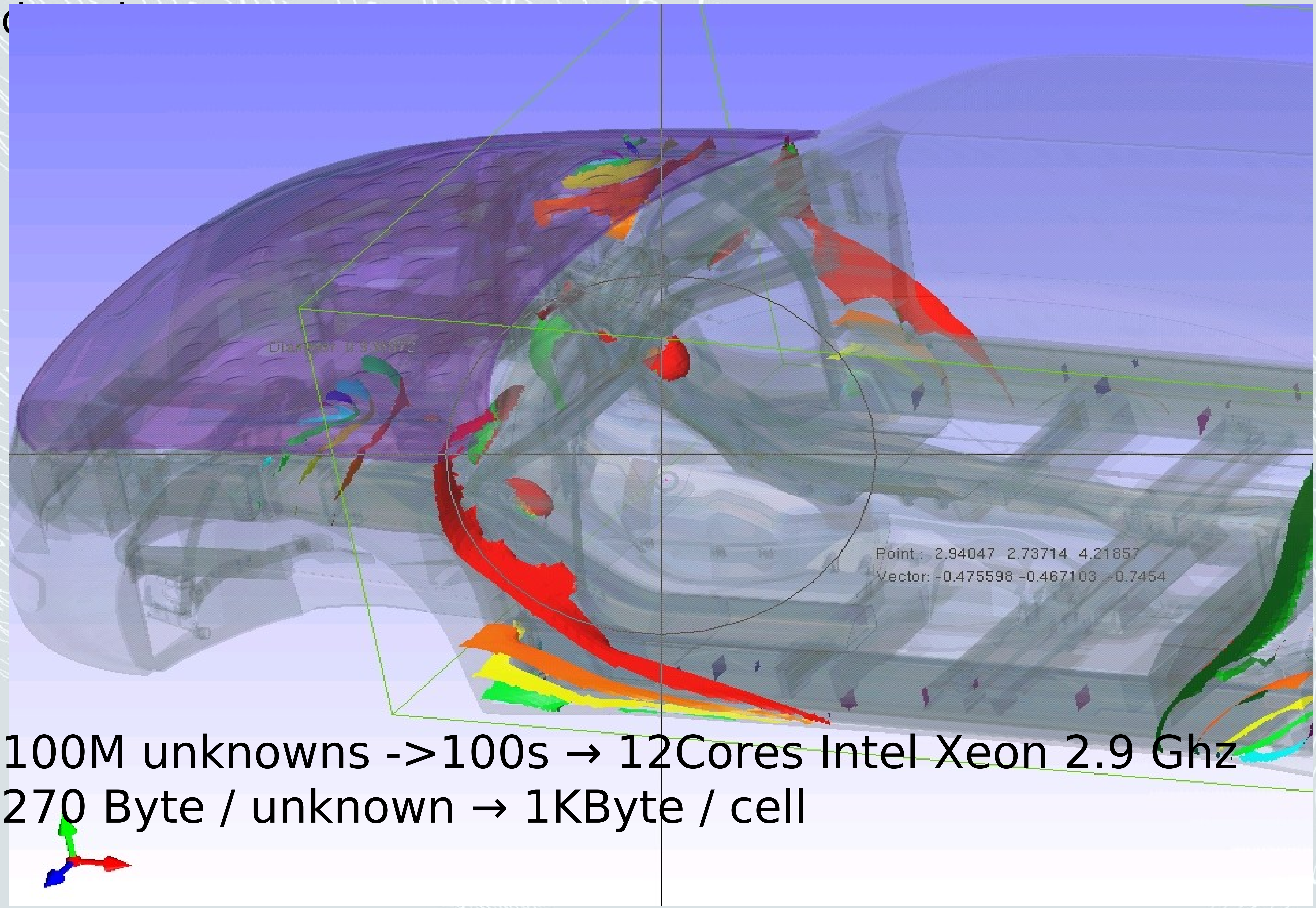
$$u=0$$

Mission: create mesh for fluid(air) domain



Using the iso-surfaces for $0 < t < 1$

Mission: create mesh for fluid(air) d



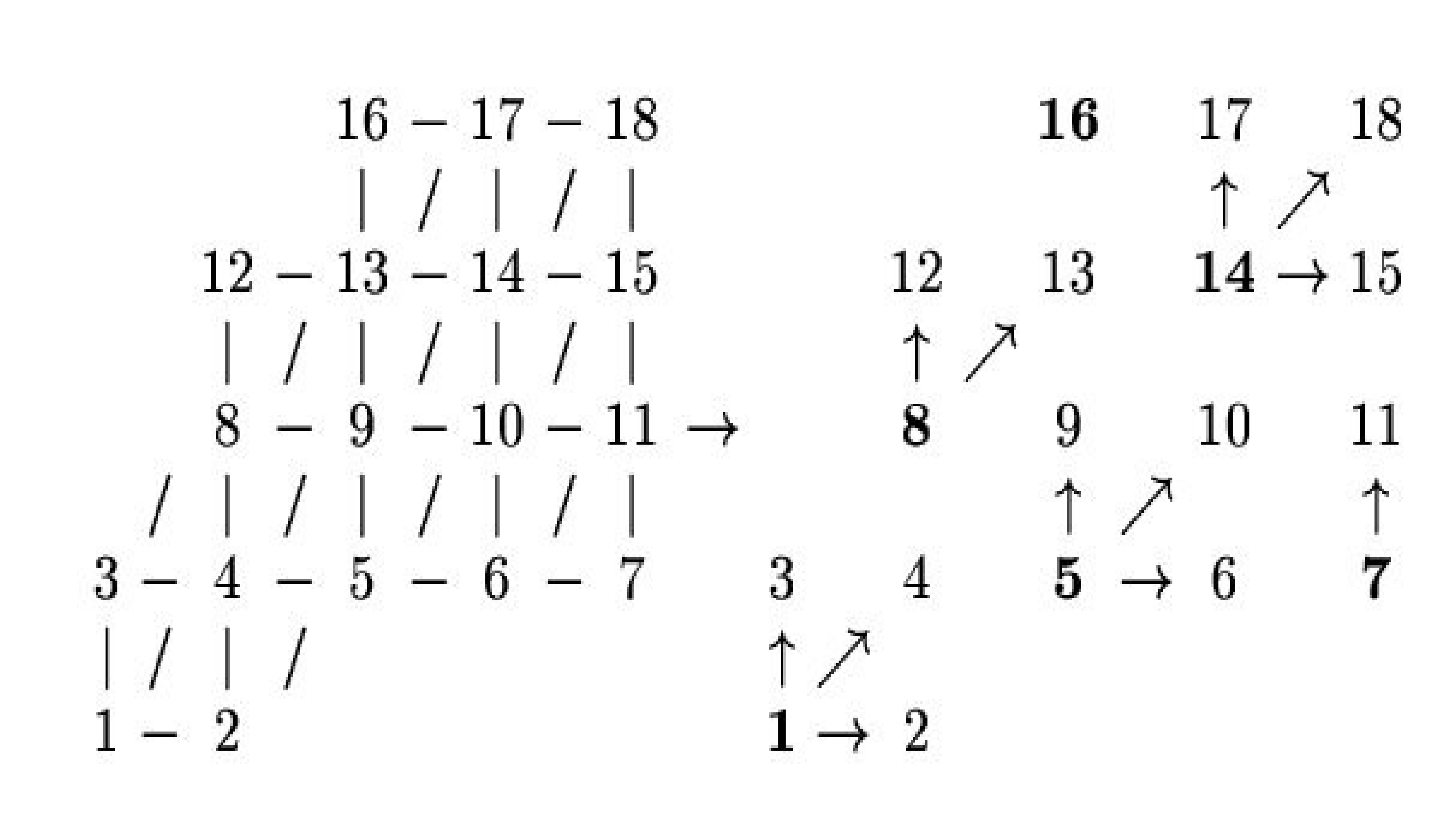
- 100M unknowns ->100s → 12Cores Intel Xeon 2.9 Ghz
- 270 Byte / unknown → 1KByte / cell

Mission: create mesh for fluid(air) domain

- Using the octree as mesh
- Mesh is non-conformal
- Using piecewise constant ansatz-functions
- Boundary description is staircase
- AMG used

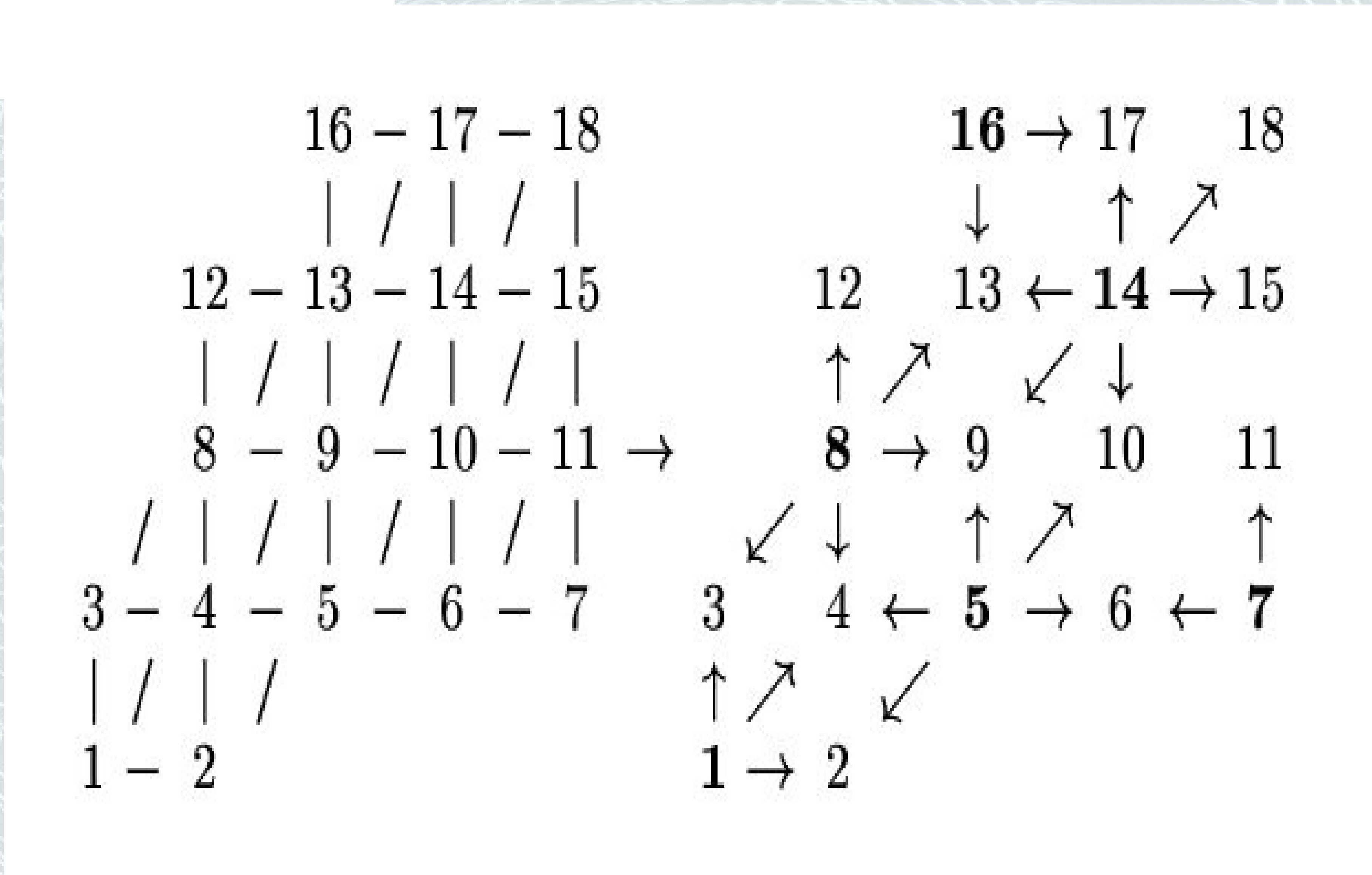
- AMG does a red-black coloring
- Does a piecewise linear interpolation (if possible)
- AMG is formulated, that geometrical MG is a special case. [Kickinger 1997]

Mission: create mesh for fluid(air) domain

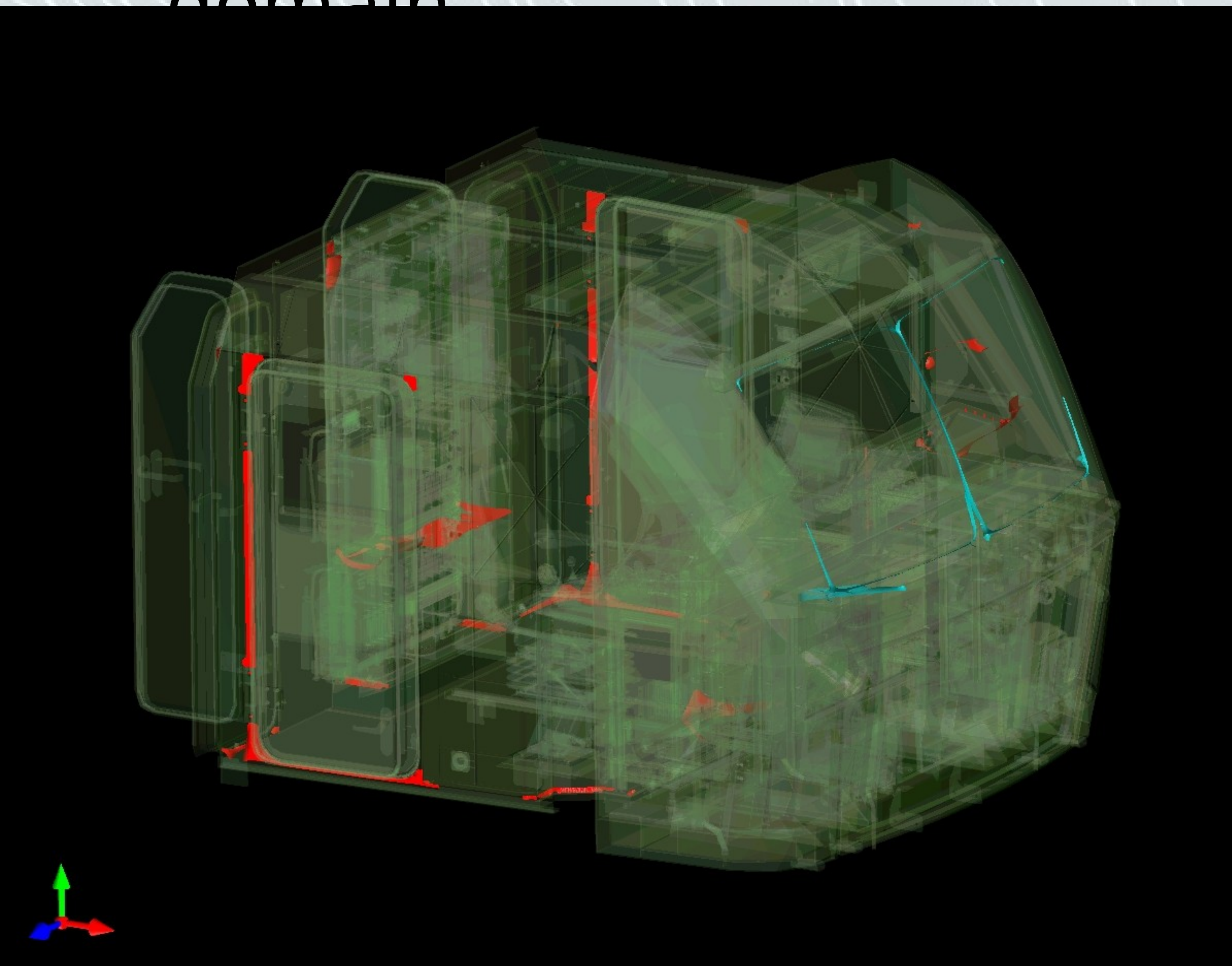


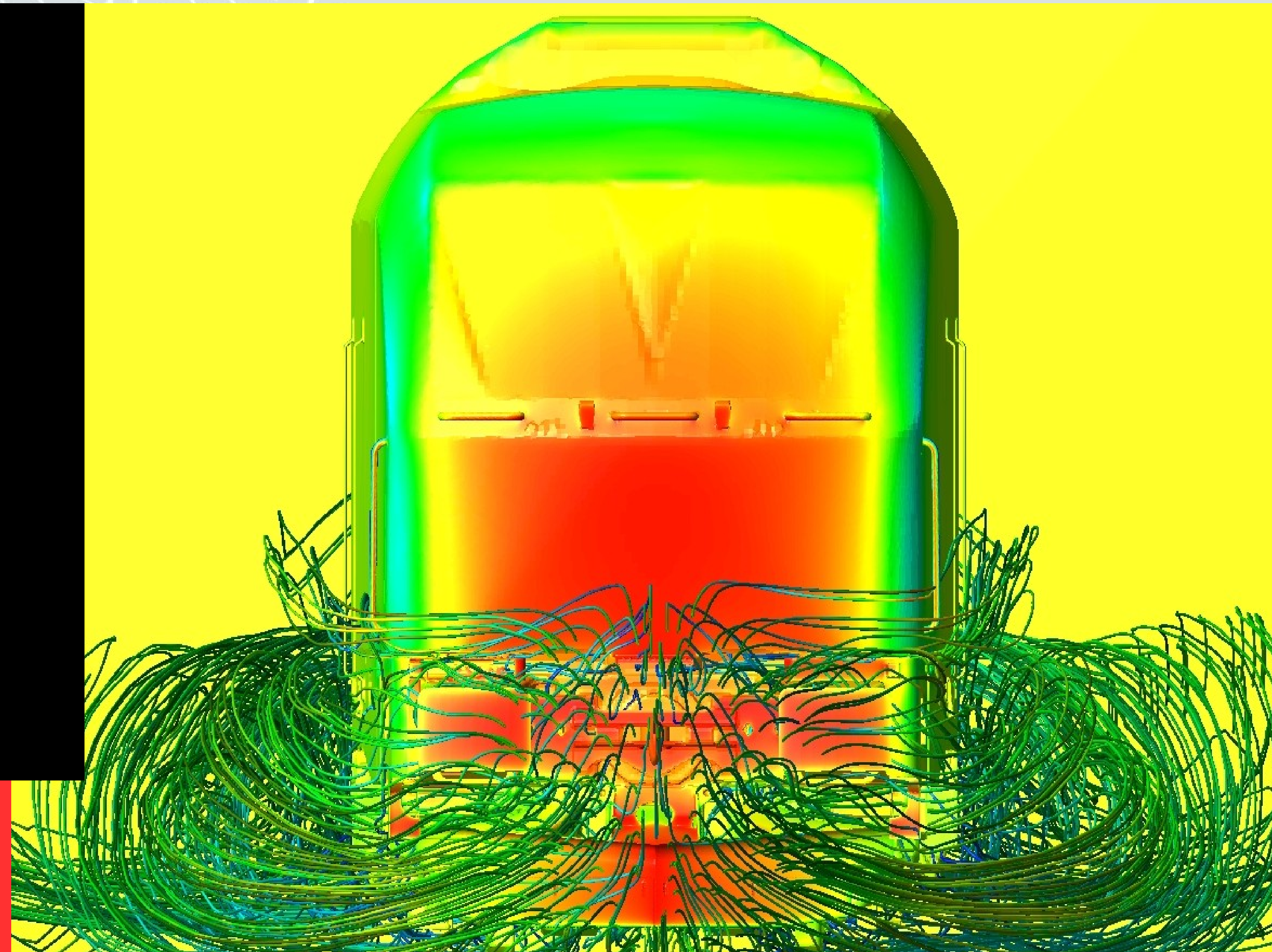
<- agglomeration

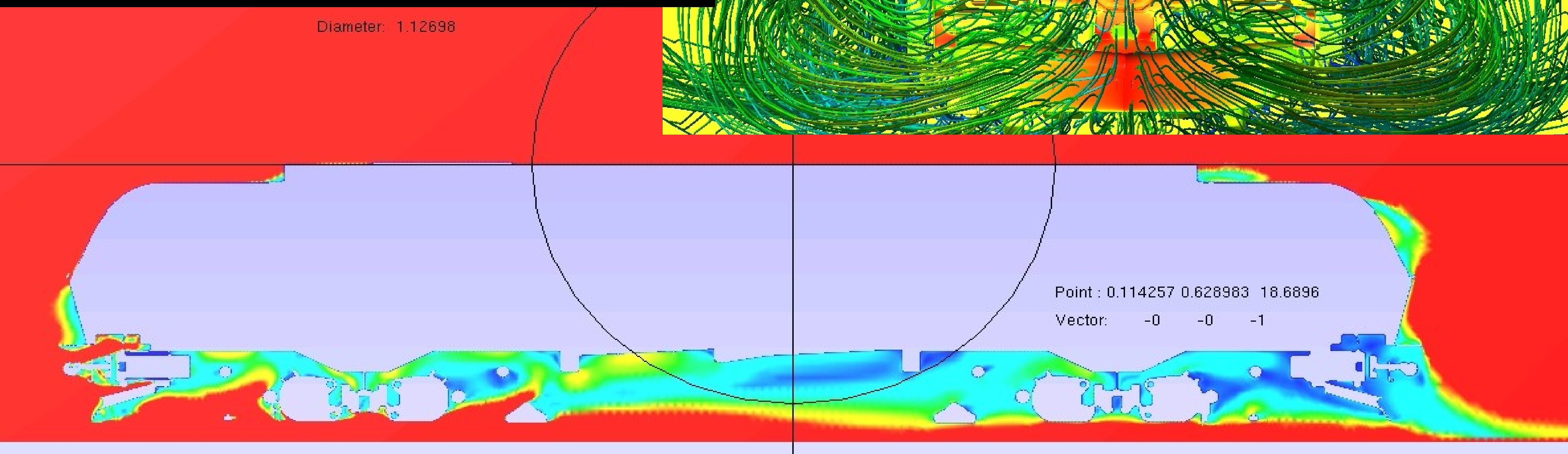
"linear extention"->



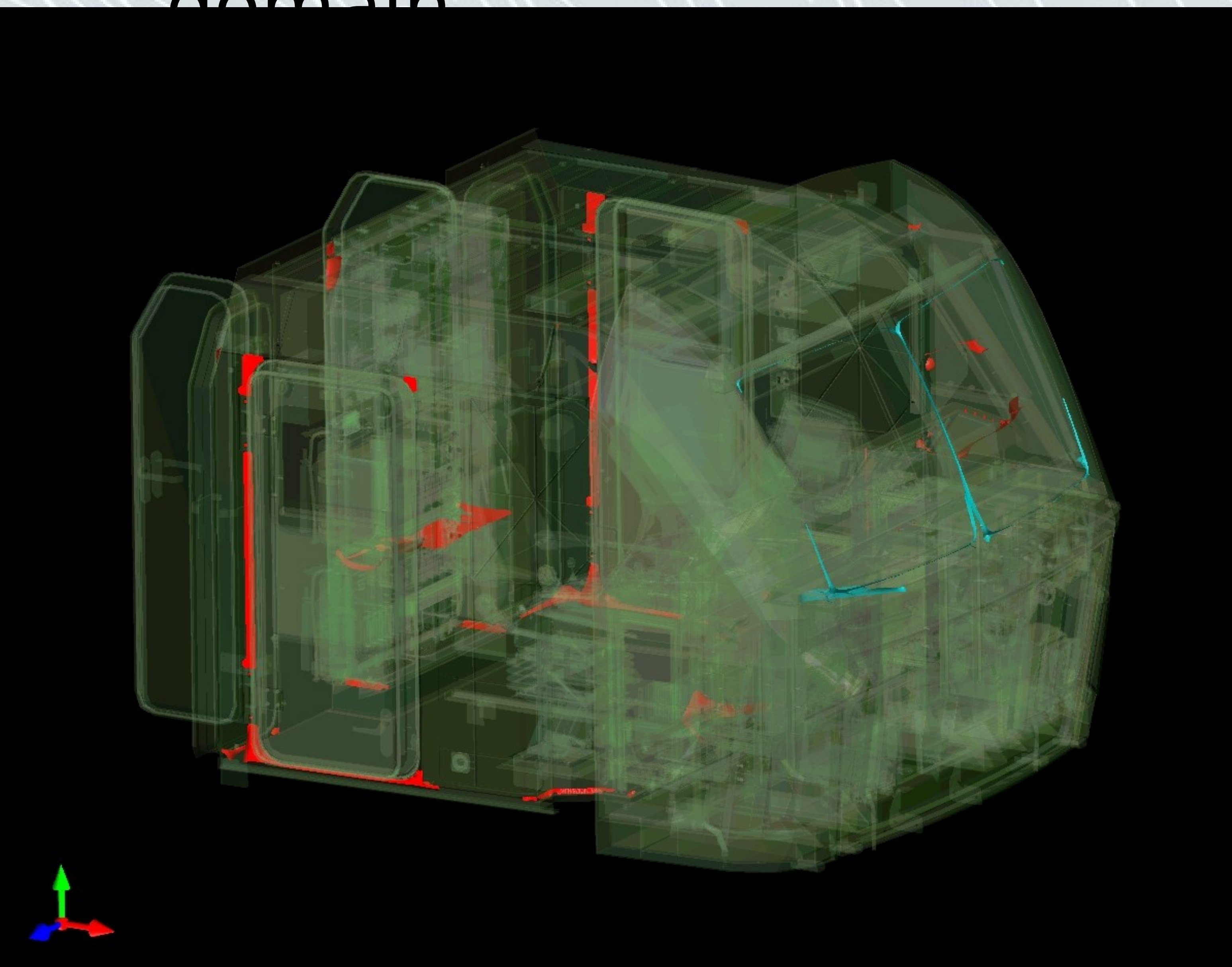
Mission: create mesh for fluid(air) domain

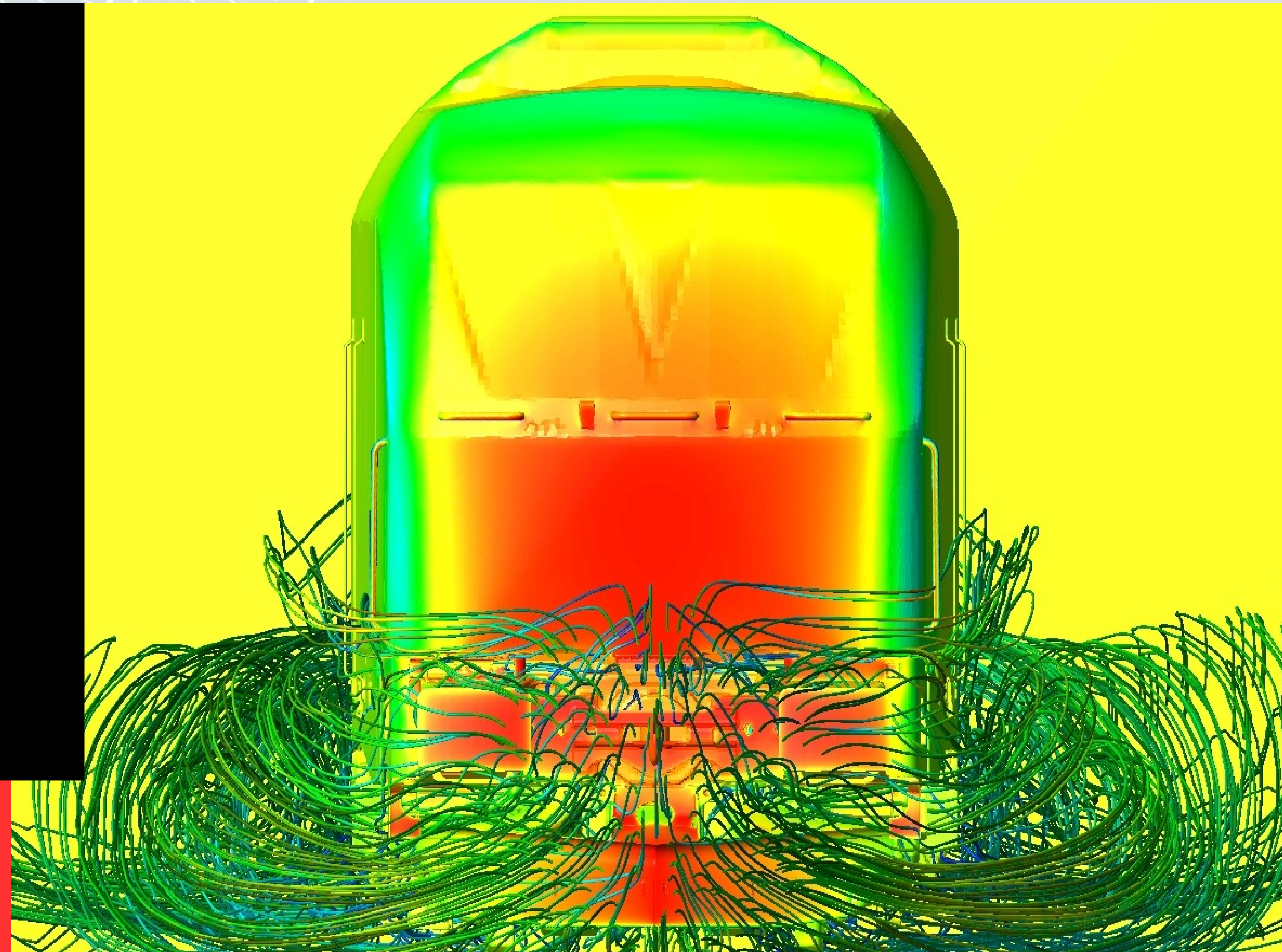


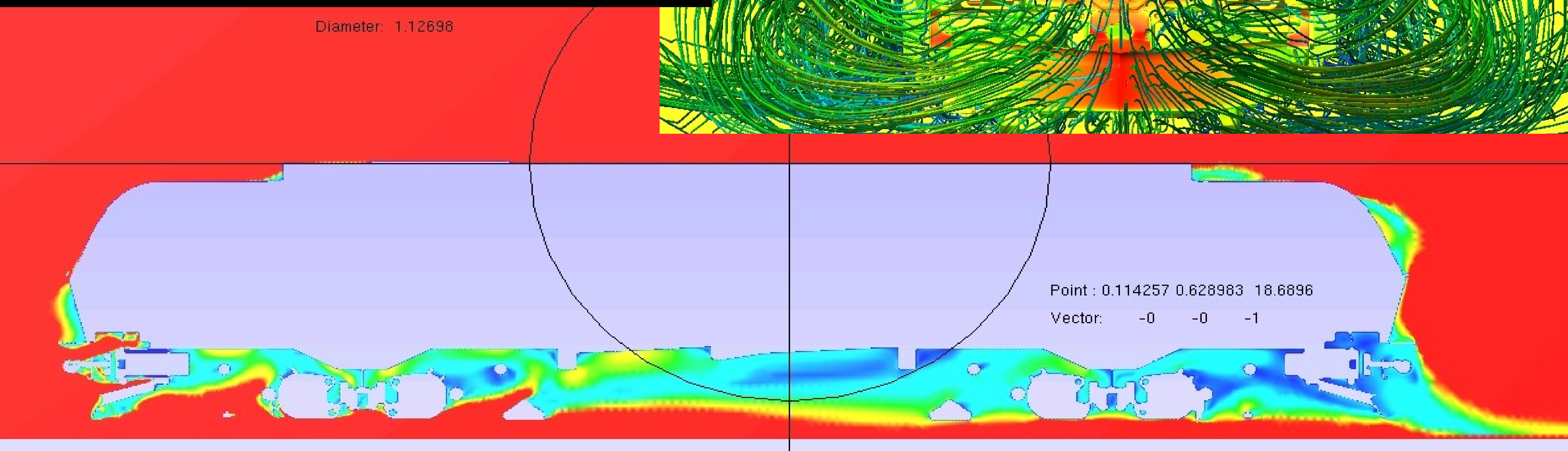




Mission: create mesh for fluid(air) domain







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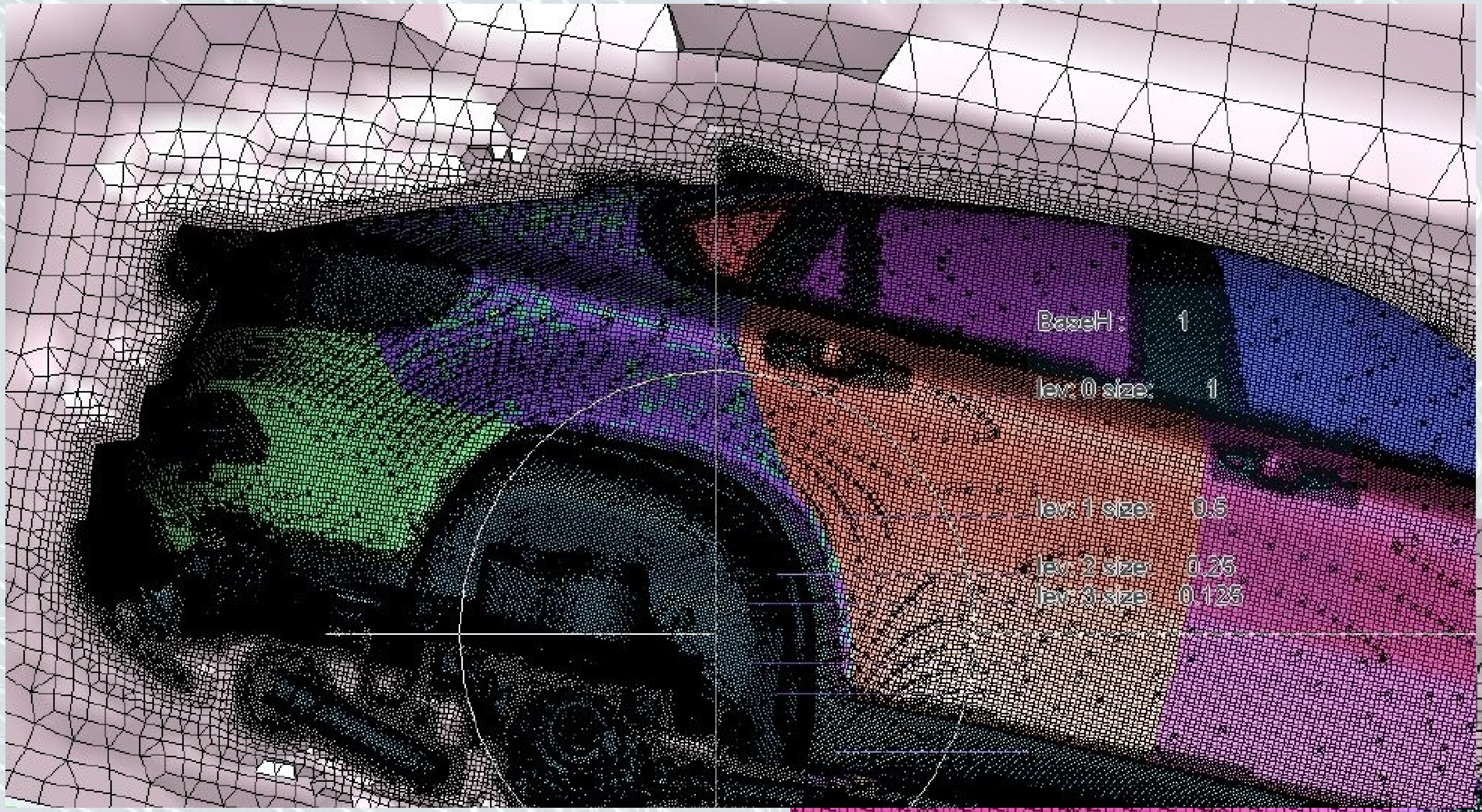
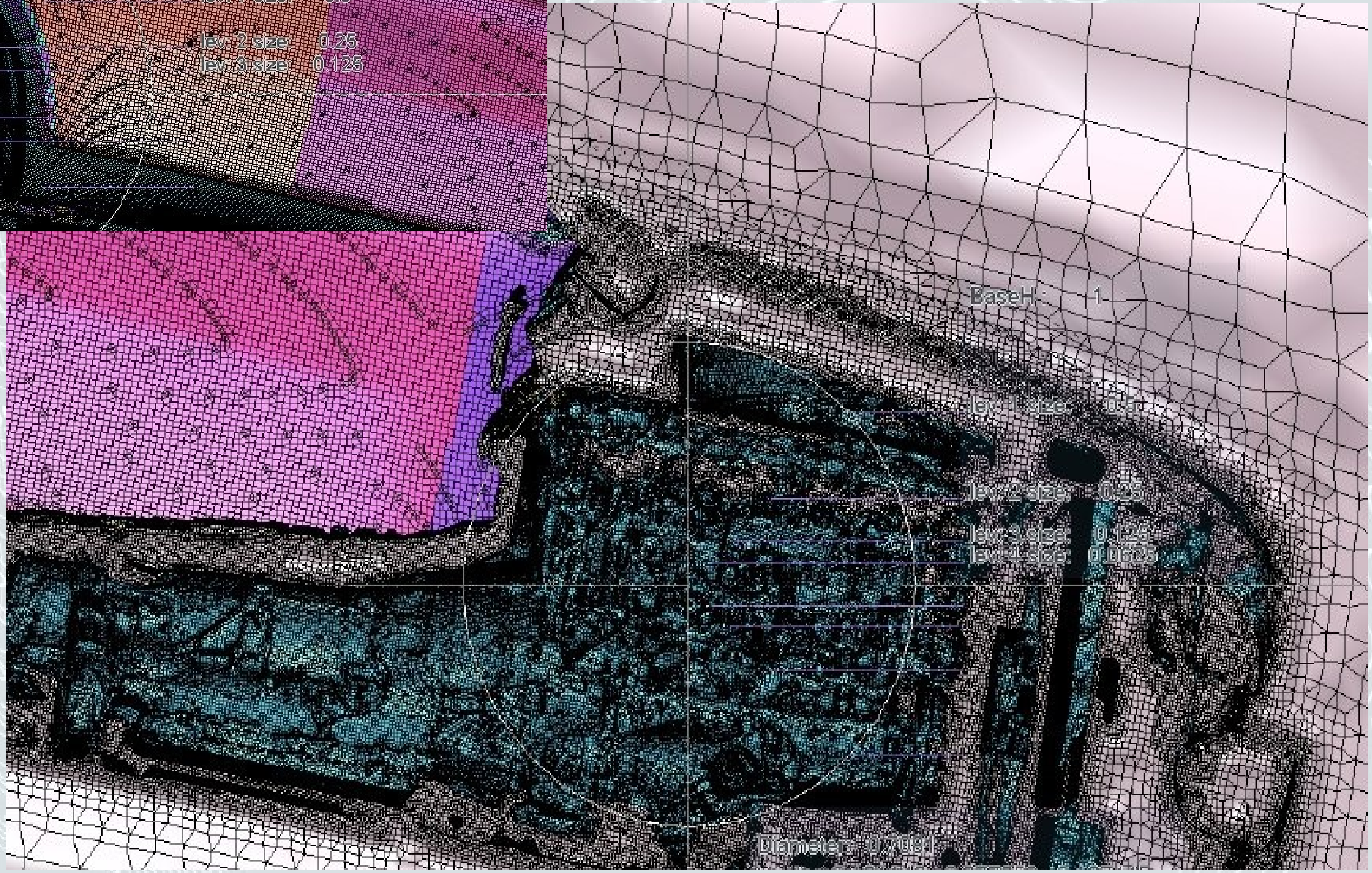
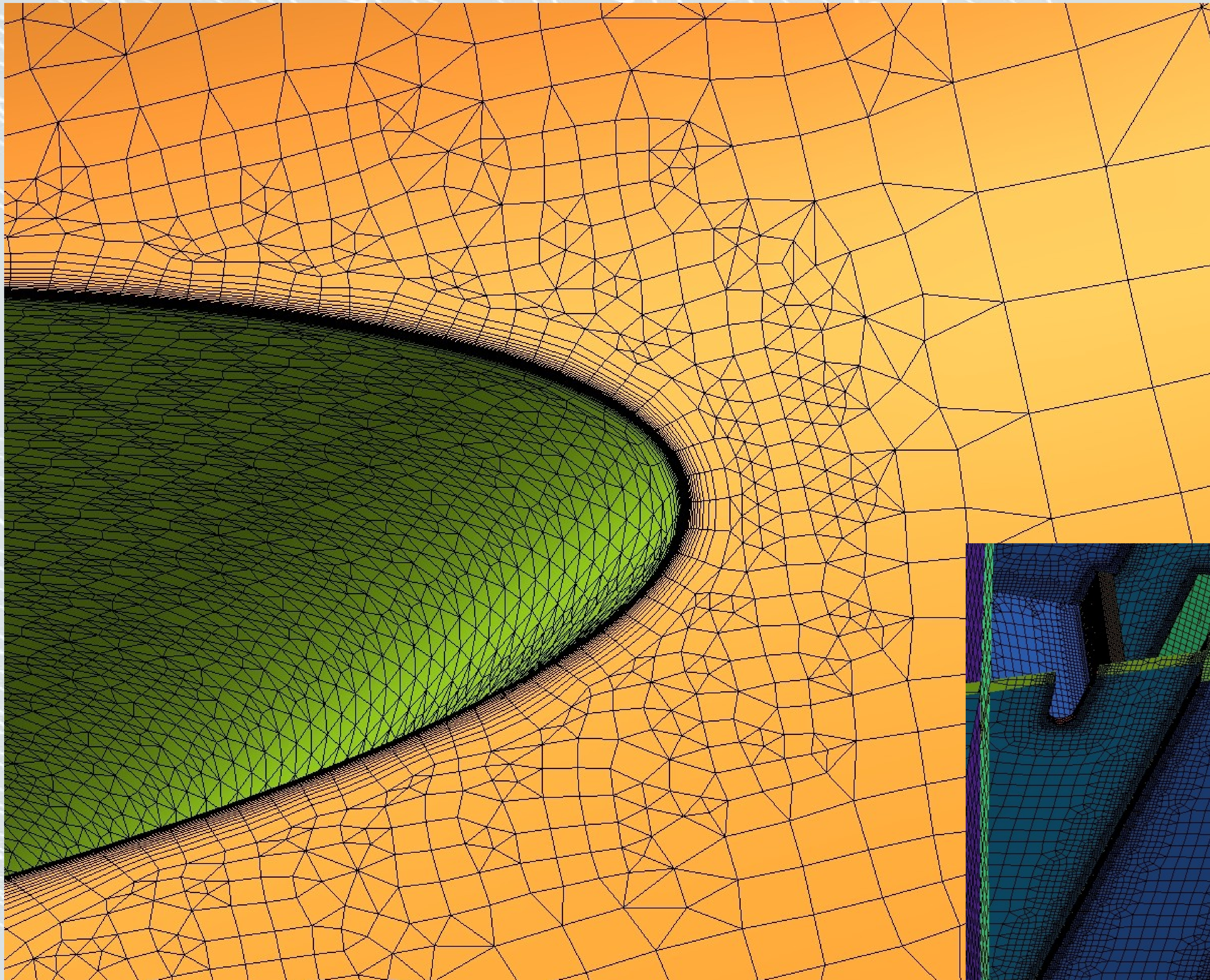
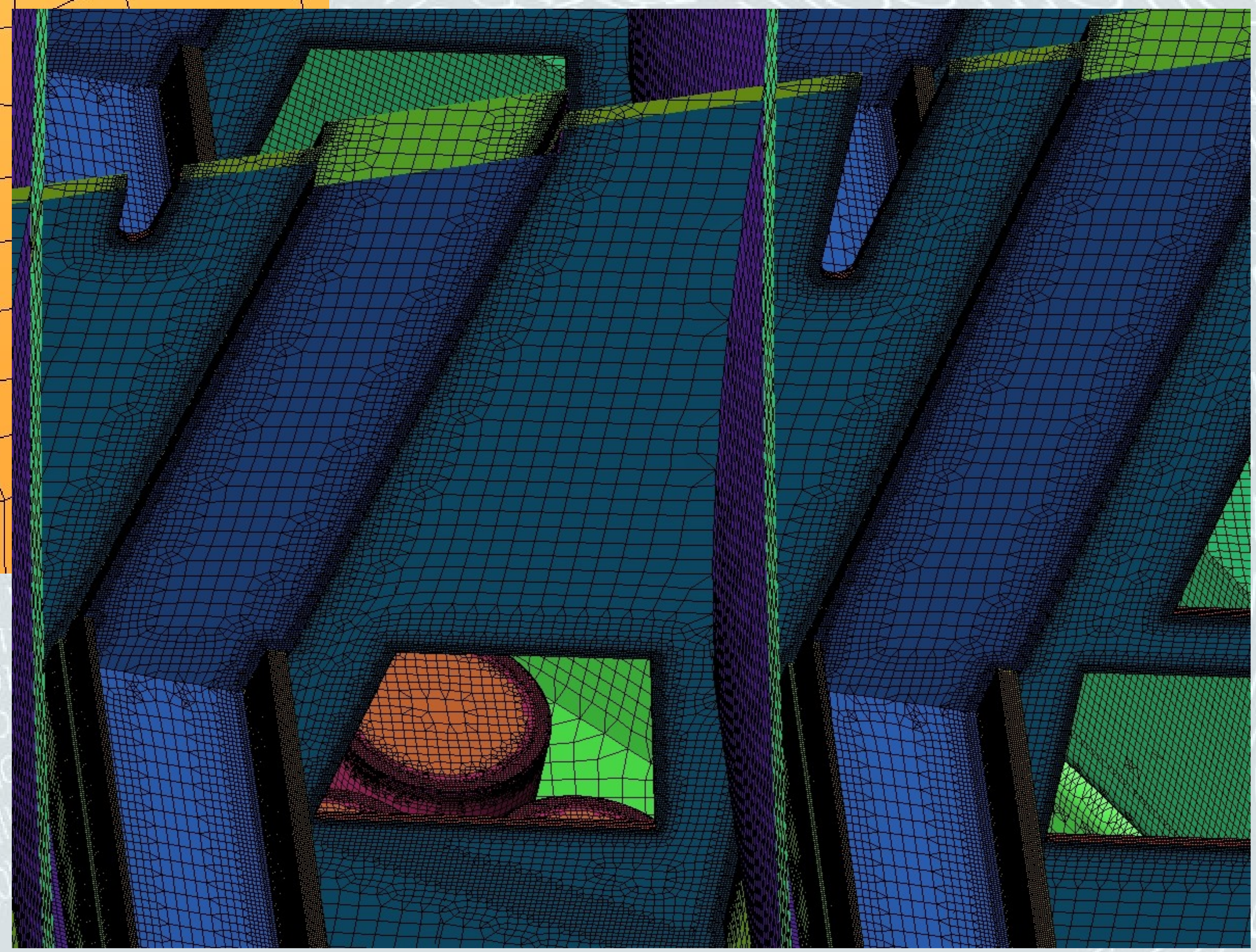




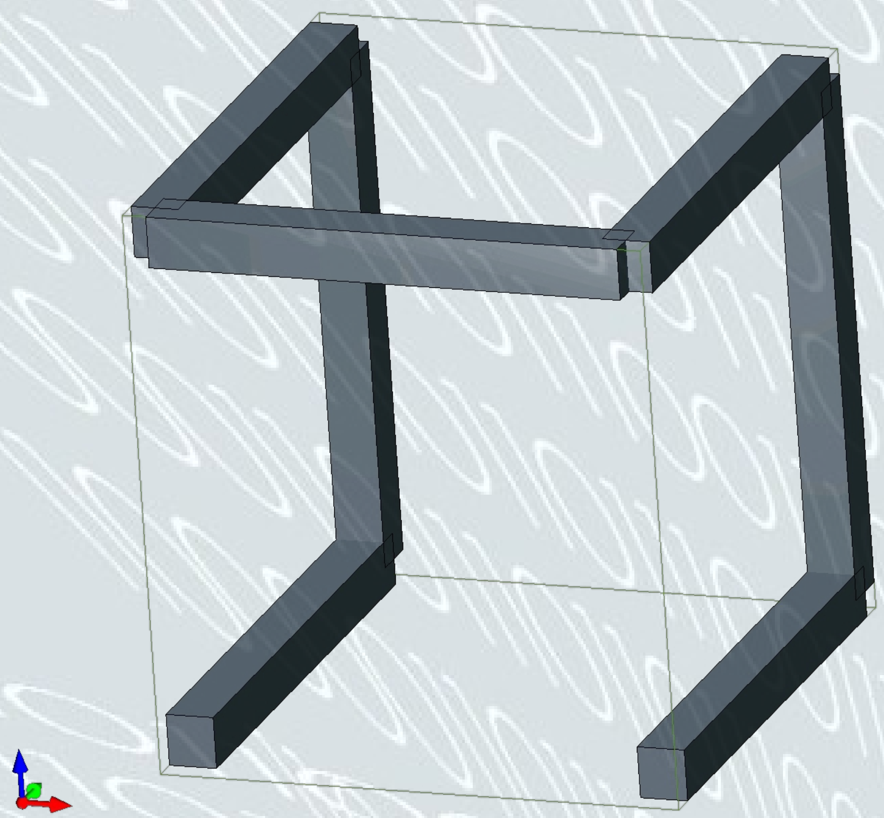
Other good reasons for $-\Delta u=f$



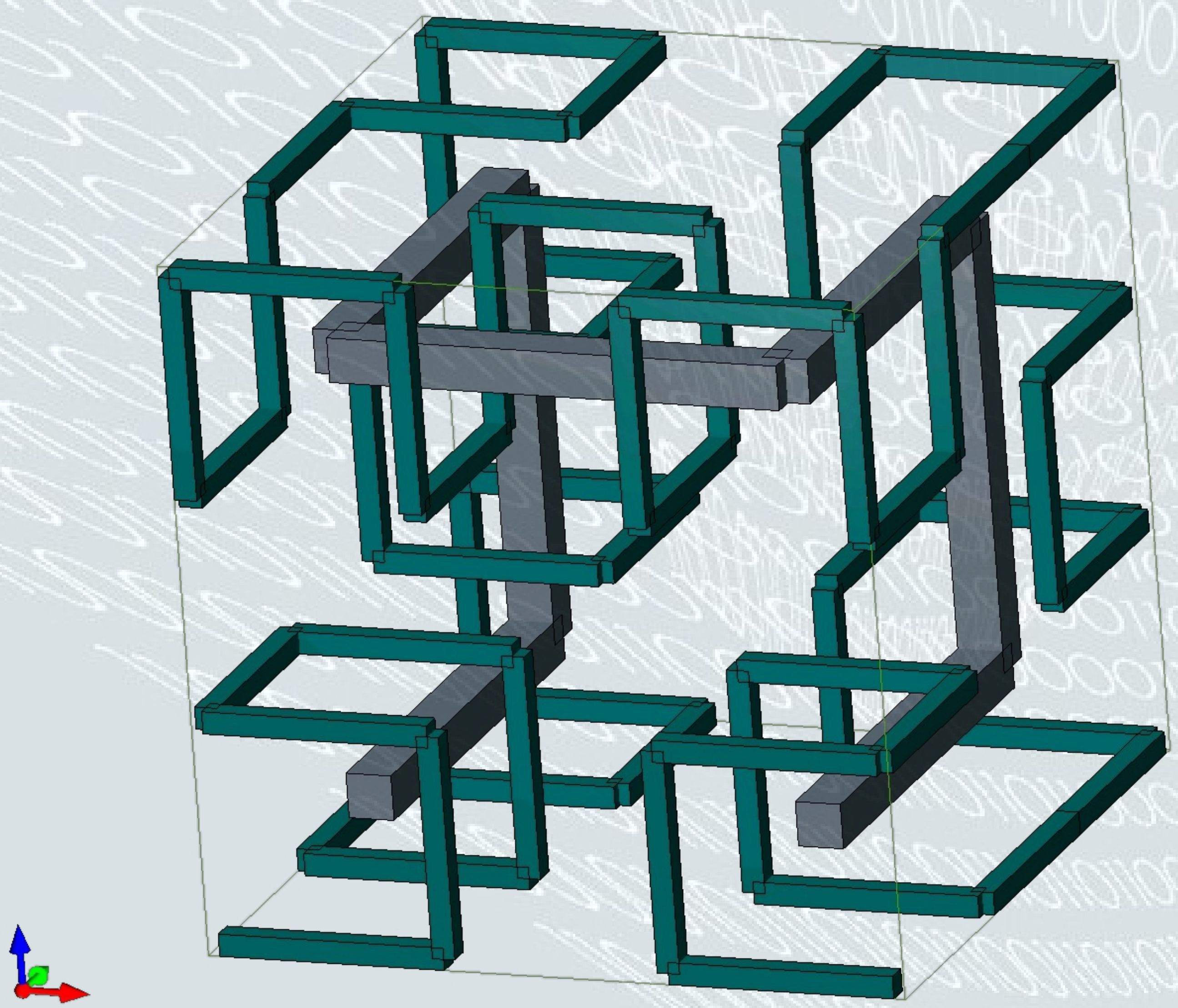
- Replace parallel markin
- Distance calculations
- “geomertry analyzer”
- ...

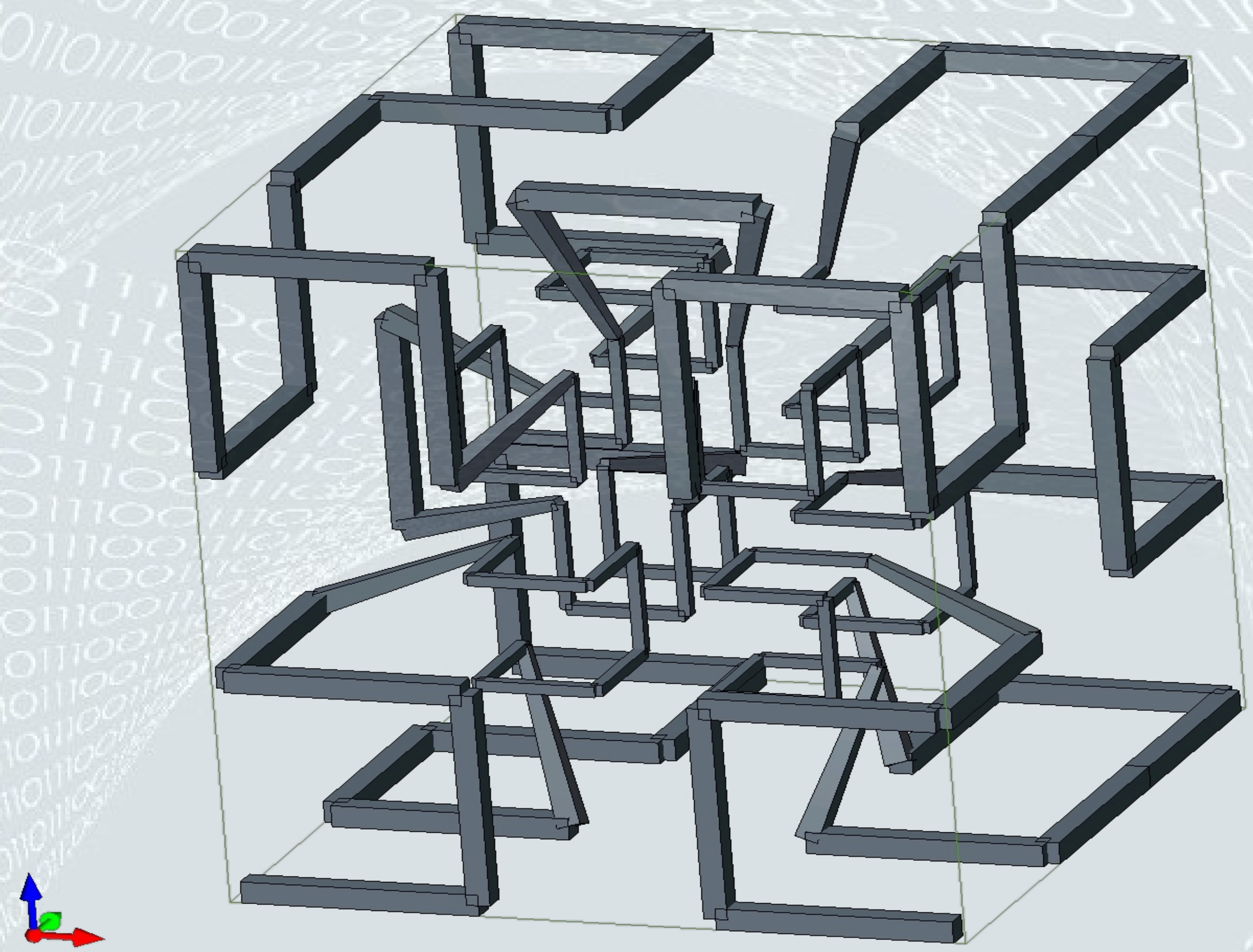


Parallel aspects

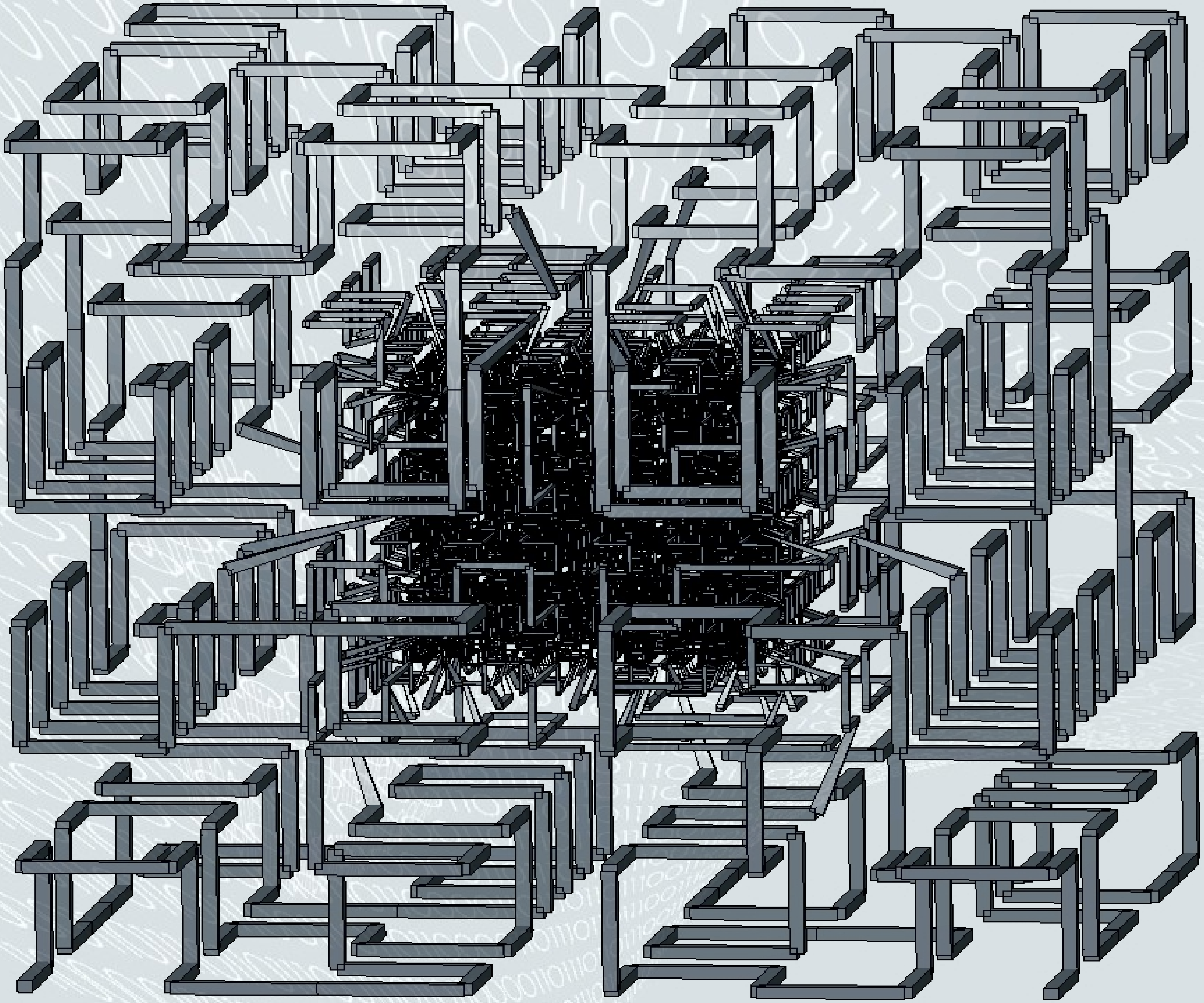


- Space filling curves
- Once you have an octree, SFC is free
- You can use SFC to partition and to load-ballance



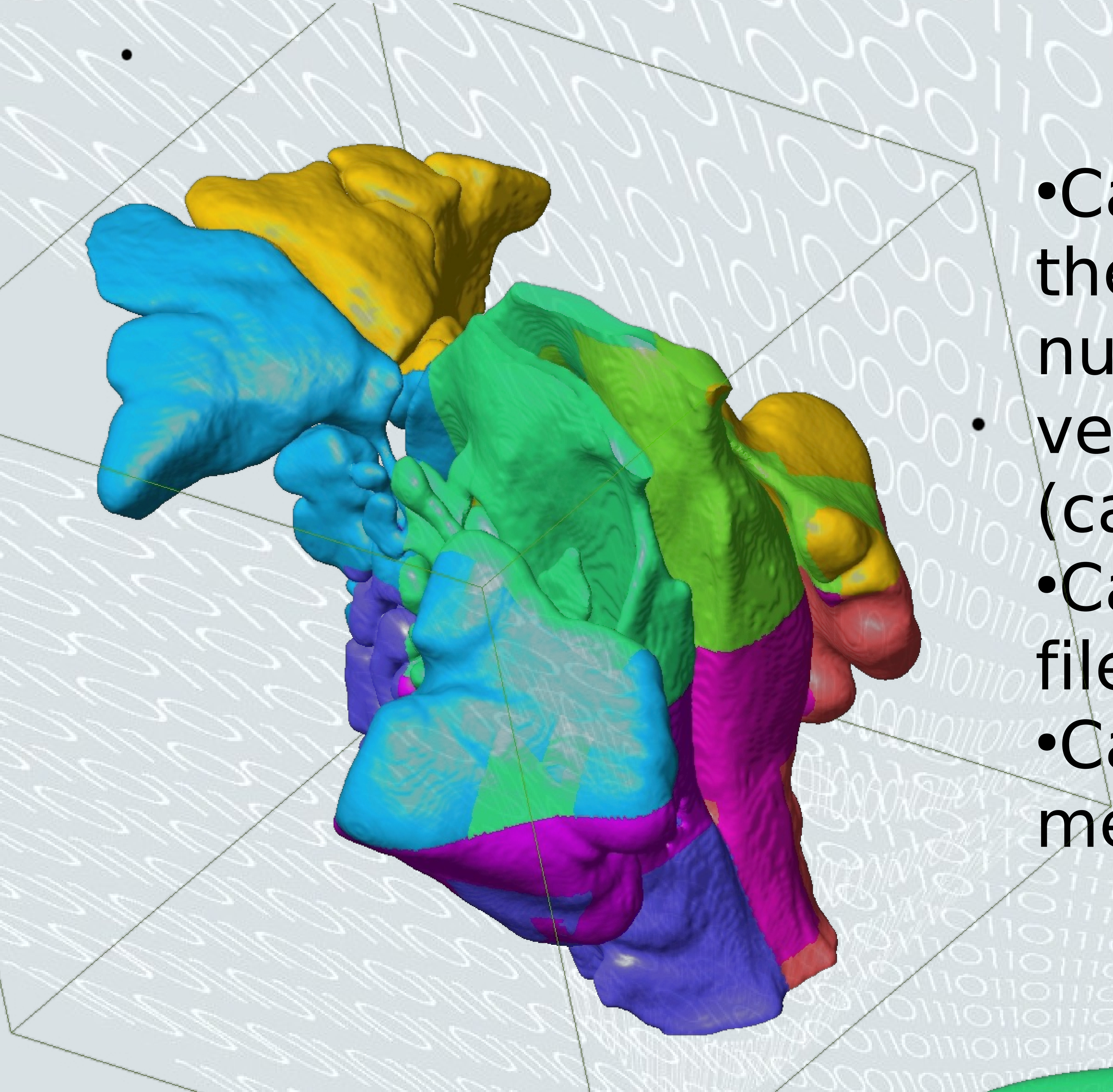


Parallel aspects

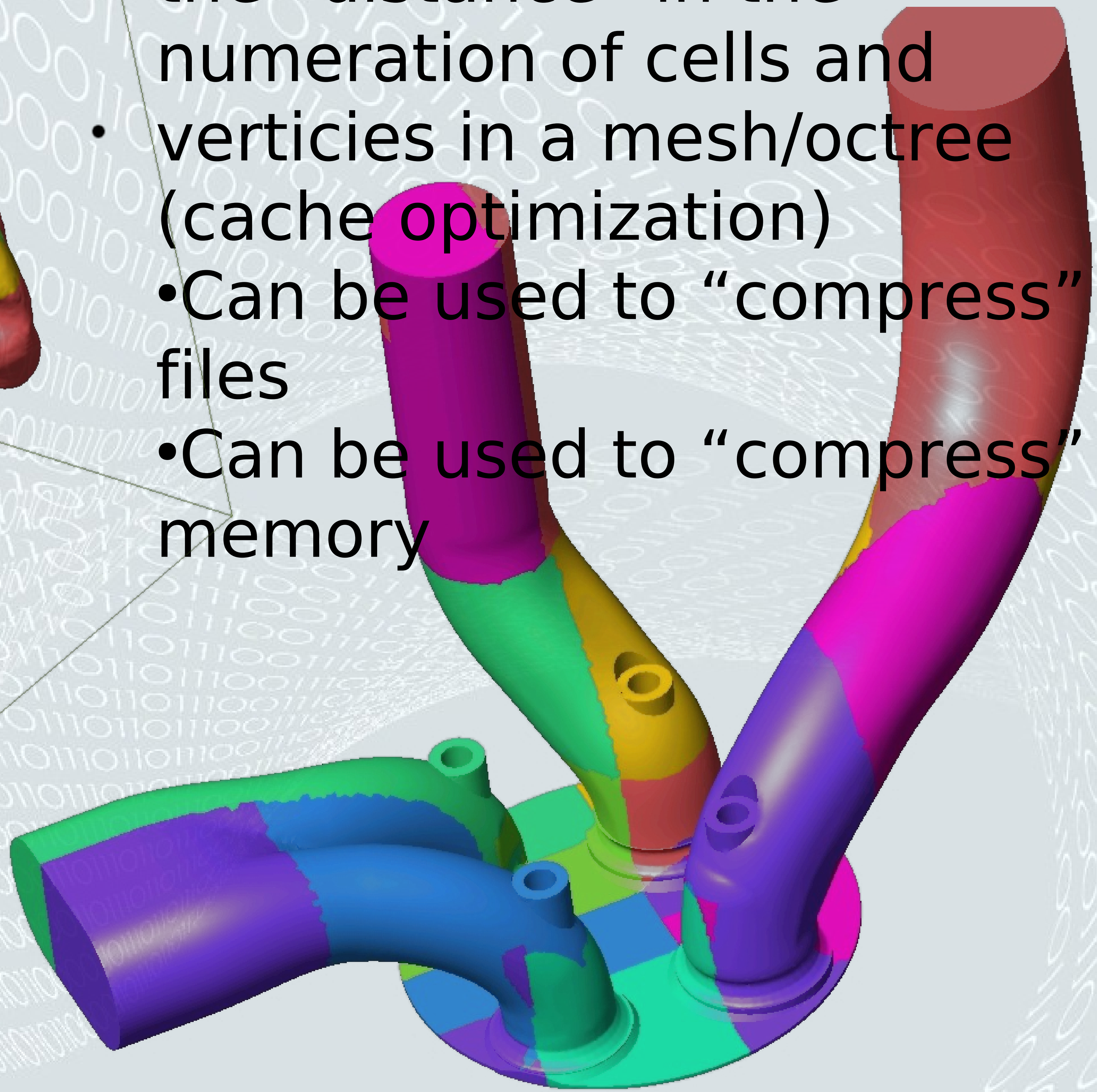


v

Parallel aspects



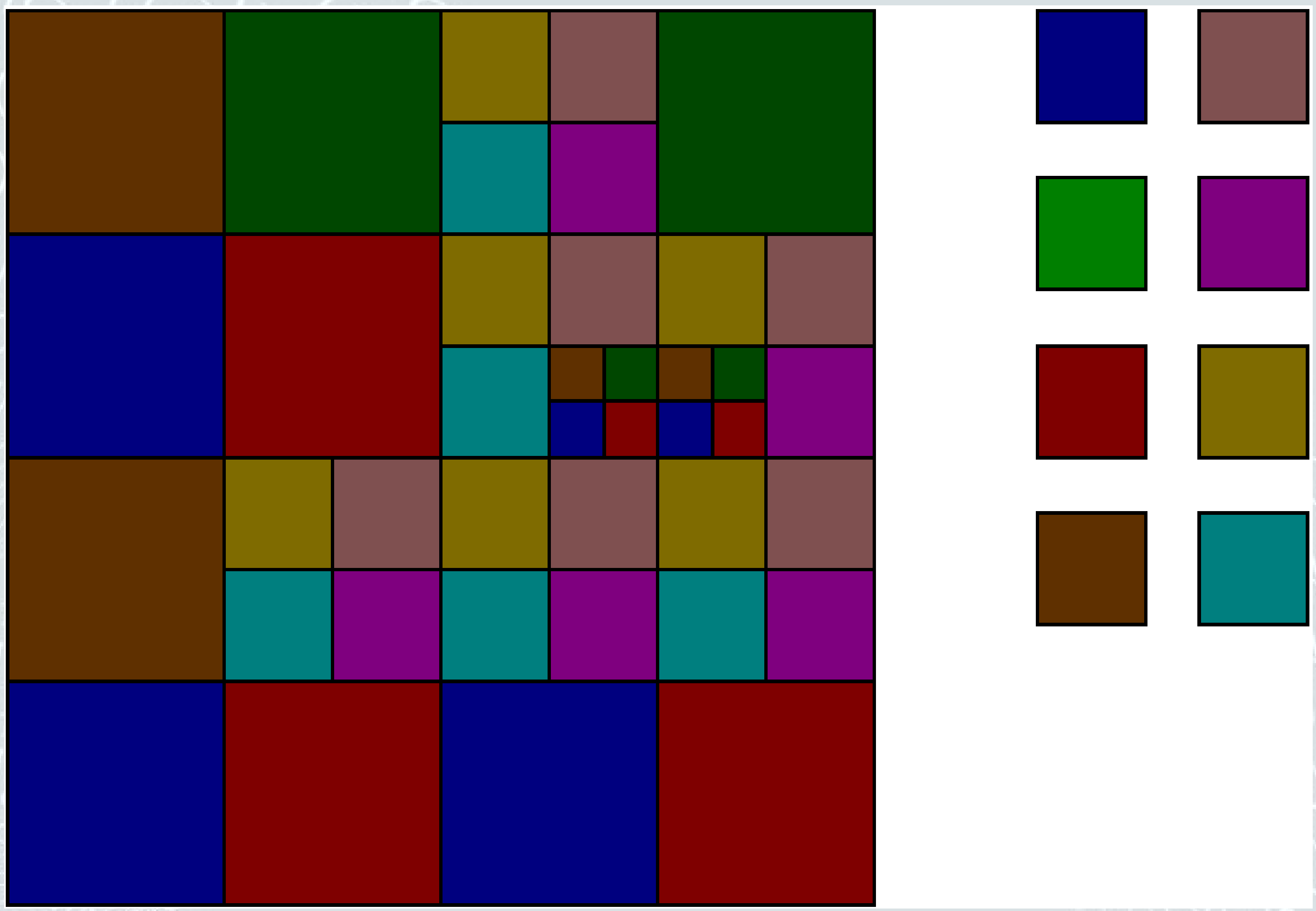
- Can also be used to minimize the “distance” in the numeration of cells and verticies in a mesh/octree (cache optimization)
- Can be used to “compress” files
- Can be used to “compress” memory



Parallel aspects

- Color-mark-tree
- Ensures deterministic parallel behaviour
- No more data races
- Same result in parallel as in serial
- GS-typed smoothers possible
- 2D 8 colors, 3D 16 colors

- Colors can be reduced to 8 in 3D



Summary

- Solving Laplace equation during mesh generation can help a lot
- Efficient solution is possible
- There are a lot more possible extentions possible
- Solution is done via OMP for shared memory machines
- Method to preserve serial result was presented