Preconditioning for large scale micro finite element analysis of 3D poroelasticity

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ABSTRACT

Poroelasticity investigates the deformation of porous media under the influence of the fluid contained within the body. The foundations of the classical theory of poroelasticity has been stated by Biot. In his theory, Biot coupled a porous Hookean solid with Darcy's law in conjunction with continuity to model the fluid passing through the pores of the solid matrix. Poroelasticity is used in a wide range of disciplines. Our interest in poroelasticity originates in osteoporosis, a disease that is a major health problem in developed countries where the risk for an osteoporotic fracture for women above 50 years is about 50%, for men it is about 20%. In this connection, we have developed a solver called ParFE to model linear *elastic* response of realistic bone structures to exterior forces. The code is highly adapted to voxel-based models generated by CT scans. It is used by researchers that focus on bone remodeling which investigates the changes on the bone structure exposed to cyclic load.

A linear *poroelastic* problem requires the simultaneous solution of elasticity equations and Darcy's law along with mass conservation where the coupling between the solid and fluid content is realized. We present a solver for large scale poroelasticity problems considering voxel models on the micro level which will later be used as a tool to analyze bone poroelasticity. The governing equations are discretized using mixed finite elements considering a formulation which uses displacements $\mathbf{u}$, flux $\mathbf{f}$, and pressure p as primary unknowns, respectively. The geometry is modeled with equal size hexagonal elements, so-called voxels, in which the approximations are piecewise trilinear (Q_1) for displacements and piecewise constant (P_0) for the pressure. The flux is approximated by lowest order Raviart-Thomas (RT_0) elements.

This mixed finite element discretization leads to a linear system of equations with a 3-by-3 block structure. All diagonal blocks are symmetric definite. The system as a whole is indefinite.

We solve the linear system by (flexible) GMRES, with block-diagonal and block-triangular preconditioners. A solve with the diagonal blocks of the preconditioner is replaced by a number of AMG V-cycles or by AMG preconditioned CG. We investigate how the iteration count of the outer iteration is affected by the accuracy of the inner iteration.

Our present highly parallel solver, an extension of ParFE, relies heavily on the Trilinos framework which defines distributed vectors and sparse matrices for parallel environments and provides state-of-the-art packages for iterative solvers as well as for preconditioners and partitioning tools. The solver is tested against benchmark problems on artificial domains and realistic bone structures, all of them composed of voxels. We report on the parallel performance observed on the Cray XT-5 at the Swiss National Supercomputing Center (CSCS).