International Workshop on

CONTROL AND OPTIMIZATION OF PDES

October 10–14, 2011

Graz, Bildungshaus Mariatrost

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LOCATION

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DEAR PARTICIPANT,

this workshop aims at bringing together researchers from all over the world working in the area of control of PDEs and PDE-constrained optimization, covering the full spectrum of the field from analysis to numerical realization and applications. The goal is to give you an opportunity to exchange results and ideas on topics at the forefront of current research.

We wish you a fruitful and stimulating time at the workshop, and hope you will enjoy your stay in Graz! If you have any questions, please don't hesitate to contact us.

The organizing commitee

CONTENTS

VENUE | TRANSPORT

The workshop is held at the Bildungshaus Mariatrost, located in a suburb of Graz (about 6 km from the city center, close to the Basilica Mariatrost).

The Bildungshaus can be reached from Graz by taking the street car line 1 towards *Mariatrost* from either Hauptbahnhof (train station), Hauptplatz (city center) or Jakominiplatz. Either exit at the stop Tannhof or the final stop Mariatrost and continue on foot (about 10 minutes, slight slope, or 5 minutes, steep slope, respectively).

From the airport, you can either take a bus (lines 630/631) to Jakominiplatz or the regional train (line S 5) to Hauptbahnhof and continue as above.

A cab will cost about €40 from the airport and €30 from the train station.

If you arrive on **Sunday** or earlier, please be advised that the reception will not be staffed, and your keys will be deposited in a safe. Please contact Melanie Moser ahead of time to obtain the combination.

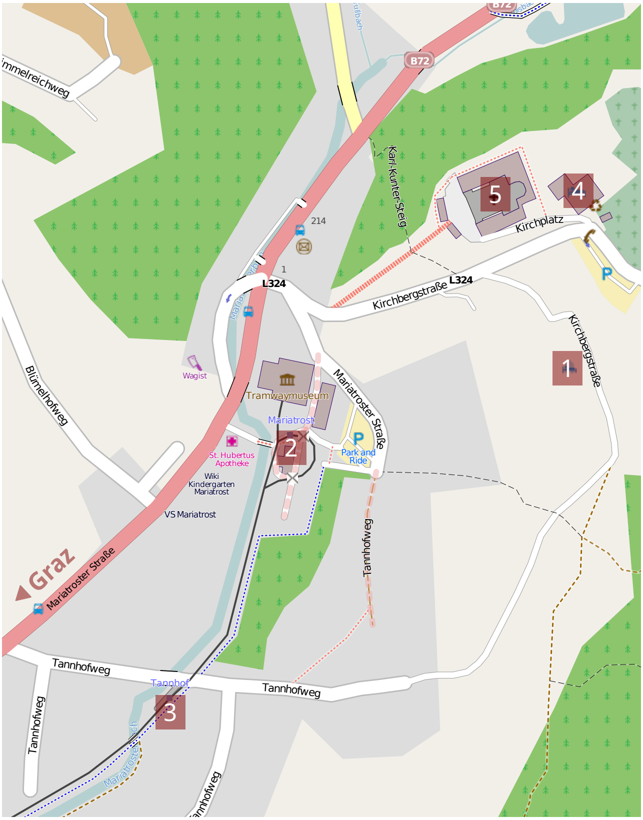
MEALS

For participants staying at the Bildungshaus, there will be a full meal service from Monday morning until Friday noon. All meals will be served in the dining hall in the basement.

- **Breakfast** will be served from 7.30.
- A warm **lunch** buffet will be available between 12.30 and 14.00, which is open to all participants. If you are not staying at the Bildungshaus and would like to join, please let us know in advance.
- **Dinner** will be buffet-style as well (warm or cold), served from 19.00.

Drinks are not included; we ask you to pay for them “Oberwolfach style”.

For participants arriving on **Sunday**, a supper of bread, cold cuts and cheese will be provided starting at 15.00 in the Cafeteria on the first floor (by European count).



1 Bildungshaus

2 Stop Mariatrost

3 Stop Tannhof

4 Hotel
"Zum Kirchenwirt"

5 Basilica Mariatrost

CONFERENCE EQUIPMENT

The conference room is equipped with a **beamer** and a **chalkboard**. A notebook with a PDF and a Powerpoint viewer will be provided. Please bring your presentation on a USB memory stick and transfer it before your session.

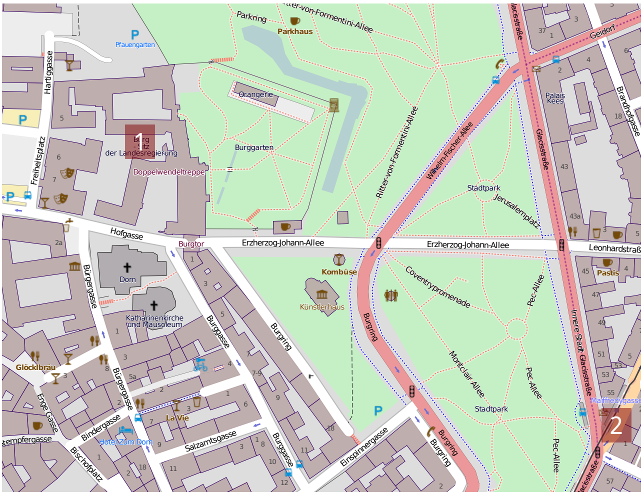
Speakers whose presentations contain animations or videos, or require a specific version of the viewer software, are strongly encouraged to use their own notebook and to test the system well in advance.

SOCIAL PROGRAM

We have planned two social activities:

- On **Tuesday evening**, you are invited to the conference dinner, held at the "Weißer Saal" in the Grazer Burg (Castle Graz, the seat of the Styrian government). We will leave after the coffee break for a short walk through downtown Graz, which will end at Castle Graz in time for the conference dinner.

  After dinner, you have two options: Take a bus that is organized for our group, or make your own way by public transport (e.g., streetcar line 1 from stop Maiffredygasse).



1 Castle Graz

2 Stop Maiffredygasse

- On **Wednesday afternoon**, we will take a tour of "Zotter", a small chocolate manufacturer famous for the quality and variety of its products (all of which you will have a chance to try). The factory is located in a very scenic countryside roughly 50 km east of Graz. We will charter a bus, which will leave at 14.00 and return to Mariatrost around 18.30.

SCHEDULE | MONDAY, OCTOBER 10

08.50 – 09.00	**Opening**	
09.00 – 09.30	**H. T. Banks**	Sensitivity and generalized sensitivity for nonlinear delay systems
09.30 – 10.00	**Fredi Tröltzsch**	A general theorem on error estimates with application to optimal control
10.00 – 10.30	**Volker Schulz**	Accelerating aerodynamic shape optimization by usage of shape Hessians
10.30 – 11.00	**Coffee break**	
11.00 – 11.30	**Xu Zhang**	Stochastic pseudo-differential operators and Calderón-type uniqueness theorem for stochastic PDEs
11.30 – 12.00	**Olivier Glass**	Prescribing the motion of a set of particles in a perfect fluid
12.00 – 12.30	**Roland Herzog**	Optimal control problems in elastoplasticity
12.30 – 15.00	**Lunch**	
15.00 – 15.30	**Asen Dontchev**	Dennis–Moré theorem revisited
15.30 – 16.00	**Winnifried Wollner**	Pointwise convergence of the feasibility violation for Moreau–Yosida regularized optimal control problems
16.00 – 16.30	**Coffee and cake**	
16.30 – 17.00	**Arnd Rösch**	Optimality conditions and error estimates for nonlinear optimal control problems
17.00 – 17.30	**Hans Josef Pesch**	New necessary conditions for elliptic optimal control problems with state constraints
17.30 – 18.00	**Aurora Marica**	Wave propagation in discrete heterogeneous media
19.00	**Dinner**	

SCHEDULE | TUESDAY, OCTOBER 11

09.00 – 09.30	**Gabriel Turinici**	An equilibrium mean field games model of transaction volumes
09.30 – 10.00	**Gerd Wachsmuth**	Optimal control of quasistatic plasticity
10.00 – 10.30	**Hasnaa Zidani**	Hamilton–Jacobi approach in singular domains
10.30 – 11.00	**Coffee break**	
11.00 – 11.30	**Jean-Pierre Puel**	Some controllability results for a system of Schrödinger equations modeling trapped ions
11.30 – 12.00	**Michael Hinze**	Bang-bang control of elliptic PDEs
12.00 – 12.30	**Sérgio Rodrigues**	On the exponential stabilization of a nonstationary solution for Navier–Stokes equations
12.30 – 14.30	**Lunch**	
14.30 – 15.00	**Jun Zou**	Recent theoretical and numerical developments in inverse obstacle scattering
15.00 – 15.30	**Alfio Borzì**	A Fokker–Planck control framework for multidimensional stochastic processes
15.30 – 16.00	**Georg Vossen**	$\mathcal{H}_{2,\alpha}$-norm model reduction for optimal control of PDEs with applications in laser processing
16.00 – 16.30	**Coffee and cake**	
17.00 – 19.00	**Walking tour of downtown Graz**	
19.00	**Conference dinner (Castle Graz)**	

SCHEDULE | WEDNESDAY, OCTOBER 12

09.00 – 09.30	**Maurizio Falcone**	A patchy dynamic programming method for the numerical solution of Hamilton–Jacobi–Bellman equations
09.30 – 10.00	**Anton Schiela**	Optimal control for cancer treatment planning in deep regional hyperthermia
10.00 – 10.30	**Takéo Takahashi**	Controllability of a simplified 1d fluid–structure system
10.30 – 11.00	**Coffee break**	
11.00 – 11.30	**Eduardo Casas**	Approximation of elliptic control problems in measure spaces with sparse solutions
11.30 – 12.00	**Ekkehard Sachs**	Numerical structure preserving POD for calibration of PIDE driven models
12.00 – 12.30	**Irena Lasiecka**	Analysis and control of coupled PDE systems arising in gas flow–structure interactions
12.30 – 14.00	**Lunch**	
14.00 – 18.30	**Excursion (Chocolate manufacturer "Zotter")**	
19.00	**Dinner**	

SCHEDULE | THURSDAY, OCTOBER 13

09.00 – 09.30	**Stefan Ulbrich**	Adaptive multilevel methods for PDE-constrained optimization based on adaptive finite element or reduced order approximations
09.30 – 10.00	**Julien Salomon**	Control through operators in quantum chemistry
10.00 – 10.30	**Michael Hintermüller**	Several approaches for the derivation of stationarity conditions for elliptic MPECs with upper-level control constraints
10.30 – 11.00	**Coffee break**	
11.00 – 11.30	**Jean-Pierre Raymond**	How to estimate the velocity of a fluid flow from pressure measurements?
11.30 – 12.00	**Daniel Wachsmuth**	Time optimal control of the wave equation, its regularization and numerical realization
12.00 – 12.30	**Gregory von Winckel**	A globalized Newton method for the optimal control of multiple interacting fermions
12.30 – 15.00	**Lunch**	
15.00 – 15.30	**Masahiro Yamamoto**	Inverse problems for a system for non-isothermal crystallization of polymers
15.30 – 16.00	**Yvon Maday**	TBA
16.00 – 16.30	**Coffee and cake**	
16.30 – 17.00	**Lionel Rosier**	On the control of the motion of a boat
17.00 – 17.30	**Boris Vexler**	A priori error analysis of the Petrov–Galerkin Crank–Nicolson scheme for parabolic optimal control problems
17.30 – 18.00	**Kazufumi Ito**	Nonsmooth control problems for semilinear control systems
19.00	**Dinner**	

SCHEDULE | FRIDAY, OCTOBER 14

09.00 – 09.30	**Claude Le Bris**	Some (possibly random) multiscale problems which are worth considering for optimization
09.30 – 10.00	**Stefan Volkwein**	POD for coupled nonlinear PDE systems
10.00 – 10.30	**Dominik Meidner**	A priori error estimates for finite element discretizations of parabolic optimization problems with pointwise state constraints in time
10.30 – 11.00	**Coffee break**	
11.00 – 11.30	**Michael Ulbrich**	Differentiability properties of unsteady incompressible Navier–Stokes flow with respect to domain variations
11.30 – 12.00	**Axel Kröner**	Adaptive finite element methods for optimal control of second order hyperbolic equations
12.00 – 12.30	**Marius Tucsnak**	Time optimal control for low Reynolds swimmers
12.30 – 14.00	**Lunch**	
14.00	**Departure**	

Sensitivity and generalized sensitivity for nonlinear delay systems

H. T. Banks, North Carolina State University

Abstract

We consider general systems of the form

$$\text{(1)}\qquad \begin{aligned} &\dot{z}(t) = f(t, z(t), z_t, z(t-\tau_1), \ldots, z(t-\tau_m), \theta) + f_2(t), \quad 0 \le t \le T, \\ &z_0 = \phi, \end{aligned}$$

where $f = f(t, \eta\psi, y_1, \ldots, y_m, \theta) \colon [0,T] \times X \times \mathbb{R}^{nm} \times \Theta \to \mathbb{R}^n$. Here $X = \mathbb{R}^n \times L_2(-r, 0; \mathbb{R}^n)$, $0 < \tau_1 < \cdots < \tau_m = r$, z_t denotes the usual function $z_t(\theta) = z(t+\theta)$, $-r \le \theta \le 0$, and $\phi \in H^1(-r, 0)$. Here the admissible parameter set Θ is a subset of a metric space (possibly infinite dimensional – i.e., some set of functions). For this system we investigate traditional sensitivities

$$\frac{\partial z}{\partial q} = \left(\frac{\partial z}{\partial \theta}, \frac{\partial z}{\partial z_0}, \frac{\partial z}{\partial \tau_i}\right)$$

as well as generalized sensitivity in the sense of Thomseth and Cobelli [7]. Fundamental theoretical results as developed in [1, 2] are given. These include existence and uniqueness of solutions and their derivatives including $\frac{\partial z}{\partial \tau_i}$ and well-posedness of the corresponding sensitivity equations along with convergence of approximations. Computational findings are discussed for examples including Hutchinson's equation (the logistic equation with delay) [3], the Minorsky [4, 5, 6] examples of delayed damping and delayed restoring force in harmonic oscillators (delayed stabilizing systems) as well as recent behavioral models in alcohol therapy [1].

References

[1] H.T. Banks, K. Rehm and K. Sutton, *Inverse problems for nonlinear delay systems*, CRSC-TR10-17, N.C. State University, Raleigh, NC, November, 2010; Methods and Applications of Analysis, 17 (2010), 331–356.

[2] H.T. Banks, D. Robbins and K. Sutton, *Generalized sensitivity functions for delay differential equations*, to appear.

[3] G.E. Hutchinson, *Circular causal systems in ecology*, Ann. N.Y. Acad. Sci., 50 (1948) 221–246.

[4] N. Minorsky, *Self-excited oscillations in dynamical systems possessing retarded actions*, J. Applied Mechanics, 9 (1942), A65–A71.

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A Fokker–Planck control framework for multidimensional stochastic processes

Alfio Borzì, Universität Würzburg

Abstract

An efficient framework for the optimal control of probability density functions (PDF) of multidimensional stochastic processes is presented. This framework is based on the Fokker–Planck equation that governs the time evolution of the PDF of stochastic processes and on tracking objectives of terminal configuration of the desired PDF. The corresponding optimization problems are formulated as a sequence of open-loop optimality systems in a model predictive control strategy. These systems are discretized efficiently by the Chang–Cooper scheme and solved by nonlinear optimization techniques. The effectivenes of the proposed computational framework is validated with a stochastic Lotka–Volterra model and a noised limit cycle model.

Approximation of elliptic control problems in measure spaces with sparse solutions

Eduardo Casas, University of Cantabria

joint work with *Christian Clason* and *Karl Kunisch*

Abstract

This talk deals with the approximation of the optimal control problem

(1) $$\text{(P)} \qquad \min_{u \in \mathcal{M}(\Omega)} J(u) = \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \alpha \|u\|_{\mathcal{M}(\Omega)},$$

where $\alpha > 0$, $y_d \in L^2(\Omega)$ and y is the unique solution to the Dirichlet problem

(2) $$\begin{cases} -\Delta y + c_0 y &= u \quad \text{in } \Omega, \\ y &= 0 \quad \text{on } \Gamma. \end{cases}$$

The controls are taken in the space of real and regular Borel measures $\mathcal{M}(\Omega)$. It has been observed that the use of measures leads to optimal controls which are sparse. This is relevant for many applications in distributed parameter control. Moreover, the support of the optimal control provides information on the optimal placements of control actuators. Formally, the same features can be achieved by using $L^1(\Omega)$ control cost. In this case, however, the optimal control problem is not well-posed in the sense of a possible lack of existence of a minimizer due to the fact that $L^1(\Omega)$ does not allow an appropriate topology for compactness arguments. The focus of the talk is to give an approximation framework which, in spite of the difficulties due to the presence of measures, leads to implementable schemes for which a priori error estimates can be provided. We show that the optimal control measure can be approximated efficiently by a linear combination of Dirac measures. This is important for practical applications because it provides a way of controlling a distributed system by finitely many point actuators, giving information on where they have to be placed.

Dennis–Moré theorem revisited

Asen L. Dontchev, AMS and the University of Michigan

Abstract

We present several generalizations of the Dennis–Moré theorem characterizing superlinear convergence of quasi-Newton methods. Our first characterization does not impose differentiability and employs instead strong metric subregularity of the function involved. Then we give a Dennis–Moré theorem for Newton differentiable mappings. Finally, we present a Dennis–Moré theorem for generalized equations.

A patchy dynamic programming method for the numerical solution of Hamilton–Jacobi–Bellman equations

Maurizio Falcone, Università di Roma "La Sapienza"

joint work with *S. Cacace, E. Cristiani* and *A. Picarelli*

Abstract

The Dynamic Programming approach for partial differential equations offers a good mathematical framework but has always suffered from the high dimension of the dynamical system corresponding to the discretization of the dynamics. Although Reduced-Basis and POD approximation techniques can help reducing the size to a dozen of variable the problem is still very difficult to solve via dynamic programming. In order to overcome this difficulty, we present an approximation method for the solution of Hamilton–Jacobi–Bellman equations which combines a patchy decomposition of the domain and a dynamic programming scheme. The patchy vector fields leading to the decomposition are inspired by a class of piecewise smooth vector fields introduced by Ancona and Bressan to study feedback stabilization problems. Since the subdomains are invariant with respect to the patchy vector fields, we can split the computation of the solution in each sub-domain and use a parallel method opening the way to the solution of HJB equations in high dimension.

Prescribing the motion of a set of particles in a perfect fluid

Olivier Glass, Université Paris-Dauphine

joint work with *Thierry Horsin (Conservatoire National des Arts et Métiers, Paris)*

Abstract

Let Ω a smooth bounded domain in the plane or in the three-dimensional space, and let Σ a nonempty open set of its boundary. We consider the Euler system for perfect incompressible fluids in Ω, controlled by means of boundary conditions on Σ. We study the possibility of driving a certain set of particles in Ω, to a certain target. This differs from the standard notion of controllability which would refer to the possibility of driving the velocity field to a given target at final time.

Acknowledgments

The authors are partially supported by the Agence Nationale de la Recherche, project CISIFS (ANR-09-BLAN-0213-02).

References

[1] O. Glass, T. Horsin, Prescribing the motion of a set of particles in a 3D perfect fluid, preprint 2011.

[2] O. Glass, T. Horsin, Approximate Lagrangian controllability for the 2-D Euler equation. Application to the control of the shape of vortex patches. J. Math. Pures Appl. 93, 2010, pp. 61–90.

Optimal control problems in elastoplasticity

Roland Herzog, Technische Universität Chemnitz

joint work with *Christian Meyer* and *Gerd Wachsmuth*

Abstract

Elastoplastic deformations are the basis of many industrial production techniques, and their optimization is of significant importance. We consider the case of infinitesimal strains as well as linear kinematic hardening. From a mathematical point of view, the quasi-static forward system in the stress-based (so-called dual) form is represented by a time-dependent variational inequality (**VI**) of mixed type [2]: given time-dependent loads ℓ, find generalized stresses $\boldsymbol{\Sigma}(t) \in S^2 = L^2(\Omega; \mathbb{S}^2)$ with $\mathbb{S} = \mathbb{R}^{3\times 3}_{\text{sym}}$ as well as displacements $\boldsymbol{u}(t) \in V = H^1_D(\Omega; \mathbb{R}^3) = \{\boldsymbol{u} \in H^1(\Omega; \mathbb{R}^3) : \boldsymbol{u} = 0 \text{ on } \Gamma_D\}$ which satisfy the constraint $\boldsymbol{\Sigma}(t) \in \mathcal{K}$ and

$$\text{(VI)} \qquad \begin{aligned} \langle A\dot{\boldsymbol{\Sigma}}(t) + B^\star \dot{\boldsymbol{u}}(t),\, \boldsymbol{T} - \boldsymbol{\Sigma}(t)\rangle_{S^2} &\geq 0 \quad \text{for all } \boldsymbol{T} \in \mathcal{K}, \\ B\boldsymbol{\Sigma}(t) &= \ell(t) \quad \text{in } V'. \end{aligned}$$

The bounded linear operators $A : S^2 \to S^2$ and $B : S^2 \to V'$ are associated with the system's energy and the balance of momentum, respectively. The convex set $\mathcal{K}$ of admissible generalized stresses is determined by the von Mises yield condition.

The optimal control of (**VI**) leads to an infinite dimensional MPEC (mathematical program with equilibrium constraints). Due to the non-differentiability of the associated control-to-state map $\ell \mapsto (\boldsymbol{\Sigma}, \boldsymbol{u})$, the derivation of necessary optimality conditions is challenging. The same is true for the re-formulation of (**VI**) as a complementarity system [5]. It is well known that for the resulting MPCC (mathematical program with complementarity constraints) classical constraint qualifications fail to hold. To overcome these difficulties, several competing stationarity concepts have been developed, see for instance [8] for an overview in the finite dimensional case.

In this presentation, we discuss recent advances concerning optimality conditions for (**VI**). For its static counterpart, these comprise B(ouligand)- and C(larke)-stationarity as well a strong stationarity in a particular situation [3, 4]. Using a passage to the limit with respect to the discretization in time, the quasi-static problem can be shown to admit weak stationarity of local minimizers [9, 10].

The classical approach to obtain C-stationarity conditions is by penalization [1, 7]. The following facts however make the pursuit of this program significantly more difficult than it is for control problems of obstacle type: the admissible set $\mathcal{K}$ is characterized by a pointwise *nonlinear* constraint function; the differentiability of the associated Nemytzki operator appearing in regularized formulations is thus a nontrivial result and it requires recent regularity results for *quasi-linear* elasticity systems [6]. The proof of B- and strong stationarity conditions is based in part on new ideas for infinite dimensional MPCCs [4].

Acknowledgments

This work was supported by a DFG grant within the Priority Program SPP 1253 (*Optimization with Partial Differential Equations*), which is gratefully acknowledged.

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Several approaches for the derivation of stationarity conditions for elliptic MPECs with upper-level control constraints

Michael Hintermüller, Humboldt-Universität Berlin

Abstract

The derivation of multiplier-based optimality conditions for elliptic mathematical programs with equilibrium constraints (MPEC) is essential for the characterization of solutions and development of numerical methods. Though much can be said for broad classes of elliptic MPECs in both polyhedric and non-polyhedric settings, the calculation becomes significantly more complicated when additional constraints are imposed on the control. In this paper we develop three derivation methods for constrained MPEC problems: via concepts from variational analysis, via penalization of the control constraints, and via penalization of the lower-level problem with the subsequent regularization of the resulting nonsmoothness. The developed methods and obtained results are then compared and contrasted.

Bang-bang control of elliptic PDEs

Michael Hinze, Universität Hamburg

joint work with *Klaus Deckelnick* and *Nicolaus von Daniels*

Abstract

We apply variational discretization to elliptic optimal control problems with bang-bang controls. We propose a numerical scheme and prove error estimates for the resulting numerical solutions. We also present a numerical example which supports our analytical findings.

Nonsmooth control problems for semilinear control systems

Kazufumi Ito, North Carolina State University

Abstract

We develop Lagrange multiplier rules for abstract optimization problems with mixed smooth and convex terms in the cost, with smooth equality constrained and convex inequality constraints. Applications are given to L^1 and L^∞-minimum norm control problems and time optimal control for a general class of optimal control problems.

Adaptive finite element methods for optimal control of second order hyperbolic equations

Axel Kröner, Technische Universität München

Abstract

In this talk a posteriori error estimates for space-time finite element discretizations for optimal control of second order hyperbolic equations are considered using the dual weighted residual method (DWR). We transfer methods developed in [1] for optimal control of parabolic equations to optimal control of hyperbolic equations; cf. [2]. The provided error estimator separates the influences of different parts of the discretization (time, space, and control discretization). This allows us to set up an adaptive algorithm which improves the accuracy of the computed solutions by construction of locally refined meshes. Numerical examples are presented.

References

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Analysis and control of coupled PDE systems arising in gas flow–structure interactions

Irena Lasiecka, University of Virginia

joint work with *Igor Chueshov* and *Justin Webster*

Abstract

We shall consider a model of flow–structure interaction which consists of a perturbed wave equation defined on unbounded domain and coupled with a nonlinear plate. The interaction between two media takes place on the edge of the plate with the dissipation occurring in a small layer near the edge of the plate. The flow equation has no damping except for natural dispersive effects. We shall consider both subsonic and supersonic case. It is known that in the latter case the static problem looses ellipticity.

Questions such as existence and uniqueness of finite energy solutions will be addressed first. The final goal is to determine geometric conditions for the configuration which would lead to existence of global attractors capturing solutions near the structure (wing of the airplane). One challenge is then to place a controller localized at such interface for the purpose of achieving a desired performance of the overall coupled system. The presence of retarded potential in the underlying PDE arises as a natural effect of the coupling with the flow. This leads to interesting mathematical questions such as the analysis of short and long time behavior of the combined PDEs and their interaction via interface. This lecture will present several new developments in this area and will also underscore open questions.

Some (possibly random) multiscale problems which are worth considering for optimization

Claude Le Bris, École des Ponts ParisTech

Abstract

We review some problems in multiscale materials science which are possibly random in nature and which could be considered as challenging optimization/inverse problems. We present a toolbox that allow the (direct) problems to be simulated very quickly and thus that could be a building brick of a full approach in inverse problems.

Wave propagation in discrete heterogeneous media

Aurora Marica, Basque Center for Applied Mathematics (BCAM)

Abstract

It is well-known that the continuous wave equation has the observability property, in the sense that, for a sufficiently large time interval, the total energy of waves can be estimated by measurements done in the observability region during the whole time interval. The observability time is the one needed by all solutions to touch the observability zone by traveling along characteristics.

However, when replacing the continuous equation by a numerical scheme, for the most part of the numerical approximations on uniform meshes, the observability inequality does not hold uniformly with respect to the mesh size. The reason for this singular behaviour is the existence of wave numbers for which the so-called group velocity, i.e. the gradient of the Fourier symbol of the discrete Laplacian, vanishes. For a global picture of the pathologies generated by high frequency spurious solutions and their remedies using Fourier truncation methods or the bi-grid filtering tecniques, see the review paper [1] for simple schemes like finite differences or linear finite elements and [2] for more complex schemes like the discontinuous Galerkin ones.

Our aim is to tackle the observability problem in the case of numerical methods on non-uniform meshes. More precisely, we will give a meaning to the notion of rays of Geometric Optics in the 1-d and 2-d simple space domains (1-d intervals, squares, the whole Euclidian space) when the non-uniform mesh $y = g(x)$ is a transform of the uniform one x. More precisely, using pseudo-differential calculus techniques (for example the homogenized limit of Wigner transforms, cf. [3]), we deduce the Hamiltonian system verified by the rays of Geometric Optics.

References

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A priori error estimates for finite element discretizations of parabolic optimization problems with pointwise state constraints in time

Dominik Meidner, Technische Universität München

joint work with *Rolf Rannacher* and *Boris Vexler*

Abstract

We present an a priori error analysis for the finite element discretization of an optimal control problem governed by a linear parabolic PDE in the presence of pointwise constraints on the distributed control variable q given as

$$q_a \leq q(t,x) \leq q_b \text{ a. e. in } (0,T) \times \Omega$$

and constraints on the state variable u which are formulated pointwise in time in the following sense:

$$G(u(t,\cdot)) \leq b \text{ in } [0,T] \qquad \text{with} \qquad G(v) := \int_\Omega v(x)\omega(x)\,dx,\ \omega \in L^2(\Omega).$$

For the numerical solution of this optimization problem, we discretize the state variable u in time by discontinuous and in space by continuous Galerkin finite element methods. The control variable q is discretized cellwise constant in time and space.

Under reasonable assumptions on the problem data, we prove the following bound for the error between the continuous optimal solution $\overline{q}$ and its discrete counterpart $\overline{q}_\sigma$:

$$\|\overline{q} - \overline{q}_\sigma\|_{L^2(0,T;L^2(\Omega))} \leq \frac{C}{\sqrt{\alpha}}\left(\ln\frac{T}{k}\right)^{\frac{1}{2}}\left\{k^{\frac{1}{2}} + h\right\}.$$

The derived estimate is illustrated by numerical results.

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New necessary conditions for elliptic optimal control problems with state constraints

Hans Josef Pesch, Universität Bayreuth

joint work with *Simon Bechmann, Michael Frey,* and *Armin Rund*

Abstract

Based on two different reformulations of the state constraints and a hypothesis on the structure of the active set, new necessary conditions for linear-quadratic elliptic optimal control problems with distributed controls are obtained which exhibit higher regularity of the multipliers associated with the state constraint. Moreover, we obtain also new jump and sign conditions. Measures are no longer an issue. For details see [1]. The two investigated approaches mimic the well-known Bryson-Denham-Dreyfus indirect adjoining method [3] which is the preferred ansatz in solving state constrained optimal control problems with ordinary differential equations numerically. Mathematically the reformulations lead to a new kind of set optimal control problem, where the active set of the state constraint, resp. the interface between the inactive and the active set are to be determined as part of the solution. Various formulations of this type of optimization problem as bilevel optimization problems are discussed which also include shape-optimization. In the end, these conditions can be formulated as free boundary value problems. Numerical results will demonstrate the performance of the resulting numerical method [2].

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Some controllability results for a system of Schrödinger equations modeling trapped ions

Jean-Pierre Puel, Université de Versailles Saint-Quentin-en-Yvelines

Abstract

In order to create quantum logic gates like the phase gate or the C-Not gate, physicists use ions trapped in an electromagnetic cavity (qubits) controlled by (three) electromagnetic waves. Each trapped ion is a two level system, the state of which correspond to an information. The problem is then to design controls which enable to drive the system from an initial state (input) to another desired final state (output). On the mathematical pojnt of view this corresponds to controlling a coupled system of Schrödinger equations. For the case of one qubit, some results of approximate controllability have been given in Ervedoza–Puel (Annales IHP, 2009) using an approximate simplified system (Law–Eberly system) which can be excatly controlled. For the case of two qubits, the situation is much more complex. In a work in progress with S.Ervedoza, we have been able to derive the corresponding "simplified" system and prove that it actually approaches the original one. But here the "simplified" system is not easy to control. We know how to obtain some configurations in a controlled time. Together with S. Ervedoza and M. Mirrahimi, we have a strategy to obtain all possible configurations but this is still not completely proved at the moment. We will present the model, the results and open questions.

How to estimate the velocity of a fluid flow from pressure measurements?

Jean-Pierre Raymond, Université Paul Sabatier

Abstract

We consider fluid flows governed by the Navier–Stokes equations and we are interested in the stabilization of a flow about an unstable stationary solution in the case of partial information. This means that we have some measurements and that we look for a control expressed in terms of an estimate of the velocity of the flow.

In the case of a control acting in a Dirichlet boundary condition and when the observation is expressed in terms of the stress tensor or of the pressure at the boundary, the observation involves the derivative of the control at the boundary. This happens not only for the continuous system but also for discrete models obtained by finite element methods. This leads to unsual filtering and control problems. We shall present a new appraoch for determining a feedback control law and for estimating the velocity of the flow.

This work is in collaboration with several colleagues and post-docs in Toulouse, J.-M. Buchot, L. Thevenet, M. Fournié, P. A. Nguyen, J. Weller, J. Tiago.

On the exponential stabilization of a nonstationary solution for Navier–Stokes equations

Sérgio S. Rodrigues, RICAM / Universidade Técnica de Lisboa

joint work with *Viorel Barbu* and *Armen Shirikyan*

Abstract

We consider the Navier–Stokes system in a bounded domain with a smooth boundary. Given a sufficiently regular global solution, we construct a finite-dimensional feedback control that is supported by a given open set and stabilizes the linearized equation. The proof of this fact is based on a truncated observability inequality, the regularizing property for the linearized equation, and some standard techniques of the optimal control theory. We then show that the control constructed for the linear problem stabilizes locally also the full Navier–Stokes system.

Keywords: Navier–Stokes system, exponential stabilization, feedback control

Acknowledgments

This work was done during my stay in the University of Cergy-Pontoise in France and I thank the supporting postdoc fellowship given to me by the CNRS-France.

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Optimality conditions and error estimates for nonlinear optimal control problems

Arnd Rösch, Universität Duisburg-Essen

joint work with *Martin Naß* and *Daniel Wachsmuth*

Abstract

Discretization or regularization error estimates for nonlinear optimal control problems are usually derived by a specific technique: An auxiliary problem is investigated in a small neighborhood of the optimal solution. Then convergence of the solutions of the discretized (or regularized) problems to the solution of the original problem is shown. Consequently, the additional constraint is inactive if the discretization (or regularization) parameter is small. Second-order sufficient optimality conditions are needed to show the main step.

This technique has several drawbacks. The exact solution of the original problem is needed to check the second-order sufficient optimality condition. This solution is of course not available. For problems with state constrainsts one needs in addition the uniqueness of dual variables. This assumption cannot be checked, too. Moreover, there is no information how small the parameter must be that the additional constraint is inactive. Consequently, the information given by that theory give only an indication for the approximation behavior.

We will propose a different way. We formulate conditions for the numerical solution which allow to derive approximation error estimates.

Acknowledgments

This work was partially supported by the DFG priority program 1253.

On the control of the motion of a boat

Lionel Rosier, Université Henri Poincaré

joint work with *Olivier Glass*

Abstract

In this paper we study the control of the motion of a boat, viewed as a rigid body S with one axis of symmetry, which is surrounded by an inviscid incompressible fluid filling $\mathbb{R}^2 \setminus S$. We take as control input the flow of the fluid through a part of the boundary of the boat. We prove that the position, orientation, and velocity of the boat are locally controllable with a bidimensional control input, even if the flow displays some vorticity.

Numerical structure preserving POD for calibration of PIDE driven models

Ekkehard Sachs, Universität Trier

Abstract

We consider as an example a partial integrodifferential equation (PIDE) which occurs in various areas of application such as pricing models for derivatives, material sciences and biological models. The calibration of these models to real data can be viewed as an optimization problem with PIDEs. Using proper orthogonal decomposition (POD) one can gain a considerable speed-up for the numerical solution of this problem. However, it turns out that a model reduction based on the mathematical equations describing the physical model is not sufficient for the design of a successful reduced order model. Moreover, the numerical scheme to solve these models has a considerable influence on the success of this method. We give several examples from finance where this observation is numerically substantiated.

Control through operators in quantum chemistry

Julien Salomon, Université Paris-Dauphine

joint work with *Gabriel Turinici* and *Herschel Rabitz*

Abstract

In this talk, we present a numerical approach to tackle the problem of finding operators that lead a trajectory to a given goal at a fixed time. Usually, control problems in quantum chemistry consist in finding relevant laser fields features so that the trajectory of a given quantum system satisfies some constrains. Such issues are well documented both at theoretical and practical levels. On the contrary, we fix in our work the "control", i.e. the laser field, and pay attention to the operators involved in the dynamic. More precisely, we are interested in computing numerically the caracteristics of the operators that achieve some control goals. In this way, we present a Newton-like algorithm, based on a particular discretization of the problem that show good results on some generic cases. This topic is also related to operator identification and inverse problems in quantum mechanics.

Optimal control for cancer treatment planning in deep regional hyperthermia

Anton Schiela, Zuse Institute Berlin

joint work with *Peter Deuflhard* and *Martin Weiser*

Abstract

The aim of hyperthermia treatment as a cancer therapy is to damage deeply seated tumors by heat. This can be done regionally by a microwave applicator and gives rise to the following optimization problem: "Find antenna parameters, such that the damage caused to the tumor is maximized, while healthy tissue is spared". Mathematically, this is a PDE constrained optimization problem subject to the time-harmonic Maxwell equations, which govern the propagation of the microwaves, and the bio heat transfer equation, a semi-linear elliptic equation, which governs the heat distribution in the human body. Further, upper bounds on the temperature in the healthy tissue are imposed, which can be classified as pointwise state constraints.

In this talk we consider a function space oriented algorithm for the solution of this problem, which copes with the various difficulties. The state constraints are tackled by an interior point method, which employs an inexact Newton corrector in function space for the solution of the barrier subproblems. Herein, discretization errors are controlled by a-posteriori error estimation and adaptive grid refinement. Importantly, the error measure and the refinement criteria fit to the structure of the non-linearity. This procedure is globalized by an affine covariant composite step method, which tackles the difficulties, caused by the non-convexity of our problem. Finally, a pointwise damping strategy is used to enhance efficiency and robustness of the method. We conclude the talk with a numerical study and show computational results that illustrate the performance of our algorithm.

Acknowledgments

This work was supported by the DFG Research Center Matheon "Mathematics for key technologies".

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Accelerating aerodynamic shape optimization by usage of shape Hessians

Volker Schulz, Universität Trier

Abstract

Shape optimization is an important topic in many applications. In particular, in aerodynamics, e.g., optimal wing shapes are required to reduce aerodynamic drag and increase energy efficiency. Parameter-free optimization is the method of choice but requires highly efficient means for computing gradients and usage of them within an optimization algorithm. Because of the high computational demand from discretized flow models, the algorithmic complexity of the optimization algorithm has to be very small. In this talk, we discuss the numerical usage of shape calculus and its benefit in the context of fast one-shot methods. Numerical results from applications are presented, as well.

Controllability of a simplified 1d fluid–structure system

Takéo Takahashi, Université Henri Poincaré

joint work with *Yuning Liu* and *Marius Tucsnak*

Abstract

We consider a controllability problem for a 1d system coupling a PDE and an ODE which models the motion of a rigid particle into a viscous fluid. We control this system by imposing the velocity of the fluid at one point of the boundary of the domain. In particular, we improve previous results where two controls were needed to obtain the same result. Our proof is based on a new method to control non linear parabolic systems by using an associated linear control problem where the nonlinearity is replaced by a source term.

A general theorem on error estimates with application to optimal control

Fredi Tröltzsch, Technische Universität Berlin

joint work with *Eduardo Casas*

Abstract

A general minimization problem (P): $\min_{u\in\mathcal{K}} J(u)$ is considered, where J is a real-valued smooth functional defined on a non-empty and closed convex set $\mathcal{K}$ of a Banach space. This problem is approximated by a family of optimization problems (P_h): $\min_{u\in\mathcal{K}_h} J_h(u)$, $h > 0$. In applications to control problems, h may stand for a mesh parameter. Let $\overline{u}$ be a local solution to (P) and $\overline{u}_h$ be a sequence of local solutions to (P_h) converging to $\overline{u}$ as $h \to 0$. Based on a natural second-order sufficient optimality condition and an appropriate requirement on the distance between J' and J'_h, the error $\|\overline{u} - \overline{u}_h\|$ is estimated. An associated theorem is discussed that deals with the different types of contributions to the error. This theorem might serve as a useful tool for deriving sharp error estimates for locally optimal controls of nonlinear optimal control problems. As an application, an error estimate is presented for locally optimal controls of a distributed control problem with quasilinear elliptic equation.

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Time optimal control for low Reynolds swimmers

Marius Tucsnak, Institut Élie Cartan, Nancy Université/CNRS/INRIA

joint work with *Jérôme Lohéac* and *Jean-François Scheid*

Abstract

The aim of this paper is to tackle the self-propelling at low Reynolds number by using tools coming from control theory. This is a challenging mathematical problem since we need to tackle a dynamical system in which the control function is the shape of a three dimensional domain. Therefore the work brings together techniques from two traditionally different fields: control of non linear finite dimensional systems and shape optimization for PDE systems. From a more practical view point, studying this problem is a step towards the understanding of the locomotion mechanism of aquatic micro-organisms or in studying the control problems raised by the design of micro or nano robotic swimmers.

More precisely we first address the controllability problem: "Given two arbitrary positions, does it exist 'controls' such that the body can swim from one position to another, with null initial and final deformations?". We consider a spherical object surrounded by a viscous incompressible fluid filling the remaining part of the three dimensional space. The object is undergoing radial and ax-symmetric deformations in order to propel itself in the fluid. Since we assume that the motion takes place at low Reynolds number, the fluid is governed by the Stokes equations. It is well known, that the governing equations can be reduced to a finite dimensional control system. By combining perturbation arguments, shape optimization techniques and Lie brackets computations, we establish the controllability property.

We next investigate the time optimal control problem for this model. This is done on a simplified problem. In the case in which state constraints are not considered we are able to solve analytically the optimal control problem. If state constraints are considered we develop a numerical method based on shooting results and we discuss the efficiency and accuracy of our method with respect to other optimization techniques. Going back to the motivating application to the swimming of microorganisms, we discuss the dependence of the minimal time of the number of modes, which is seen as a measure of organism's "intelligence".

Acknowledgments

The work of Marius Tucsnak was partially supported by the "Agence Nationale de la Recherche" (ANR), Project CISIFS, grant ANR-09-BLAN-0213-02.

An equilibrium mean field games model of transaction volumes

Gabriel Turinici, Université Paris-Dauphine

joint work with *Min Shen*

Abstract

We present in this talk an equilibrium model of transactions volumes in a Mean Field Games framework. Each market participant estimates the future value of the underlying financial security but this estimation has some uncertainty. Transaction volume will be a collective trade-off of an individual cost versus precision optimization of each agent.

We prove fundamental properties of the model: monotonicity of offer and demand, existence of an equilibrium, etc. and give some applications.

Differentiability properties of unsteady incompressible Navier–Stokes flow with respect to domain variations

Michael Ulbrich, Technische Universität München

joint work with *Christian Brandenburg, Florian Lindemann,* and *Stefan Ulbrich*

Abstract

We consider shape optimization problems governed by the time-dependent incompressible Navier–Stokes equations and focus on the domain transformation approach (method of mappings). Here, the design is described by a transformation that maps a reference domain to the current domain. Transforming the Navier–Stokes equations to this reference domain converts the shape optimization problem to a nonlinear PDE-constrained optimal control problem. The main question we will address is the Fréchet differentiability of the fluid velocity with respect to domain variations. In contrast to elliptic equations and the stationary Navier–Stokes equations, the standard implicit function theorem is not applicable in this context. The main complications come from too weak time regularity properties of the velocity and the pressure and from the fact that divergence-freeness is not invariant under transformation. We will show how these difficulties can be surmounted to prove Fréchet differentiability of the velocity with respect to domain variations in a suitable setting. This forms a rigorous basis for adjoint-based shape optimization techniques. Numerical results for drag minimization will also be presented.

Acknowledgments

The support of the DFG is gratefully acknowledged.

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Adaptive multilevel methods for PDE-constrained optimization based on adaptive finite element or reduced order approximations

Stefan Ulbrich, Technische Universität Darmstadt

joint work with *J. Carsten Ziems*

Abstract

We present an adaptive multilevel generalized SQP-method for optimal control problems governed by nonlinear PDEs with control constraints. During the optimization iteration the algorithm generates a hierarchy of adaptively refined discretizations, which can be based on adaptive finite element approximations or on reduced order methods like Reduced Basis Methods or POD. The adaptive refinement strategy is based on error estimators for the PDE, adjoint PDE and a criticality measure. We consider first the case of an adaptive finite element discretization and discuss then the extension of the algorithm to adaptive approximations by reduced order models. We conclude the talk by showing numerical results.

A priori error analysis of the Petrov–Galerkin Crank–Nicolson scheme for parabolic optimal control problems

Boris Vexler, Technische Universität München

joint work with *Dominik Meidner*

Abstract

In this talk a finite element discretization of an optimal control problem governed by the heat equation and subject to pointwise control constraints is considered. The temporal discretization is based on a Petrov Galerkin variant of the Crank Nicolson scheme, whereas the spatial discretization employs usual conforming finite elements. With a suitable post-processing step, a discrete solution is obtained for which error estimates of optimal order are proved. A numerical result is presented for illustrating the theoretical findings.

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POD for coupled nonlinear PDE systems

Stefan Volkwein, Universität Konstanz

joint work with *Oliver Lass*

Abstract

In this talk coupled nonlinear PDE systems are considered. In particular, a parabolic-elliptic sytem arrising in models for lithium ion batteries serve as an application. Efficient POD Galerkin schemes are developed, where the nonlinearities are realized computationally by different empirical interpolation schemes. Residual based error control ensures sufficient accuracy for the reduced-order solutions.

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A globalized Newton method for the optimal control of multiple interacting fermions

Gregory von Winckel, Karl-Franzens-Universität Graz

Abstract

A fast globalized Newton method is presented for computing the optimal time-dependent amplitude of a laser field to induce a desired energy state transition in a system of several confined fermions with Coulomb interaction. Because standard many-body approximation techniques such as Hartree–Fock tend to be only valid for many particles, the full multi-body Hamiltonian is used for systems of two through seven particles. While the computational cost of solving PDEs with several independent variables typically becomes prohibitively high, we show that an efficient discretization strategy can be implemented for which the numerical solution of the multi-body problem becomes computationally tractable. This is critical for PDE-constrained optimization problems, where the state and adjoint equations must typically be solved many times.

Acknowledgments

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$\mathcal{H}_{2,\alpha}$-norm model reduction for optimal control of PDEs with applications in laser processing

Georg Vossen, RWTH Aachen

Abstract

A numerical solution method for optimal control problems subject to parabolic and hyperbolic equations is presented. The partial differential equation is semi-discretized in space to obtain a large-scale linear time invariant dynamical system with the boundary or distributed controls as input and those parts of the discretized state appearing in the cost functional as output variables. The corresponding transfer function is approximated optimally with respect to the $\mathcal{H}_{2,\alpha}$-norm leading to a reduced dynamical system. This induces an optimally reduced optimal control problem which can be solved by first-discretize-then-optimize or first-optimize-then-discretize methods. A-posteriori error analysis is applied to quantify the approximation quality of the solution of the optimally reduced optimal control problem. Different algorithms on basis of the error analysis will be presented which ensure a prescribed error in the optimal control. The methods are illustrated by numerical examples arising in the context of laser processing and compared to other reduction methods such as Balanced Truncation and Proper Orthogonal Decomposition.

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Time optimal control of the wave equation, its regularization and numerical realization

Daniel Wachsmuth, Radon Institute (RICAM)

Abstract

An approximation procedure for time optimal control problems for the linear wave equation is analyzed. Its asymptotic behavior is investigated and an optimality system including a transversality conditions for the regularized and unregularized problems are derived. The regularized problem also serves as a starting point for numerical techniques. For this purpose a family of parameterized optimal control problems, depending on the time horizon τ as parameter, is introduced. Each member of this family can be solved by a locally superlinearly convergent semi-smooth Newton algorithm. It is verified that the derivative of its minimal value functional of the parameterized problems coincides with the transversality condition of the time optimal problems. Selected numerical examples are given.

Optimal control of quasistatic plasticity

Gerd Wachsmuth, Technische Universität Chemnitz

Abstract

An optimal control problem is considered for the variational inequality representing the stress-based (dual) formulation of quasistatic elastoplasticity. The linear kinematic hardening model and the von Mises yield condition are used. By showing that the VI can be written as an evolutionary variational inequality, we obtain the continuity of the forward operator. This is the key step to prove the existence of minimizers.

In order to derive necessary optimality conditions, a family of time discretized and regularized optimal control problems is analyzed. By passing to the limit in the optimality conditions for the regularized problems, necessary optimality conditions of weakly stationarity type are obtained.

We present a solution method which builds upon the optimality system of the time discrete and regularized problem. Numerical results which illustrates the possibility of controlling the springback effect.

Pointwise convergence of the feasibility violation for Moreau–Yosida regularized optimal control problems

and applications to problems involving pointwise constraints on the gradient of the state on non smooth polygonal domains

Winnifried Wollner, Universität Hamburg

Abstract

In this talk we are concerned with an analysis of Moreau–Yosida regularization of pointwise state constrained optimal control problems. As recent analysis has already revealed the convergence of the primal variables is dominated by the reduction of the feasibility violation in L^∞, see, e.g., [1].

We will use a new method to derive convergence of the feasibility violation in L^∞ giving improved convergence rates compared to the results of [1] and [3].

Finally we will employ these techniques to analyze optimal control problems governed by elliptic PDEs with pointwise constraints on the gradient of the state on non smooth polygonal domains. For these problems, standard analysis, see, e.g., [2], fails because the control to state mapping does not yield sufficient regularity for the states to lie in C^1, nonetheless these problems are well posed, see [4]. In particular we will derive convergence rates for the primal variables of the regularized problem.

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Inverse problems for a system for non-isothermal crystallization of polymers

Masahiro Yamamoto, University of Tokyo

Abstract

We consider a coupled governing system for non-isothermal crystallization of polymers. We discuss the unique continuation and coefficient inverse problems and show the uniqueness and the stability.

Stochastic pseudo-differential operators and Calderón-type uniqueness theorem for stochastic PDEs

Xu Zhang, Chinese Academy of Sciences and Sichuan University, China

joint work with *Xu Liu*

Abstract

We introduce the notion of stochastic pseudo-differential operators, and discuss the corresponding symbol calculus, asymptotic expansion, pseudo-local property, boundedness, elliptic operators and Gårding-type inequality. As its application, we present a Calderón-type uniqueness theorem for general stochastic PDEs. Our uniqueness result improves a little the classical one when it degenerates to the deterministic case. The proof of this uniqueness result is based on a new Carleman-type estimate.

Hamilton–Jacobi approach in singular domains

Hasnaa Zidani, ENSTA ParisTech & INRIA Saclay

joint work with *C. Imbert* and *R. Monneau*

Abstract

The talk will focus on the study of some first order Hamilton–Jacobi equations on non-smooth domains and with discontinuous Hamiltonians. The work aims at giving a precise notion of solution and proving uniqueness results.

Recent theoretical and numerical developments in inverse obstacle scattering

Jun Zou, The Chinese University of Hong Kong

joint work with *Jingzhi Li* and *Hongyu Liu*

Abstract

In this talk we shall review some significant theoretical and numerical developments in inverse obstacle scattering in the recent years. The mathematical justifcations of reconstructions using both near-field data or far-field data are discussed and numerical simulations are presented.

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