Computing signal amplitudes for MR sequences with equally spaced RF pulses with arbitrary excitation and refocusing flip angle and phase

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The Generating Function formalism (also known as z-transform) can be used to calculate MR echo amplitudes for repetitive pulse sequences. As commonly done in signal processing the z-transform is applied to solve a linear difference equation. After solution of the Bloch equations for nutation, precession and relaxation, a difference equation relating the magnetization vector $\vec{M} = [M_+, M_+^*, M_z]^T$ after the n^{th} pulse to the magnetization before the n^{th} pulse can be established. Generally the solution looks as follows:

$$\vec{M}^{(n+1)} = \mathbf{A}\vec{M}^{(n)} + \vec{B} \tag{1}$$

where $\mathbf{A}$ is a matrix accounting for free precession, relaxation and RF nutation, and vector $\vec{B}$ models the recovery of longitudinal magnetization. Upon solving this equation using the z-transform, one arrives at a closed-form expression for the echo amplitudes. For unbalanced sequences one further has to average over all isochromates (by integration around the unit circle using the residue theorem) to yield the correct echo emplitudes. In this work this procedure was carried out for excitation and refocusing pulses with arbitrary flip angle and phase in a multi-echo spin-echo sequence. The resulting formula is given by

$$F_0(z) = M_0 e^{i\phi} \sin\theta_e \left[\frac{\cos(\phi_e - \phi)}{2} \left(1 + \sqrt{\frac{X_+(z)}{X_-(z)}} \right) + i \frac{\sin(\phi_e - \phi)}{2} \left(1 + \sqrt{\frac{X_-(z)}{X_+(z)}} \right) \right]$$

$$X_+(z) = (1 + z\kappa_2)(1 - z(\cos\alpha\cos^2\theta + \sin^2\theta)(\kappa_1 + \kappa_2) + z^2\kappa_1\kappa_2)$$

$$X_-(z) = (1 - z\kappa_2)(1 - z(\cos\alpha\cos^2\theta + \sin^2\theta)(\kappa_1 - \kappa_2) - z^2\kappa_1\kappa_2)$$

Echo amplitudes can be computed by inverse DFT of eq. $(z = e^{i\psi})$ and can be used for accurate T_2 mapping or model based reconstruction.