Compressed Sensing in MRI: how the Maximum Undersampling Factor depends on the Image Size and the SNR

C. Lazarus[1], N. Chauffert[1], A. Coste[2], A. Vignaud[2], P. Ciuciu[1]
[1]Gif-sur-Yvette,FR,CEA,DRF/I2BM/NeuroSpin/UNATI/Parietal,
[2]Gif-sur-Yvette,FR,CEA,DRF/I2BM/NeuroSpin/UNIRS

Since the work of Lustig et al. on Sparse MRI, Compressed Sensing (CS) has promised great opportunities to drastically shorten the acquisition time in MRI by reconstructing images from undersampled Fourier data. Although CS theories provide upper bounds relating the number of required measurements m to the image sparsity and its number of pixels $N \times N$ to guarantee exact recovery in the noise-free case, in practice it remains unclear to which extent MRI acquisitions can be accelerated while preserving image quality. More precisely, finding the relationship linking the maximum achievable undersampling factor $R = N^2/m$ to the image resolution in a noisy context is still an open question. In this numerical study, we propose quantitative hints that may guide CS-MRI users in their choice of an appropriate undersampling factor as a function of the image size for different SNR.

Simulations were performed on a 2D brain image for increasing image sizes and noise levels characterized by their input SNR. Fourier data was randomly undersampled ($R \leq 30$) according to a non-Cartesian variable density and reconstructed with a nonlinear algorithm solving the penalized CS L_1-minimization problem. To assess the image quality of the reconstructed images, we compared the SSIM index (Wang et al. 2004), measuring the similarity in structure with a full k-space reference image.

On the one hand, noise-free simulations showed that for a given N and a chosen image quality characterized by a SSIM threshold ε, there exists a maximum undersampling factor $R_\varepsilon^{max}(N, SNR = \infty)$, increasing with N, above which the desired image quality cannot be preserved. On the other hand, in the noisy context, performances are significantly reduced as the maximum achievable undersampling factors $R_\varepsilon^{max}(N, SNR < \infty)$ are diminished. However, our simulations showed that there is a minimum SNR above which it is possible to reach the targeted quality with the maximum undersampling factor $R_\varepsilon^{max}(N, SNR = \infty)$. In-house experiments performed on an ex-vivo baboon brain with a 7T scanner seemed to corroborate these results quantitatively.

In practice, our study provides CS-MRI users with quantitative guidance in the maximum undersampling factor that should be used to reach a desired image quality, not only based on the image size but also on the available SNR in the original fully sampled image.