Interpolation and Extrapolation in the Space of Images

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We consider the space of images as a Riemannian manifold using the metamorphosis approach by Trouvé and Younes, where the underlying Riemannian metric simultaneously measures the cost of image transport and intensity variation. We study a robust and effective variational time discretization of geodesics paths, which requires to minimize a discrete path energy consisting of a sum of consecutive image matching functionals over a set of image intensity maps and pairwise matching deformations.

For square-integrable input images the existence of discrete geodesic paths defined as minimizers of this variational problem is shown. Furthermore, the Γ-convergence of the underlying discrete path energy to the continuous path energy is proven. A spatial discretization via finite elements combined with an alternating descent scheme in the set of image intensity maps and the set of matching deformations is presented to approximate discrete geodesic paths numerically and, to further speed up the computational time, a GPU acceleration is employed.

As an application, a time discrete geodesic path of a human eye suffering from an age-related macular degeneration is computed, where the input data emerge from a combination of laser ophthalmoscopy and optical coherence tomography.

As a further application, we propose an image extrapolation method to approximately recover the geodesic paths from an initial variation by using a fixed point iteration.