Parallel Imaging with Phase Scrambling

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Introduction: Phase-scrambled (PS) MRI allows for alias-free image reconstruction of under-sampled data by reformulating the reconstruction as a convolution with a chirp-kernel [1]. The resulting low-resolution image can be used to calibrate Parallel Imaging (PI) techniques like SENSE [2] or GRAPPA [3], which is a basis of Parallel Imaging with Phase Scrambling (PIPS) [4]. In this work we present the PIPS approach and investigate possible extensions.

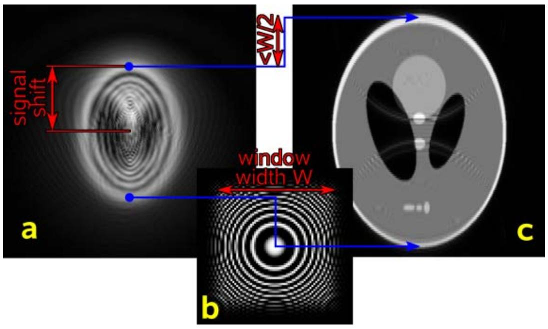

Fig. 1. *Convolution reconstruction with a windowed kernel. Quadratic phase modulation causes position-dependent shifts of MRI signals (a). Convolution kernel of width W (b) is able to transfer signals to their true positions in image space (c) if the signals in k-space matrix are within W/2 from their source in the image.*

Theory: Consider without loss of generality the MRI signal of a sample $\rho(x)$ in a 1D case:

$$s(k) = \int \rho(x)\exp(-\mathrm{i2\pi kx})\,dx = \int \rho(x)\exp\left(-i2\pi\frac{k}{k_{max}}\frac{x}{\Delta x}\right)dx \qquad (1)$$

With the substitution $2ab = a^2 + b^2 - (a-b)^2$ Eq. 1 transforms to:

$$s(k) = exp\left(-i\pi\left(\frac{k}{k_{max}}\right)^2\right)\int \rho(x)exp\left(-i\pi\left(\frac{x}{\Delta x}\right)^2\right)exp\left(-i\pi\left(\frac{k}{k_{max}}-\frac{x}{\Delta x}\right)^2\right)dx \qquad (2)$$

This expression can be simplified by defining $g(\vartheta) = \exp(i\pi\vartheta^2)$, and upon following substitutions $\eta = k/k_{max}$, $\xi = x/\Delta x$, $\rho'(\xi) = \rho(\xi\Delta x)\Delta x g^*(\xi)$, $s'(\eta) = s(\eta k_{max})g(\eta)$, as:

$$s'(\eta) = \int \rho'(\xi)exp(i\pi(\eta-\xi)^2)d\xi, \text{ or } s' = \rho' \otimes g \qquad (3)$$

Eq. 3 states that the FT of the modified spin density $\rho'(x)$ can be reformulated as a convolution with a chirp function $g(\vartheta) = exp(i\pi\vartheta^2)$. The reconstruction can therefore be defined by a convolution with the complex conjugate of the chirp function:

$$\rho' = s' \otimes g^* \qquad (4)$$

In order to allow for an alias-free reconstruction, a quadratic phase has to be added to the object prior to data acquisition. An increase in quadratic phase variation causes a relocation of k-space signals leading to a k-space that resembles the image (Fig. 1a). In case of quadratic phase modulation the displacement d_k of the k-space echo positions is proportional to the distance to the modulation center, l. As Fig. 1 illustrates, convolution-based reconstruction is only possible with a windowed kernel with a window width w obeying the relationship $d_k + \frac{w}{2} < l$.

Methods: MRI Scans were performed on a 3T MR (Siemens Healthcare, Germany) in a healthy volunteer using a 3D GRE sequence to acquire fully encoded raw data with a 256^2 matrix, FOV=256mm, 16 2mm thick partitions, TR=150ms, FA=15°.

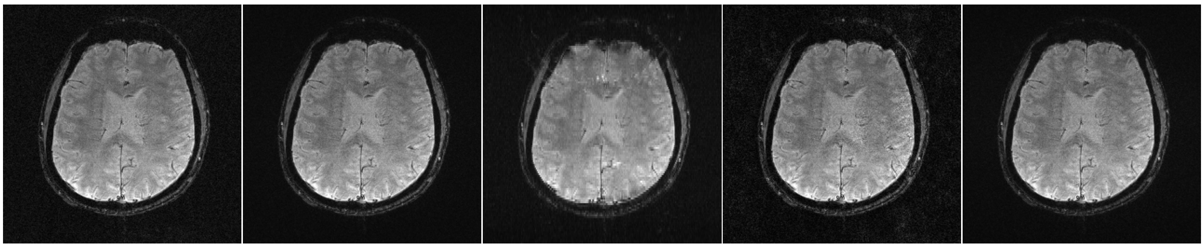

Fig 2. *In vivo experiment results. (a) SENSE reconstruction based on 128 k-space lines and coil sensitivity profiles. (b) GRAPPA reconstruction based on 128 k-space and 32 ACS lines (not included in final k-space). (c) PS-MRI reconstruction based on 128 k-space lines with reduced spatial resolution and residual high-frequency ghosting. (d) PIPS-SENSE reconstruction with coil sensitivities calculated from low-resolution PS-MR images. (e) PIPS-GRAPPA reconstruction with 32 synthetic ACSs (not included in final k-space) simulated by Fourier-transforming low-resolution PS-MR images.*

Results and Discussion: In Fig. 2 in vivo images are shown. A slice was retrospectively undersampled by a factor of 2. Figs. 2a and 2b are traditional SENSE and GRAPPA reconstructions with coil sensitivities and ACS lines extracted from the original data. PS-MRI single coil images in Fig. 2c were used directly to calculate coil sensitivities for SENSE resulting in Fig. 2d. However, due to the high-frequency ghosts and less apparent local phase errors in the calculated sensitivities, image quality in Fig. 2d is degraded. Images in Fig. 2c can also be used to simulate a central part of the fully acquired k-space to serve as ACS for GRAPPA (Fig. 2e). Apparently GRAPPA weights calculation procedure is less sensitive to the residual artifacts in Fig. 2c. In future work we will investigate methods to directly derive GRAPPA weights from PS data. Advanced coil sensitivity extraction algorithms shall be able to improve the PIPS-SENSE reconstruction. Image-space GRAPPA [5] may also offer an interesting option as reconstruction weights in this case can be derived directly from the PS images. Further reconstruction techniques like SPIRiT [6] or methods based on compressed sensing or low-rank modeling of local k-space neighborhoods (LORAKS [7]) may also profit from PS. As the usual k-space relationships do not hold anymore with PS data, further investigation of the data properties may lead to new insights and further applications.

There are different ways to induce quadratic phase modulations. The approach used here of offsetting the second order shim currents is limited by the presently available hardware. At this juncture further options based on using tailored RF pulses [8] or 2D/3D selective excitation seem attractive as well as dedicated hardware capable of producing quadratic gradient fields [9].

References: [1] Ito, MRM-08 60:422-3; [2] Pruessmann, MRM 1999 42:952-962; [3] Griswold, MRM 2002 47:1202-1210; [4] Zaitsev, MRM 2014 (published online); [5] Breuer, MRM 2009 62:739-746; [6] Lustig, MRM 2010 64:457-471; [7] Haldar, IEEE 2014 33:3; [8] Pipe, MRM-95 33:24-33; [9] Hennig, MAGMA 2008: 21(1-2): 5-14