Preconditioned Alternating Direction Method of Multipliers for Non-Smooth Regularized Quadratic Problem

Kristian Bredies Hongpeng Sun

Institute of Mathematics and Scientific Computing, University of Graz
{kristian.bredies, hongpeng.sun}@uni-graz.at

In this talk, we will introduce the proposed preconditioned ADMM (alternating direction method of multipliers) for the following kind of problem,

$$\min_{u \in X} F(u) + G(p), \quad \text{subject to} \quad Ku = p,$$

where K is a linear bounded operator, G is proper, convex, lower semi-continuous and F is a quadratic function with the form,

$$F(u) = \langle Su, u \rangle / 2 - \langle f_0, u \rangle, \quad \forall u \in X.$$

This model is frequently employed for L^2 problem with non-smooth regularization including a lot linear inverse problems in imaging. ADMM is a popular first order method for general constrained optimization problems. However, as a variant of Douglas-Rachford splitting method, it also suffers from solving linear subproblems for the general quadratic problems in various applications. Inspired by the preconditioned Douglas-Rachford splitting method introduced in [1], we give a preconditioned ADMM methods by writing ADMM as a kind of Douglas-Rachford splitting method, and prove the weak (or strong) convergence in infinite (or finite) dimensional space. Various efficient preconditioners could be used in preconditioned ADMM framework and any finite number of preconditioned iterations could guarantee the weak convergence of the solutions. Preconditioned ADMM is not covered by the framework of preconditioned Douglas-Rachford splitting method in [1]. Comparisons between preconditioned ADMM and the PDRQ in [1], which are both unconditional stable and are both variants of the Douglas-Rachford splitting method, are presented.

References

[1] K. Bredies, H. P. Sun, *Preconditioned Douglas-Rachford splitting methods for saddle-point problems with applications in image denoising and deblurring*, SFB-Report-2014-002, Univerisity of Graz.