Sparse and Co-Sparse Tomographic Recovery from Few Projections

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Compressed Sensing (CS) exploits the sparsity of signals and allows accurate reconstruction from a few linear, but random and non-adaptive measurements. Moreover, for such measurement ensembles, bounds on the required signal sparsity depending on the undersampling ratio can be derived, also known as Donoho-Tanner phase transitions, that guarantee exact recovery via l1-minimization. Unfortunately, sampling patterns as used in industrial tomographical set-ups with limited numbers of projections fall far short of common assumptions in CS. Thus, for such scenarios, CS provides neither theoretical guarantees of accurate reconstruction, nor any relation between sparsity and a sufficient number of measurements for recovery of sparse images from few tomographic projections. We investigate conditions for unique signal recovery based on sparse and cosparse signal models and present a relation between image co-/sparsity and sufficient number of tomographic measurements for exact recovery similar to the settings in CS. Numerical recovery through linear programming reveals a high accuracy of the theoretical predictions. Additionally, we show that the transition from non-recovery to recovery is sharp for specific sparse images. The signal class covered by both sparse and cosparse models seems broad enough to cover relevant industrial applications of non-standard tomography, like contactless quality inspection.