Imaging with Modulated/Undersampled Data 2014
SBF Workshop, Graz, Austria

Low-rank plus sparse (L+S) matrix reconstruction for accelerated dynamic MRI

Ricardo Otazo

NYU School of Medicine
NYU LANGONE MEDICAL CENTER

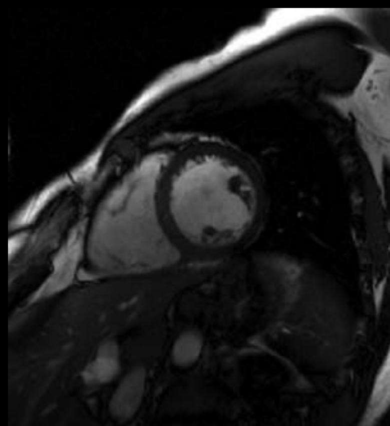
NYU CENTER FOR BIOMEDICAL IMAGING

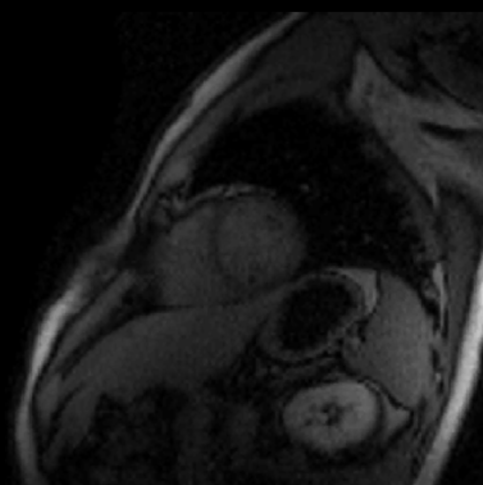
Need for speed in dynamic MRI

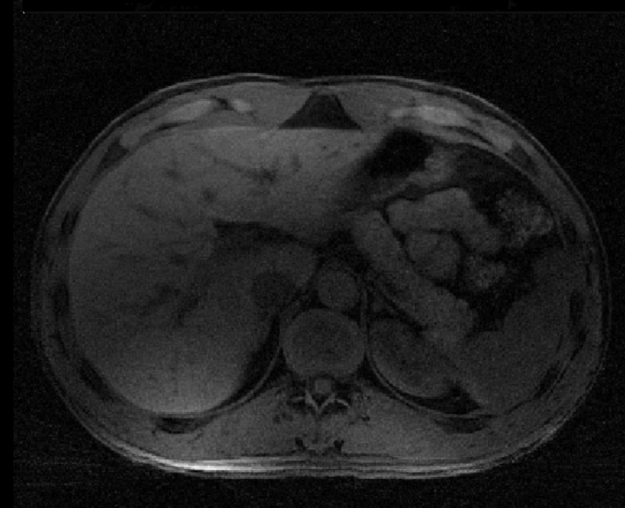
I feel the need … the need for speed – Top Gun movie

Dynamic MRI

- Time-series of images

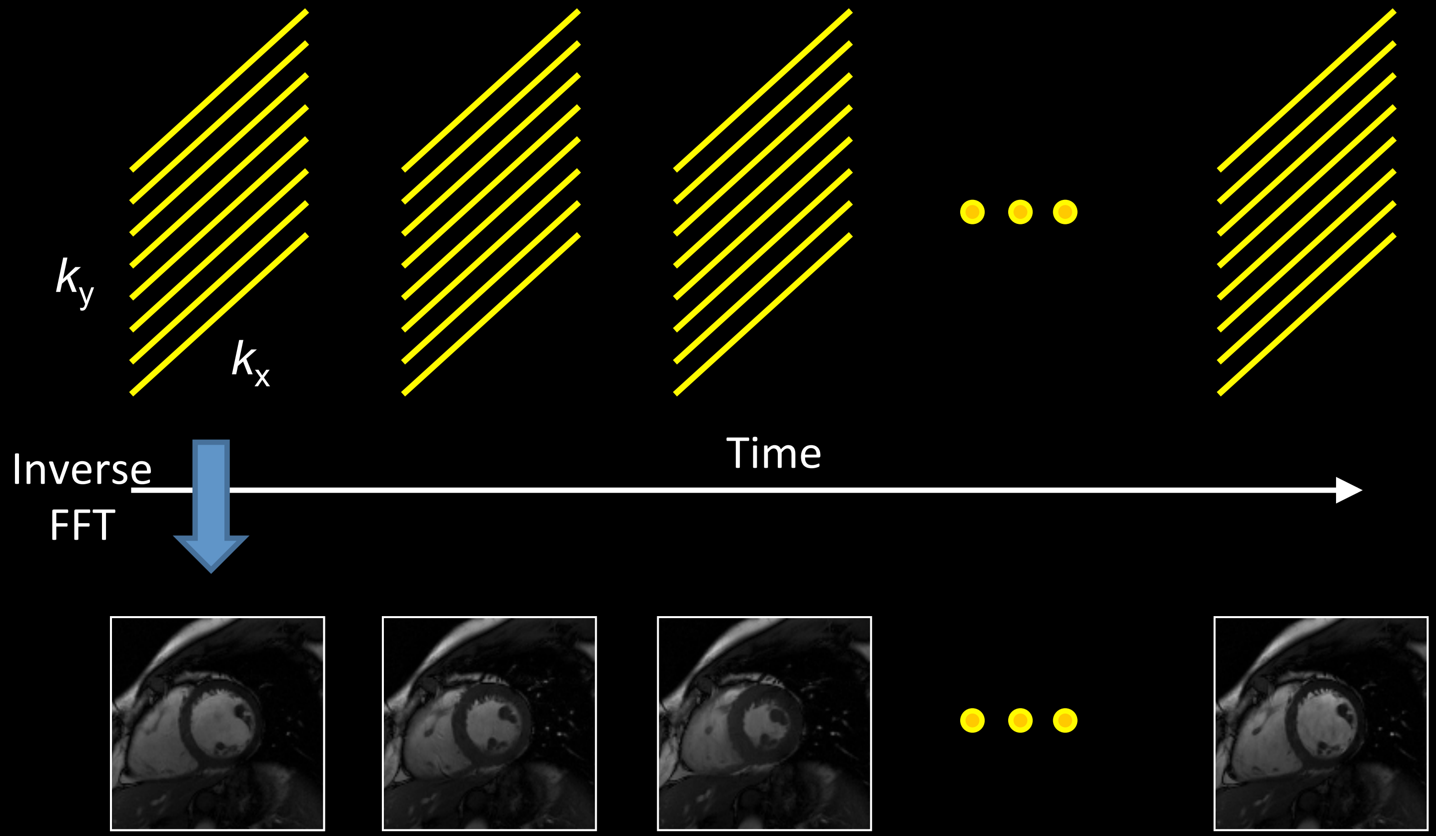






Dynamic MRI

- k-t data acquisition



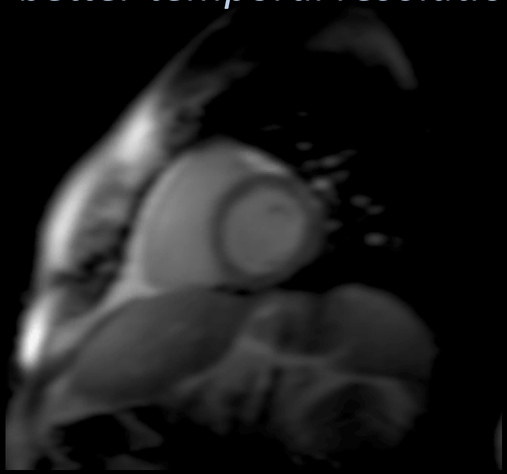
Time-series of images

Dynamic MRI feels the need for speed

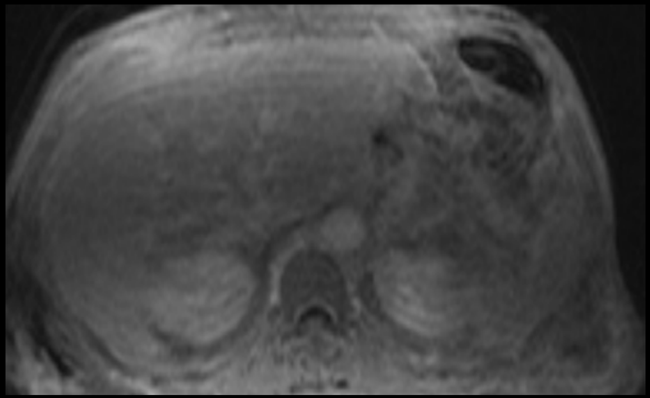
- Current dynamic MRI techniques are slow
  - Switching rate of magnetic field gradients
  - Noise amplification in parallel imaging

- Consequences

Sacrifice spatial resolution for better temporal resolution

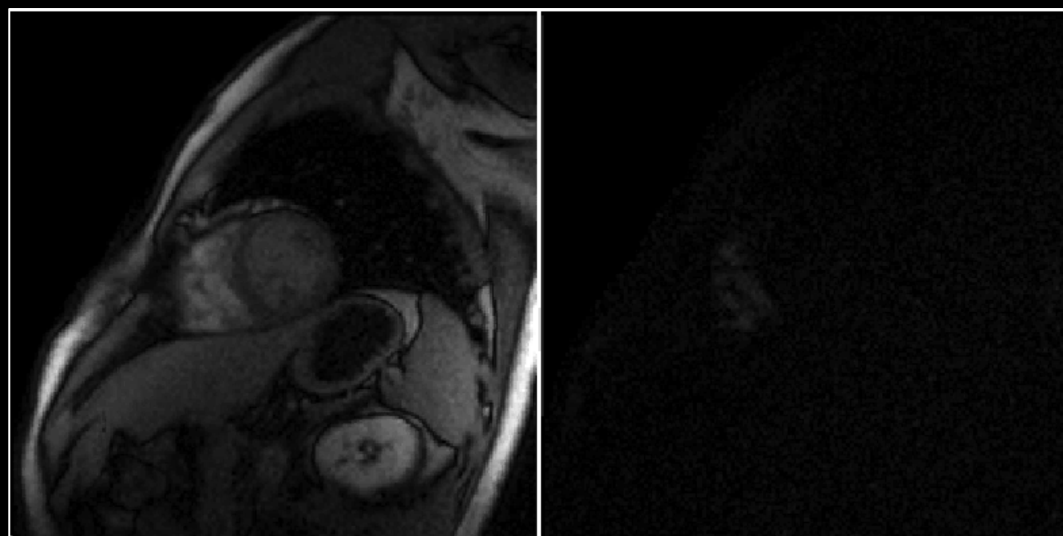


Motion-related artifacts



Dynamic MRI feels the need for speed

- Full k-space sampling for each time point is redundant



- Shouldn't we be just sampling the differences from frame-to-frame?
  - Sampling rate should match the information rate

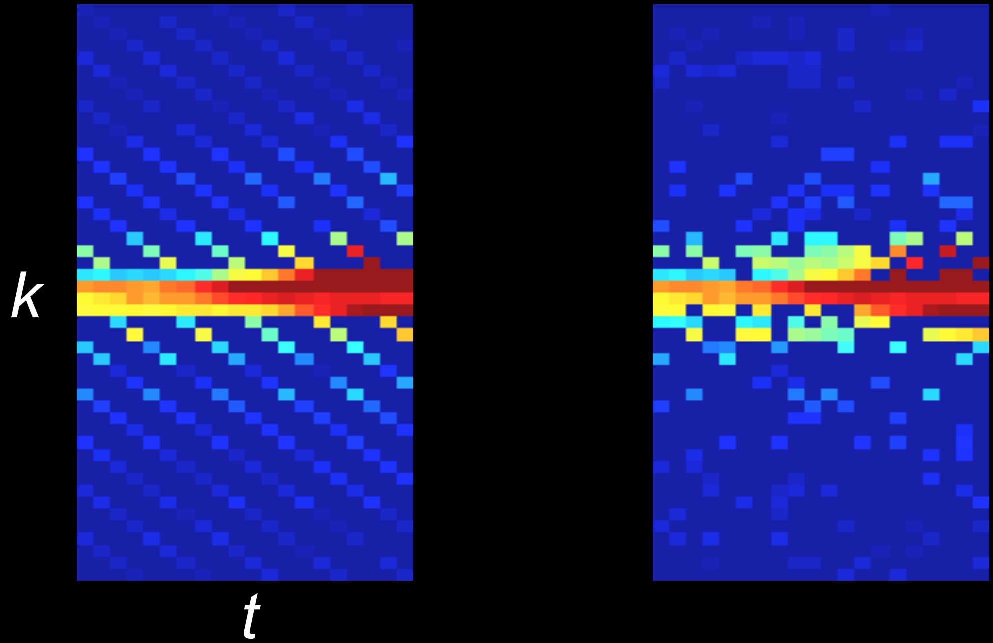
Accelerated dynamic MRI

k-t undersampling

- Different sampling pattern for each time point

Uniform

Random



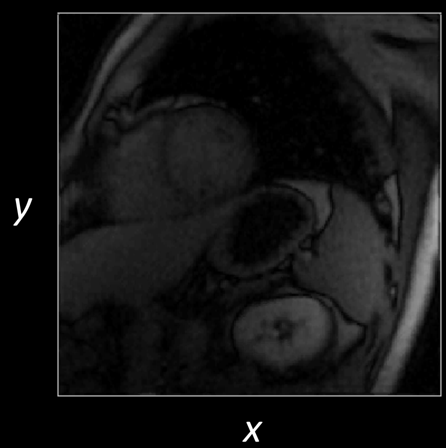
UNFOLD
k-t BLAST/SENSE

Compressed sensing
Matrix completion

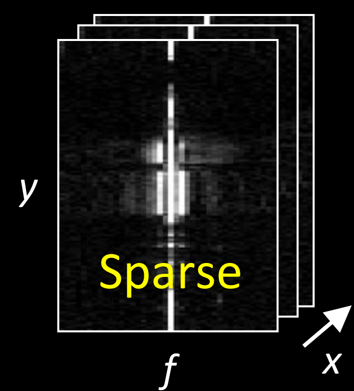
Reconstruction of undersampled k-t data

- Exploit spatiotemporal correlations
- Dynamic MRI data are compressible/sparse
  - *A movie is easier to compress than an image*

Original



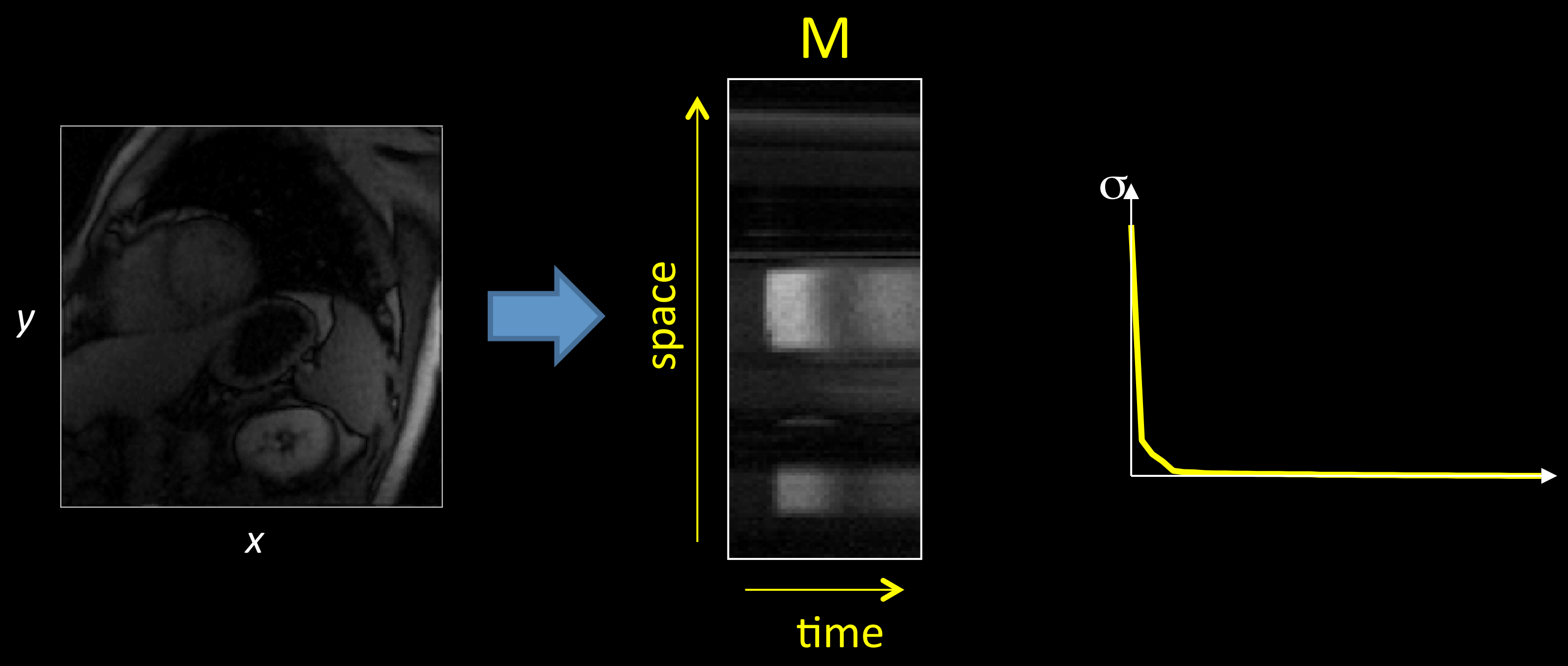
Temporal frequencies



x20 Compressed

Reconstruction of undersampled k-t data

- Exploit spatiotemporal correlations
- Dynamic MRI data are low-rank



Reconstruction of undersampled k-t data

- Compressed sensing (transform sparsity)

$$\text{minimize} \left\| Tx \right\|_1 \ \text{subject to}\ Ex = y$$

Lustig M et al. ISMRM 2006
Gamper U et al. MRM 2008
Jung H et al. MRM 2009
Otazo R et al. MRM 2010

- Matrix completion (low-rank)

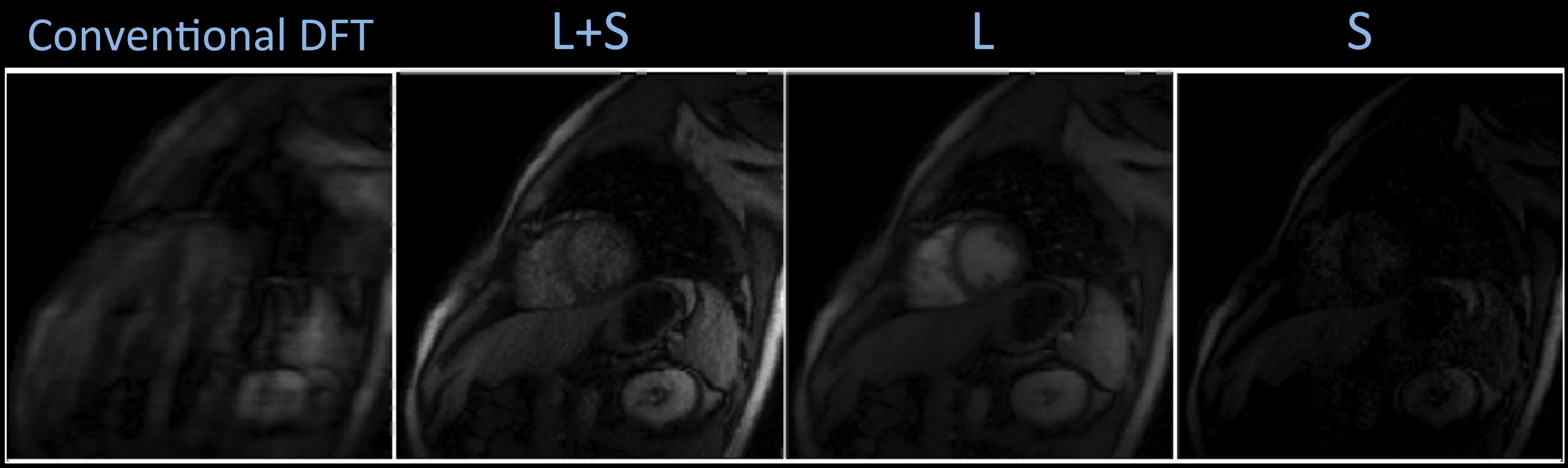
$$\text{minimize} \left\| M \right\|_* \ \text{subject to}\ EM = y$$

Haldar J et al. IEEE ISBI 2010
Otazo R et al. ISMRM 2012

Low-rank plus sparse reconstruction

L+S reconstruction of undersampled k-t data

- Combination of matrix completion and compressed sensing
- Based on L+S matrix decomposition (a.k.a. robust PCA)



8-fold undersampling

Otazo R et al. Magn Reson Med. 2014

Low-rank plus sparse matrix decomposition

- a.k.a. robust principal component analysis (RPCA)

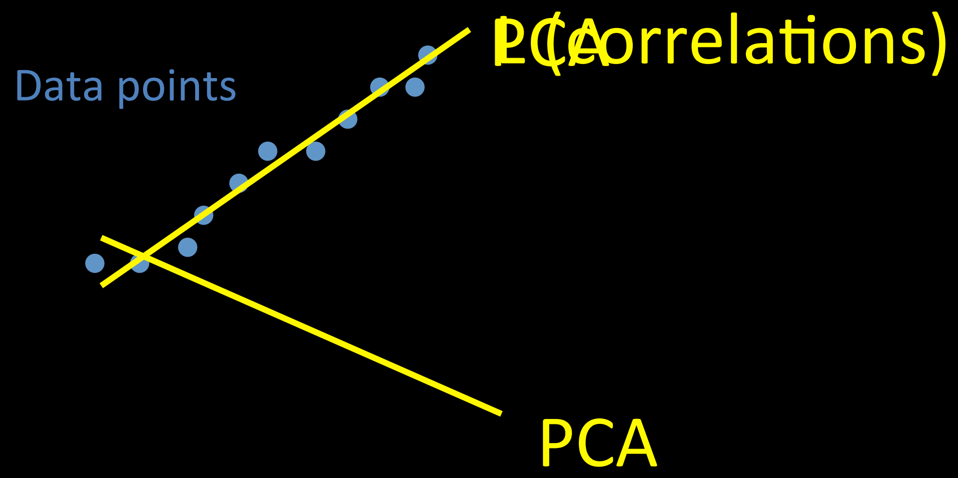
$$\mathrm{M} = \mathrm{L} + \mathrm{S}$$

M: data matrix
L: low-rank matrix
S: sparse matrix

Candès E et al. ACM 2011; 58:1-37

Low-rank plus sparse matrix decomposition

- Motivation: standard PCA is sensitive to outliers



S (outliers)

Candès E et al. ACM 2011; 58:1-37

Low-rank plus sparse matrix decomposition

- Convex optimization problem

- Find L and S to

$$\begin{aligned}&\text{minimize } \|\mathrm{L}\|_* + \lambda \|S\|_1 \\ &\text{subject to } M = L + S\end{aligned}$$

$\|\cdot\|_*$: nuclear-norm (sum of singular values)

$\|\cdot\|_1$: l_1-norm (sum of absolute values)

Candès E et al. ACM 2011; 58:1-37

Low-rank plus sparse matrix decomposition

- Requirements
  - Singular vectors of L cannot be sparse
  - S cannot have a low-rank representation

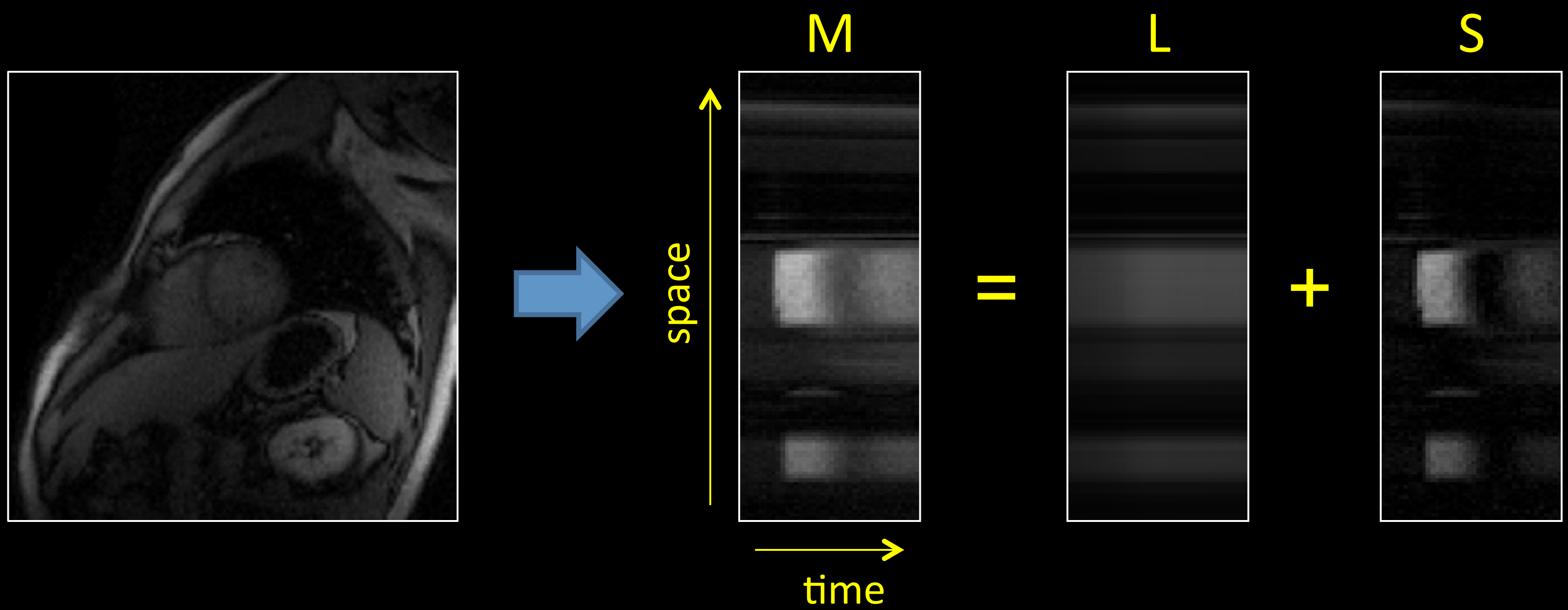
  Rank-sparsity incoherence

- No need to know the rank of L or the sparsity of S

Candès E et al. ACM 2011; 58:1-37

L+S decomposition of dynamic MRI data

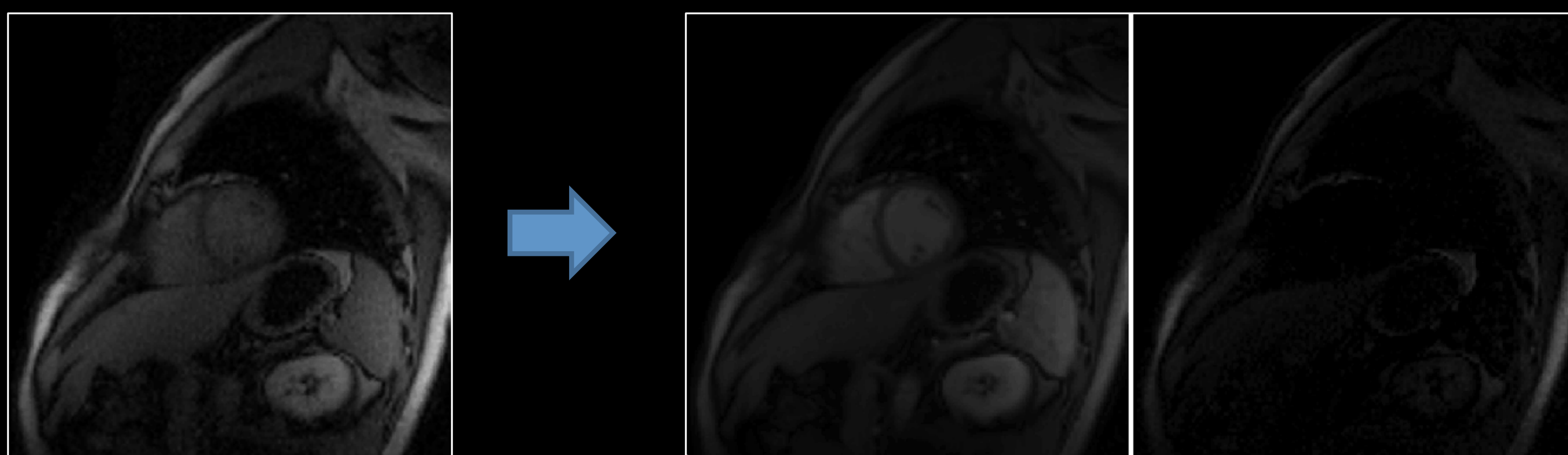
- Cardiac perfusion example



L+S decomposition of dynamic MRI data

- Cardiac perfusion example

L S

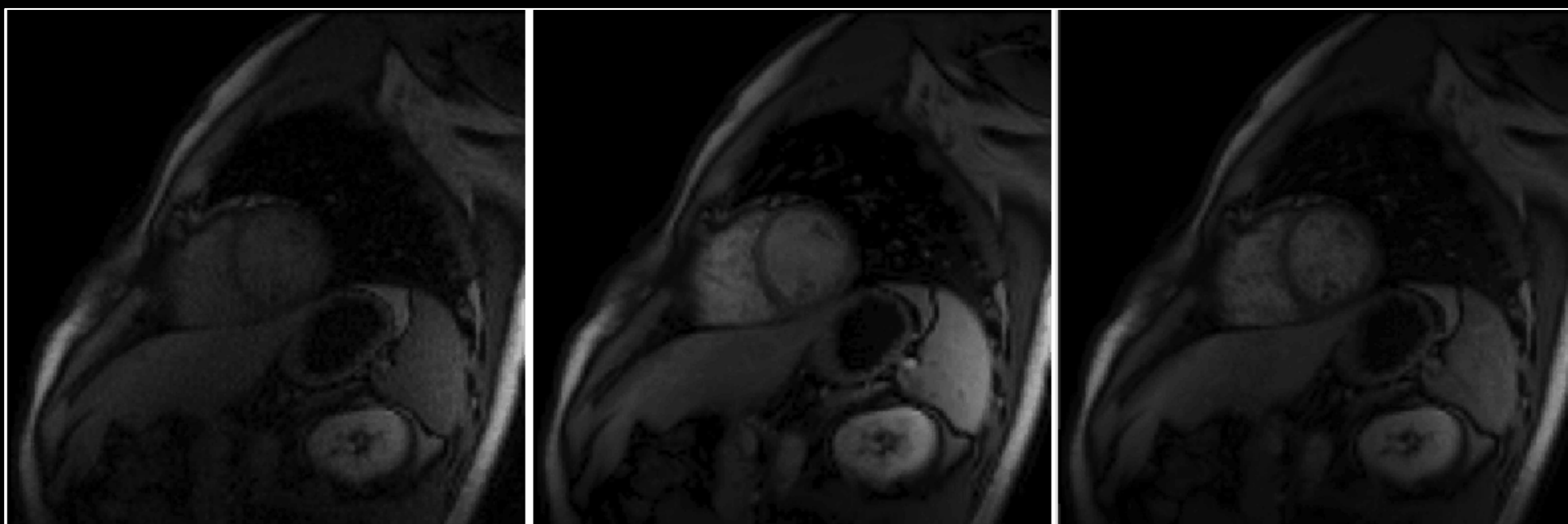


Background Innovations

L+S decomposition of dynamic MRI data

- Increased compressibility

No compression   X30 S-only compression   X30 L+S compression



L+S reconstruction of undersampled k-t data

- Find L and S to

$$\text{minimize} \|L\|_* + \lambda \|TS\|_1$$
$$\text{subject to } E(L+S) = d$$

L: low-rank component
S: sparse component
T: temporal sparsifying transform
E: encoding operator (Fourier transform + coil sensitivities)
d: undersampled k-t data

L+S reconstruction of undersampled k-t data

- Incoherence between
  - Undersampled k-t and L
  - Undersampled k-t and TS
  - L and TS

Remove aliasing

Background/dynamic separation

L+S reconstruction of undersampled k-t data

- Proximal gradient algorithm (iterative soft-thresholding)

$$[L,S]=\min_{L,S}\left\|E(L+S)-d\right\|_2^2+\lambda_L\left\|L\right\|_*+\lambda_S\left\|TS\right\|_1$$

initial solution: $M_0=E^*d,\ S_0=0$

while not converged **do:**

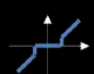
$$L_k=SVT_{\lambda_L}\left(M_{k-1}-S_{k-1}\right)$$

$$S_k=T^{-1}\left(\Lambda_{\lambda_S}\left(T\left(M_{k-1}-L_{k-1}\right)\right)\right)$$

$$M_k=L_k+S_k-E^*\left(E\left(L_k+S_k\right)-d\right)$$

end while

Soft-thresholding: $\Lambda_\lambda(x)=\frac{x}{|x|}\max\left(|x|-\lambda,0\right)$

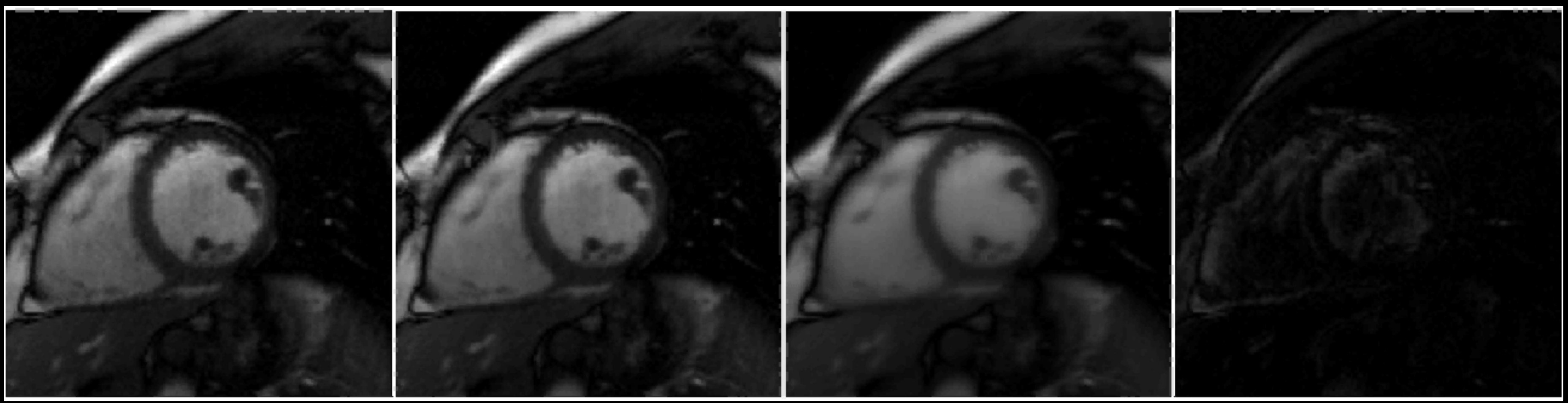


Singular value thresholding: $\mathrm{SVT}_\lambda(M)=U\Lambda_\lambda(\Sigma)V^H$

L+S reconstruction of cardiac cine

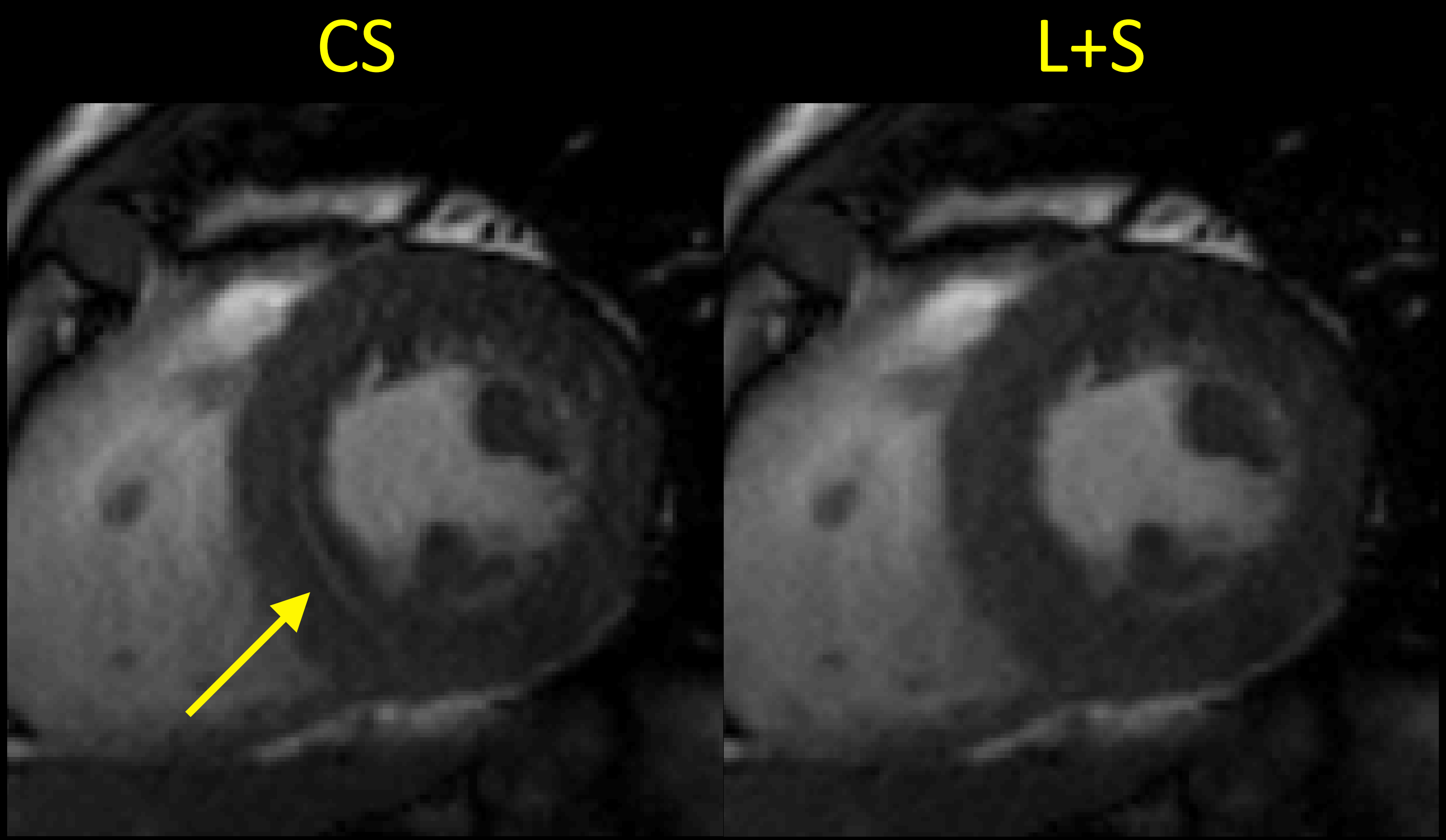
- 6-fold acceleration (k_y-t random undersampling)
- Temporal resolution = 40 ms
- Spatial resolution = $1.3x1.3x3mm^3$
- T: temporal FFT

CS L+S L S



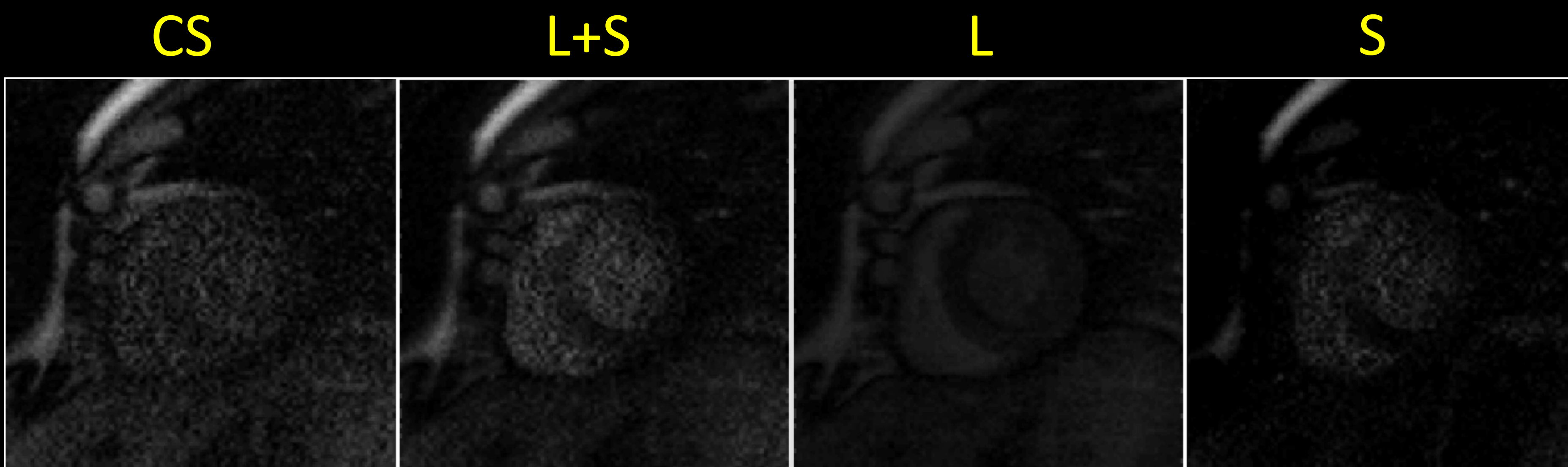
CS: standard compressed sensing using T

L+S reconstruction of cardiac cine



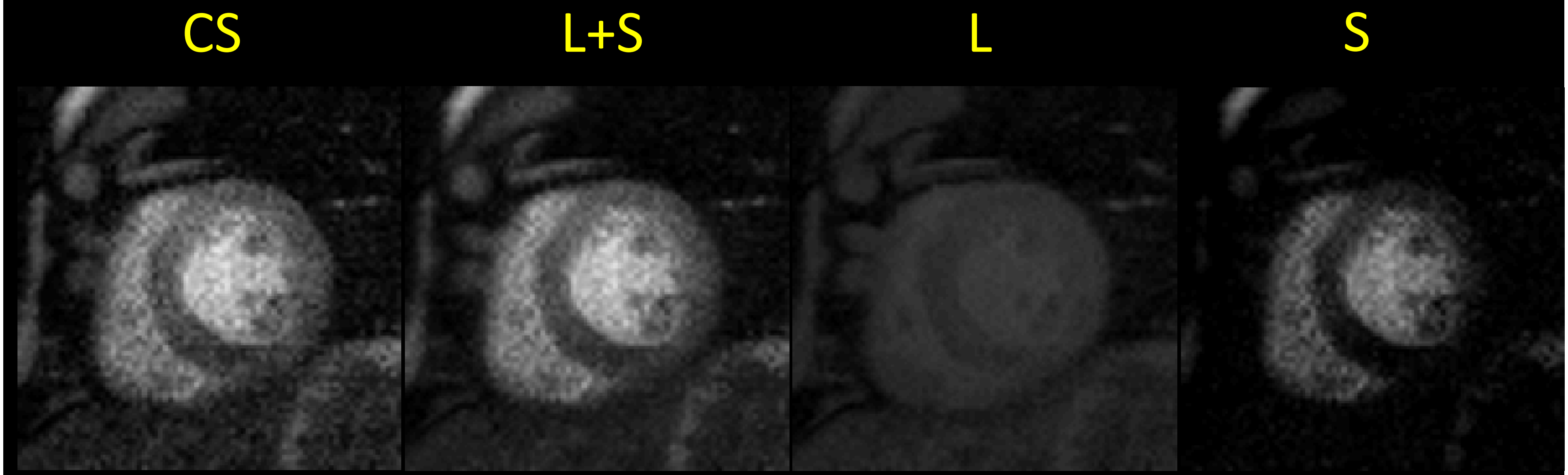
L+S reconstruction of cardiac perfusion (patient)

- 8-fold acceleration (k_y-t random undersampling)
- Temporal resolution = 60 ms
- Spatial resolution = 1.7x1.7x3mm^3
- T: temporal FFT



Patient with coronary artery disease

L+S reconstruction of cardiac perfusion (patient)



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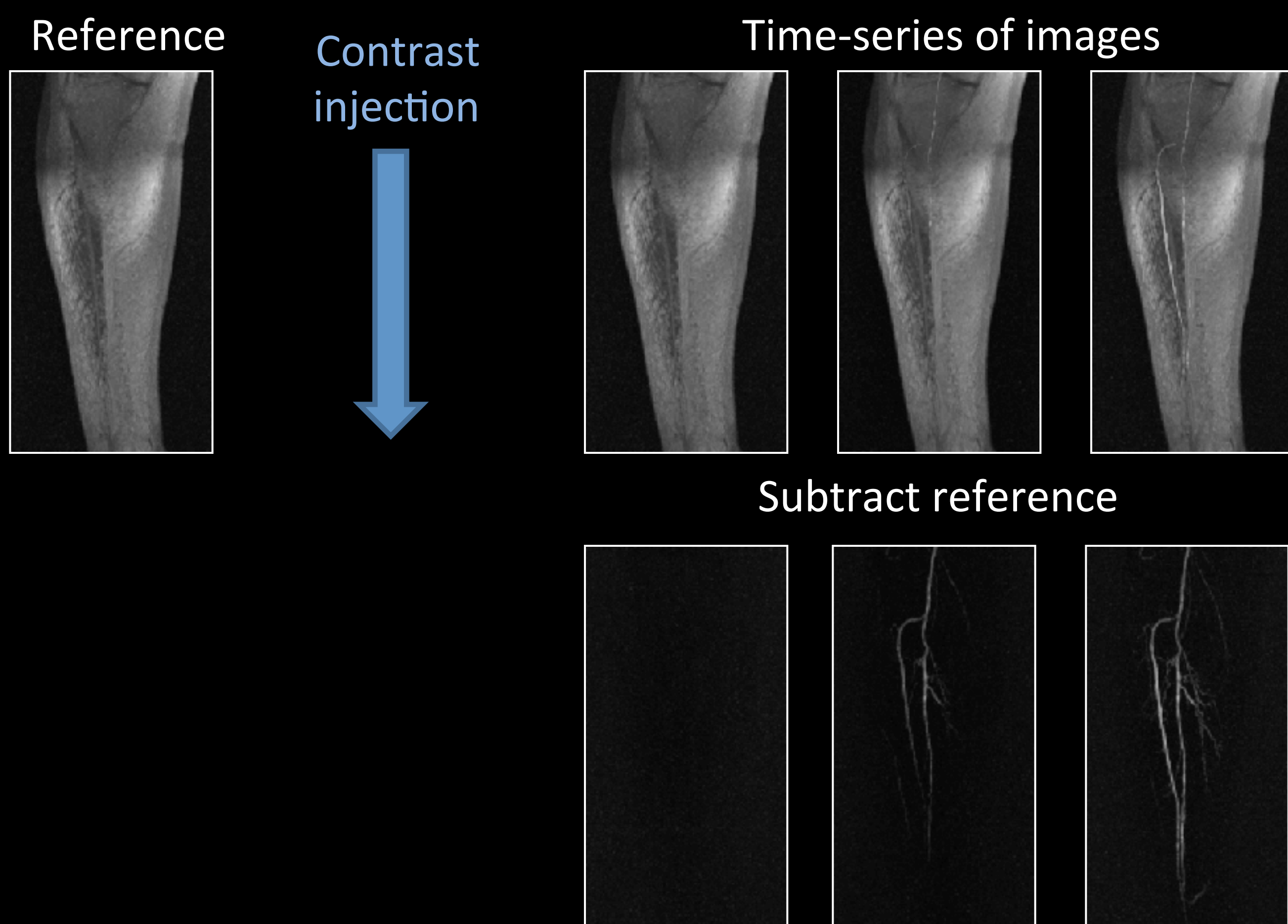
L+S reconstruction of time-resolved MRA

- It requires background suppression

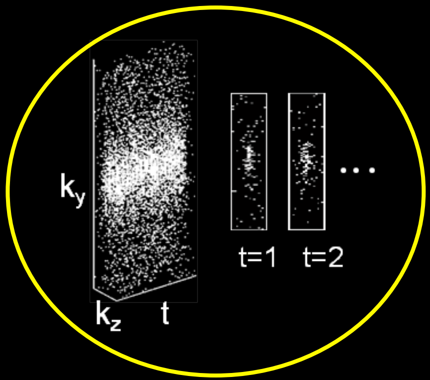


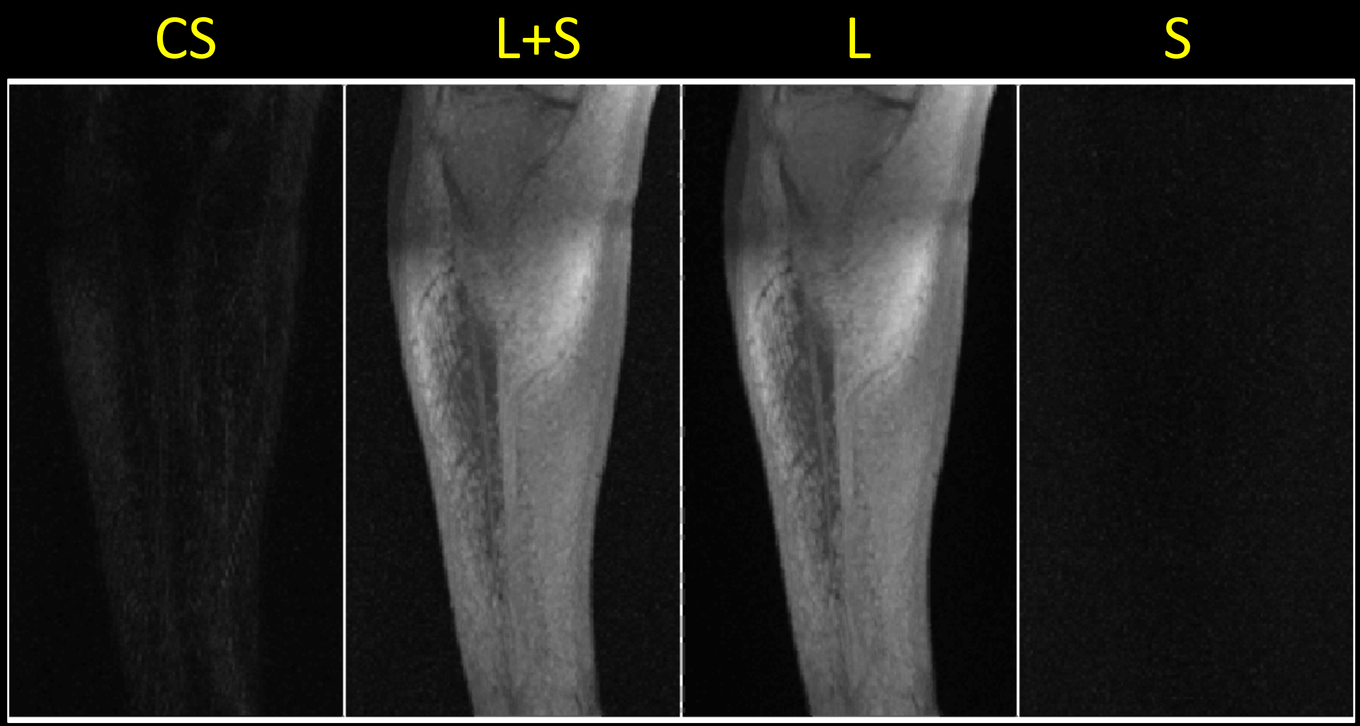
L+S reconstruction of time-resolved MRA

- Inconsistencies between reference and dynamic data
  - Residual background
  - Incorrect angiogram

L+S reconstruction of time-resolved MRA

- 7.5-fold acceleration
  - k_y-k_z-t random undersampling
- T: identity (angiograms are already sparse)

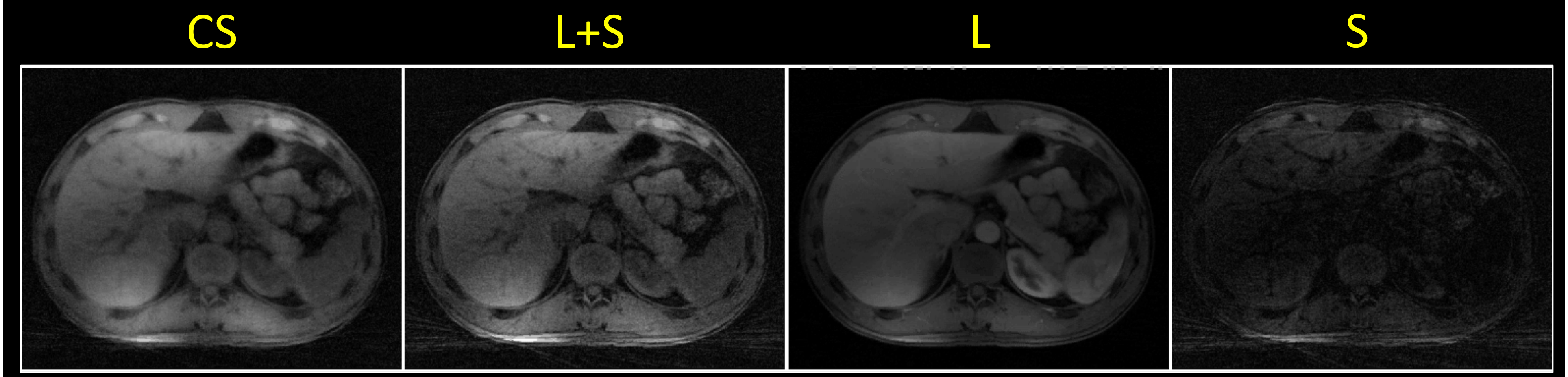




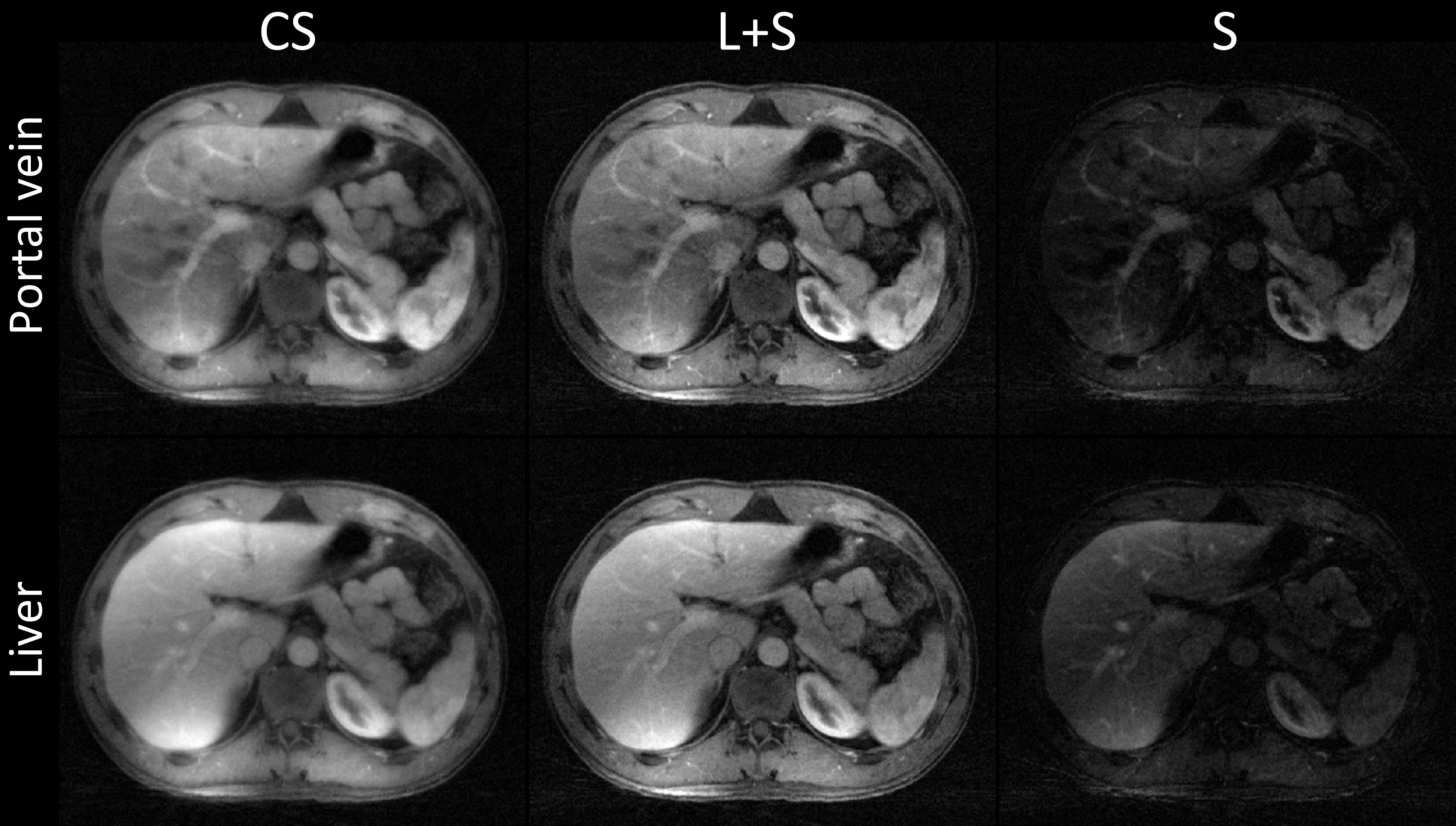
CS uses data subtraction as sparsifying transform

L+S reconstruction of radial abdominal DCE-MRI

- Continuous golden-angle radial acquisition
- NUFFT (gridding and density compensation)
- Only 8 spokes/temporal frame
  - About 50 fold-acceleration
- T: temporal finite differences



L+S reconstruction of radial abdominal DCE-MRI



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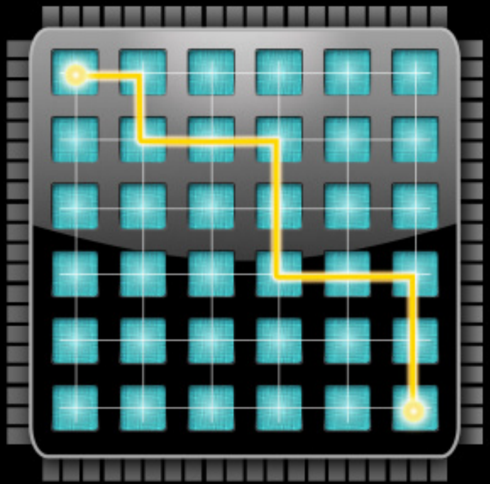
L+S reconstruction using parallel computing

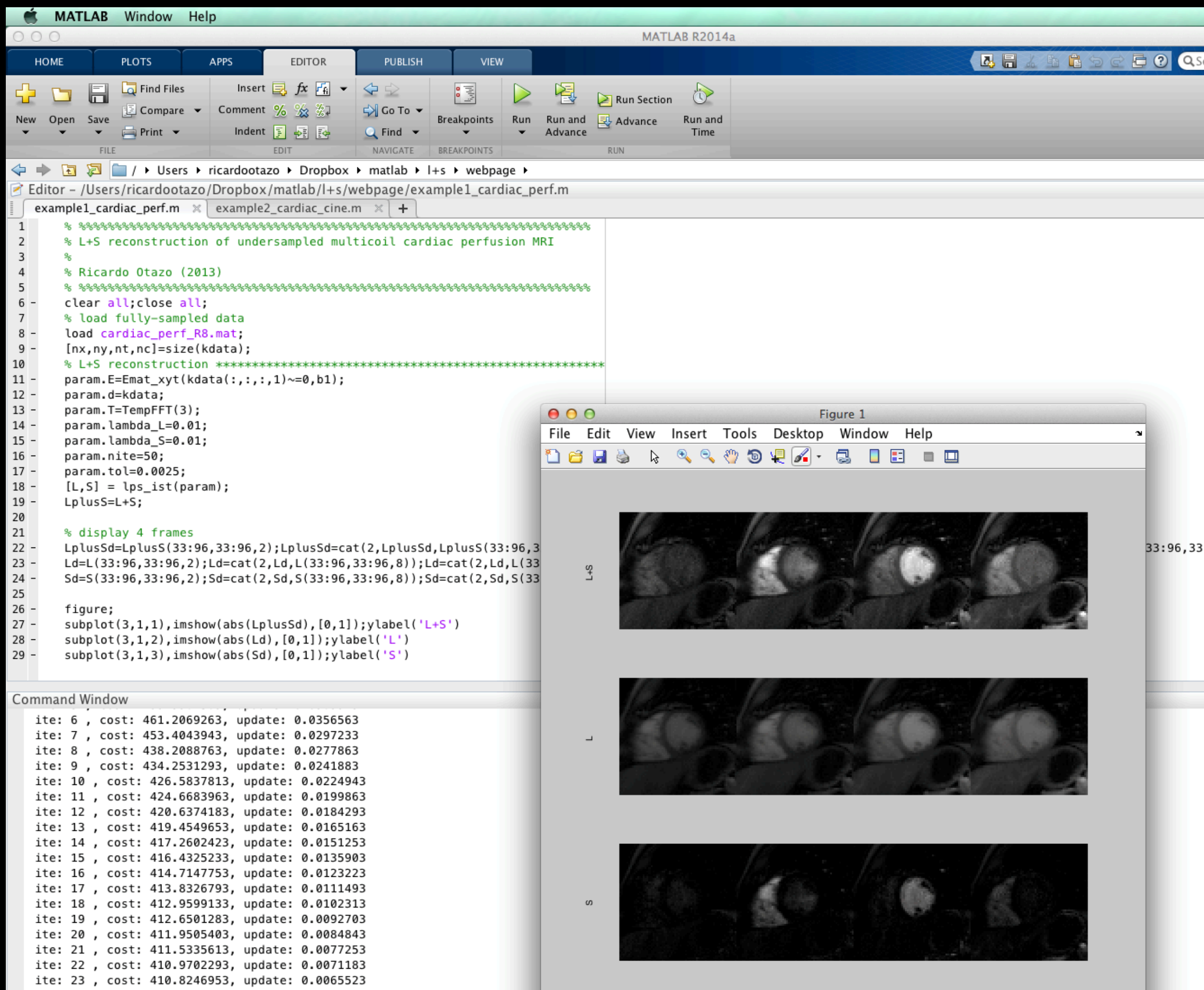


- Speed up the reconstruction process
- Parallelization
  - Separate reconstruction for each slice
  - Singular value soft-thresholding
  - Fourier transform (time and coil dimensions)
- 96-core computer, reconstructed matrix = 384x384, 40 slices, 12 coils, 48 time points
  - 5 seconds/slice
  - 4D reconstruction under 5 minutes

Joint work with Jack Poulson

Matlab code online

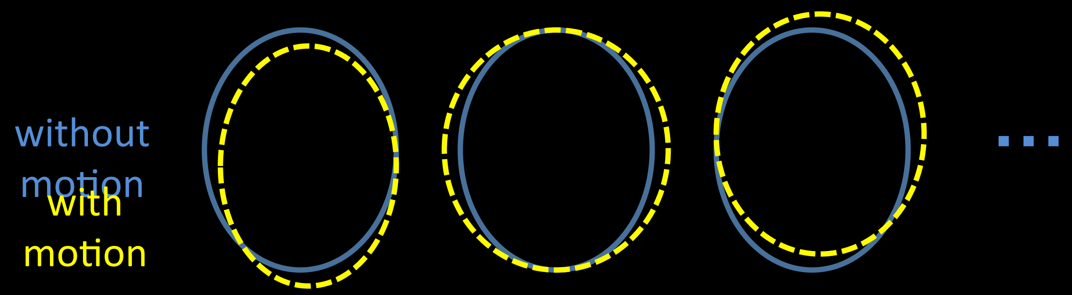
http://cai2r.net/resources/software/ls-reconstruction-matlab-code

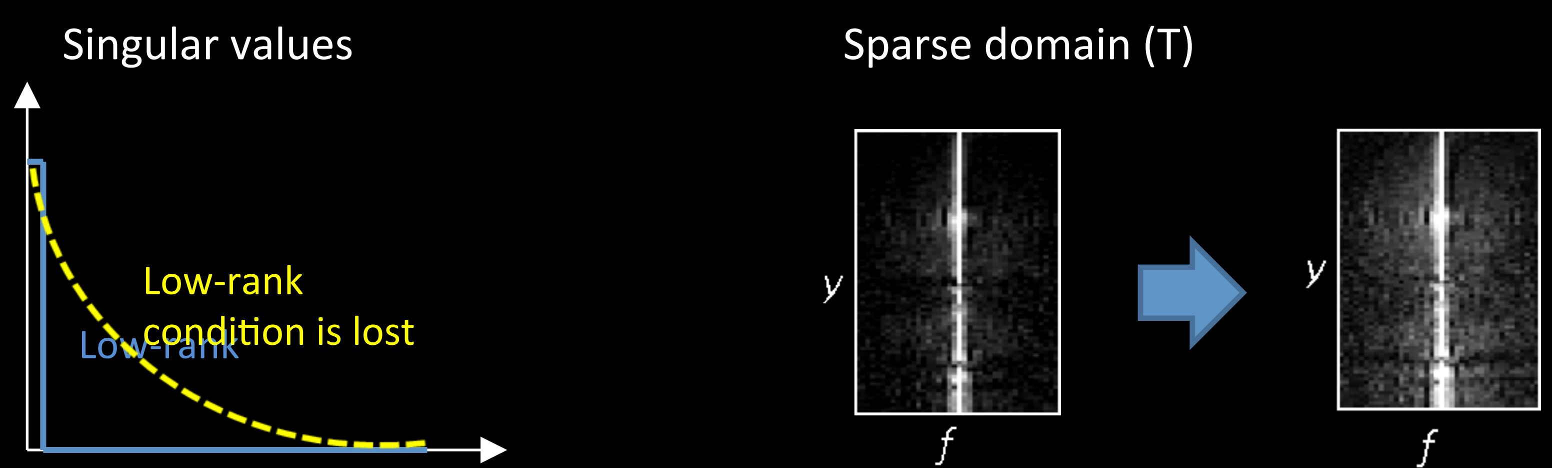


Motion-guided L+S reconstruction

Inter-frame motion issues

- Low-rank plus sparse model breaks down





Motion reduces sparsity

Inter-frame motion issues

- Motion model into L+S decomposition

$$W = \begin{bmatrix} W_1 & W_2 & W_3 & \cdots \end{bmatrix}$$

(frame-by-frame warping operator)

$$L+S = W(M)$$

low-rank + sparse model is back!

Motion-guided L+S reconstruction

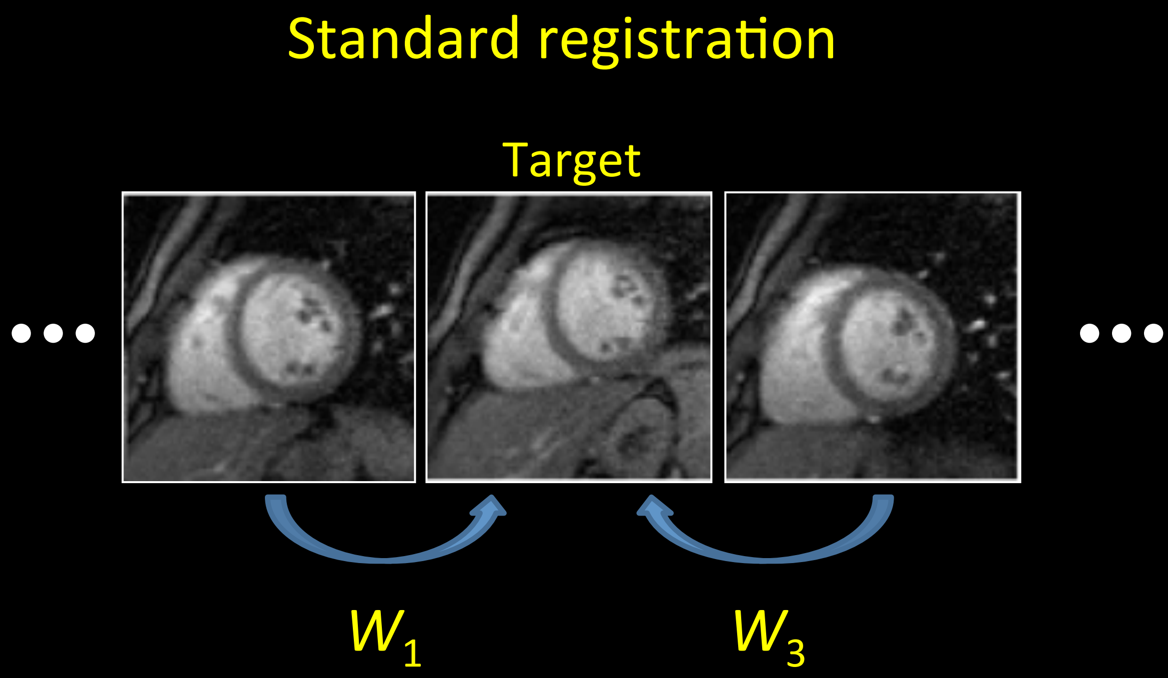
- L + S reconstruction with self motion estimation and compensation
  - Find *L*, *S* and *W* to

$$\text{minimize} \|L\|_* + \lambda \|TS\|_1$$
$$\text{subject to } EW(L+S) = d$$

L: low-rank component
S: sparse component
T: temporal sparsifying transform
W: warping operator
E: encoding operator (Fourier transform + coil sensitivities)
d: undersampled k-t data

Motion-guided L+S reconstruction

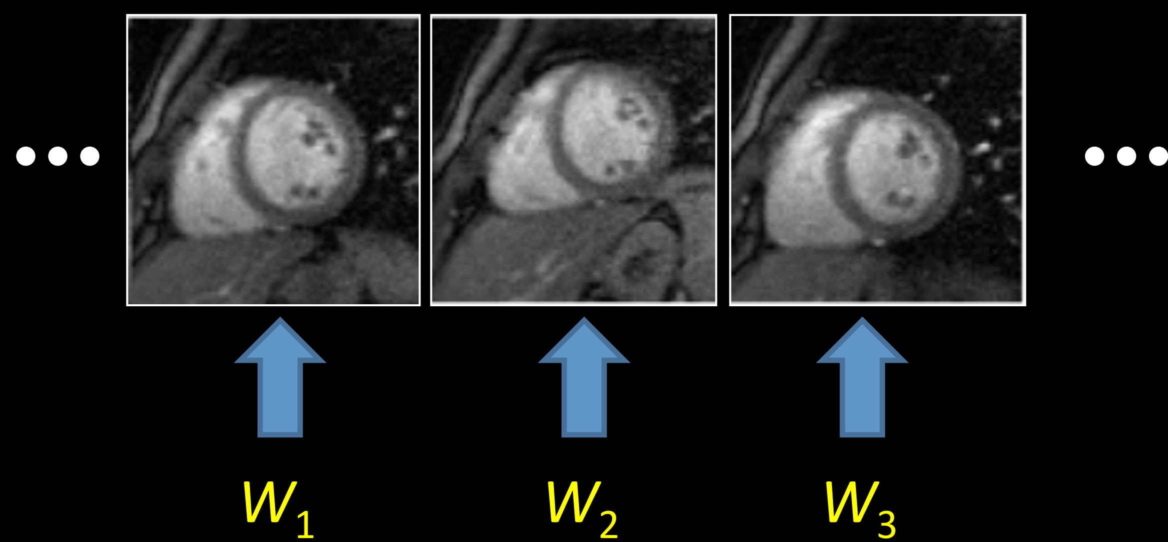
- L+S model as a tool for image-series registration
  - Different from standard image registration



Motion-guided L+S reconstruction

- L+S model as a tool for image-series registration
  - Matrix rank-sparsity as a measure of image similarity
  - Does not require similar intensity between frames

L+S registration



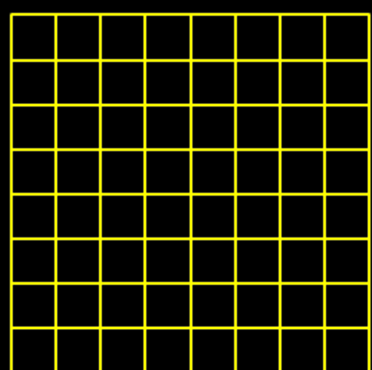
Warping operator

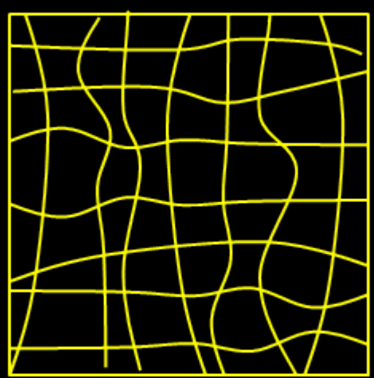
- Regridding in the image domain

$$(x,y) \xrightarrow{W} (x+v_x, y+v_y)$$

Original coordinates

Transformed coordinates
v_x, v_y: motion vectors





(non-rigid motion)

Motion vectors

- Linear approximation of the warping operator
  - Optical flow
  - Pixel-by-pixel motion information

$$W(M) = M + \frac{\partial M}{\partial x} v_x + \frac{\partial M}{\partial y} v_y$$

Gradients of M

Motion vectors

Baker S et al. Int J on Comp Vision. 2004
Dawood et al. IEEE Trans Med Imag. 2008

Motion-guided L+S reconstruction

- L + S reconstruction with self motion estimation and compensation
  - Find *L*, *S* and *v* to

$$\text{minimize} \left\|L\right\|_* + \lambda \left\|TS\right\|_1$$
$$\text{subject to } E\left(L + S + G_{L+S}v\right) = d$$

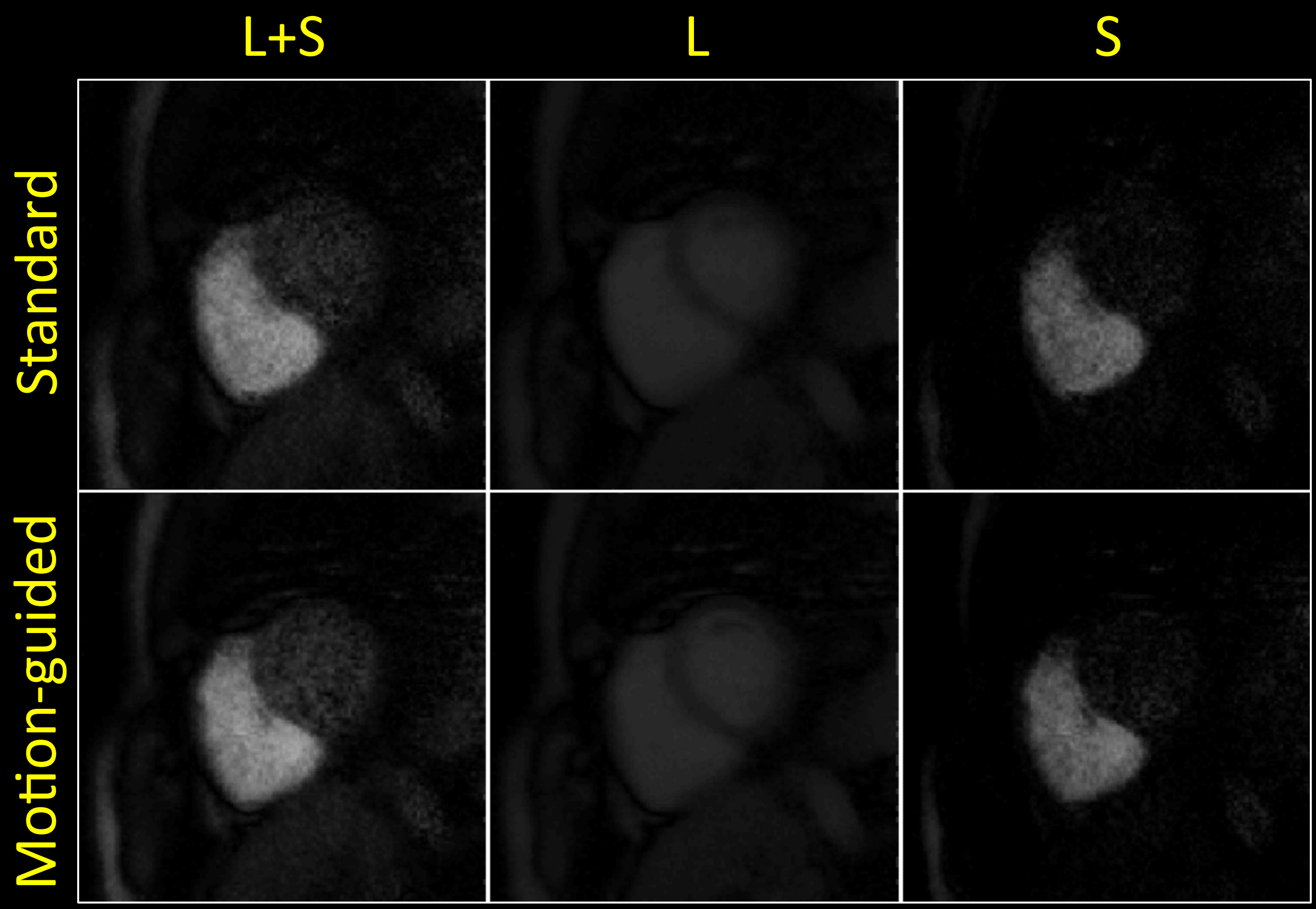
G_{L+S} : gradient of L+S
v : motion vectors

Motion-guided L+S reconstruction

- Initial solution (coil-combined inverse FFT)

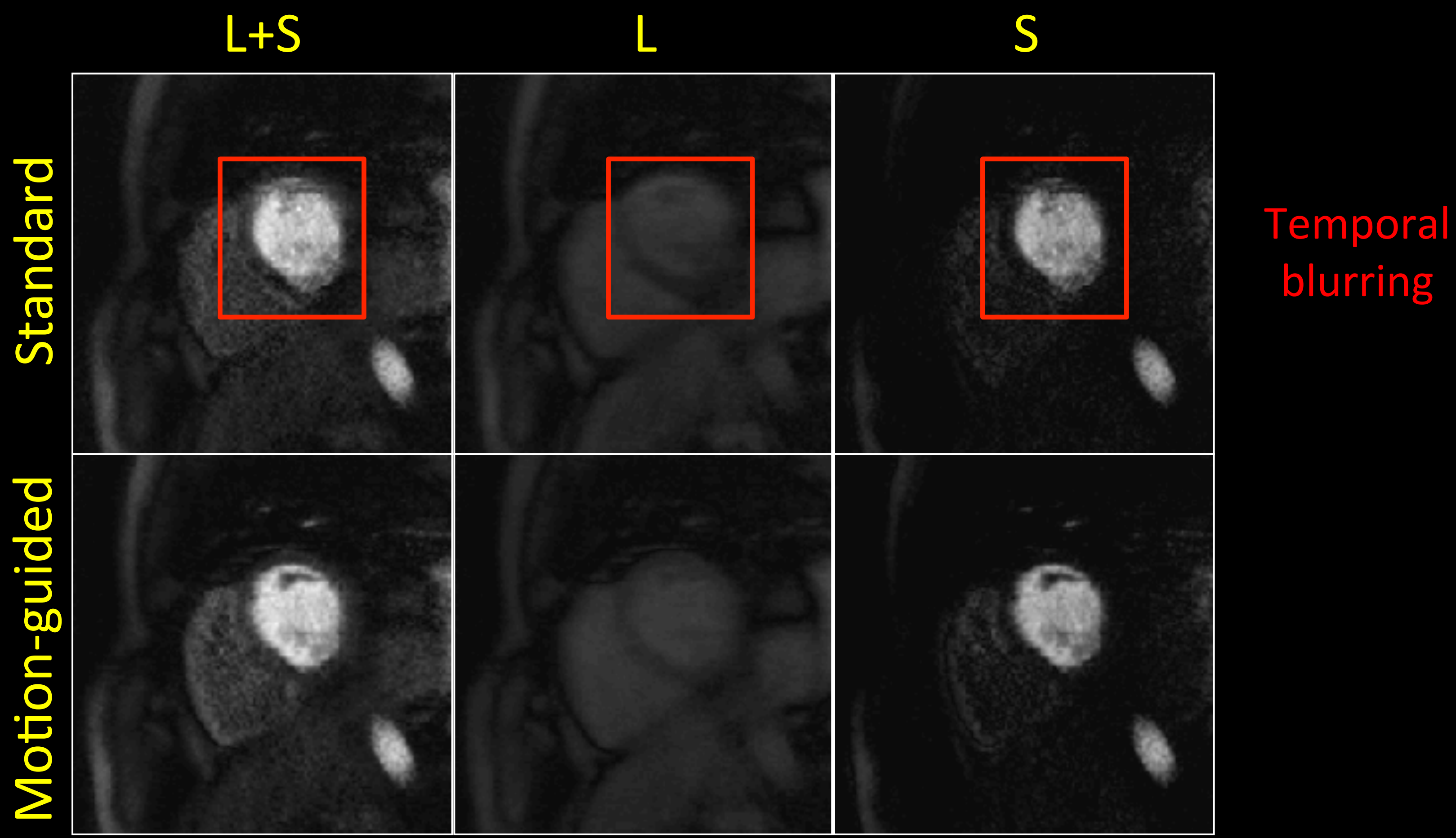
- Iterations

Free-breathing cardiac perfusion



Siemens 3T scanner
TurboFLASH sequence with 8-fold acceleration (k_y-t random undersampling)
Temporal resolution = 60ms
Spatial resolution = 1.7x1.7 mm^2
T: temporal FFT

Free-breathing cardiac perfusion



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Temporal resolution = 60ms
Spatial resolution = 1.7x1.7 mm^2
T: temporal FFT

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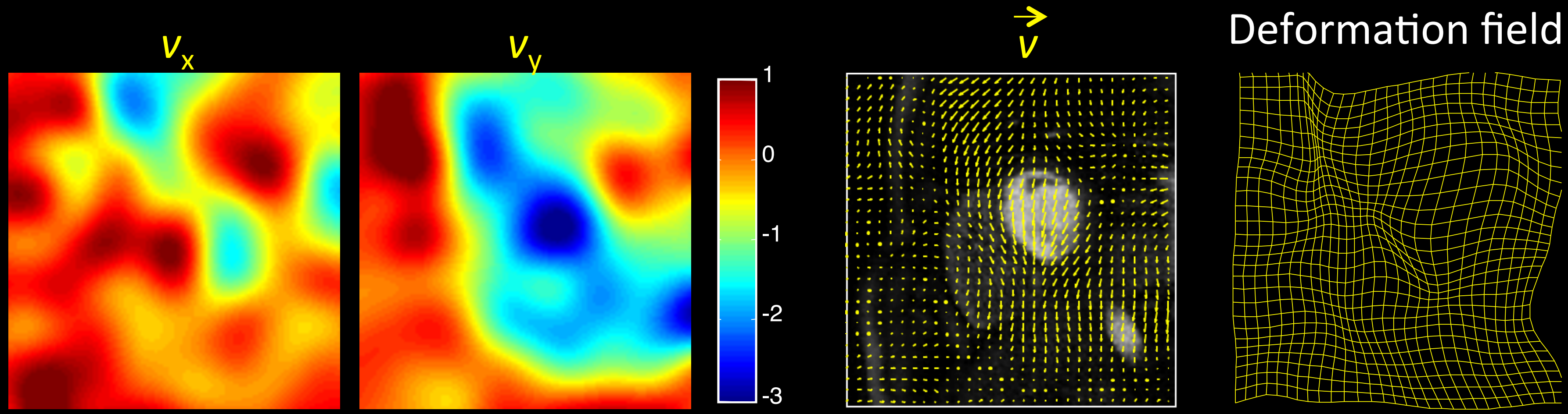
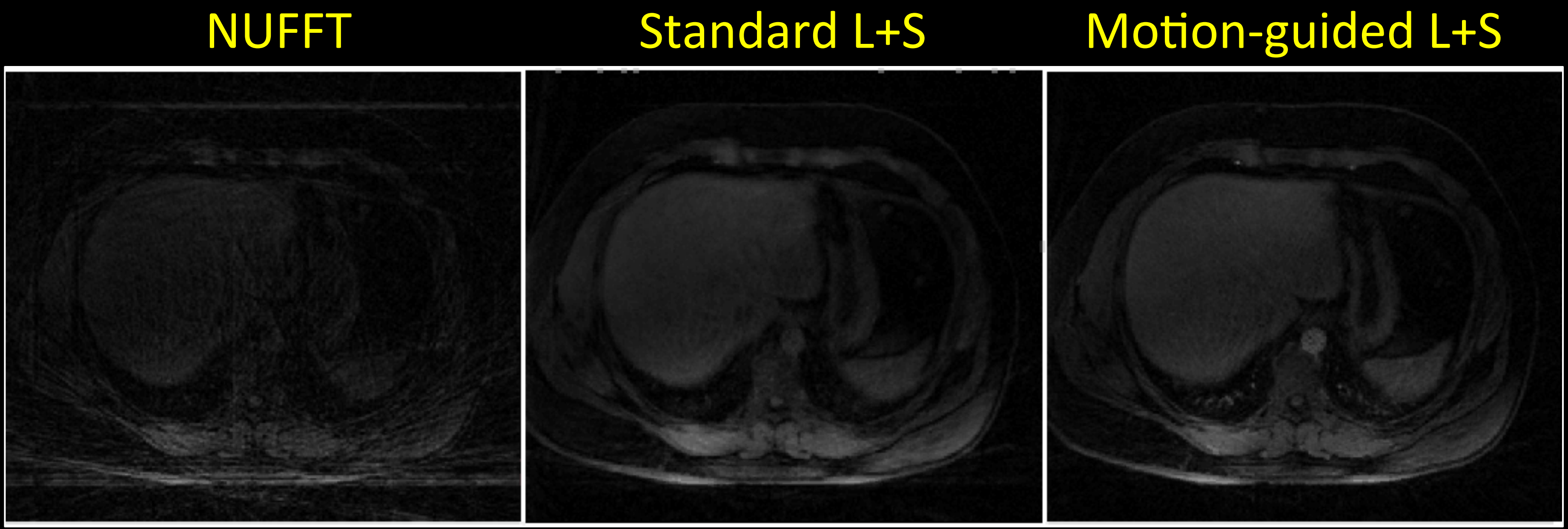
Free-breathing cardiac perfusion

- Motion vectors
  - non-rigid motion estimation



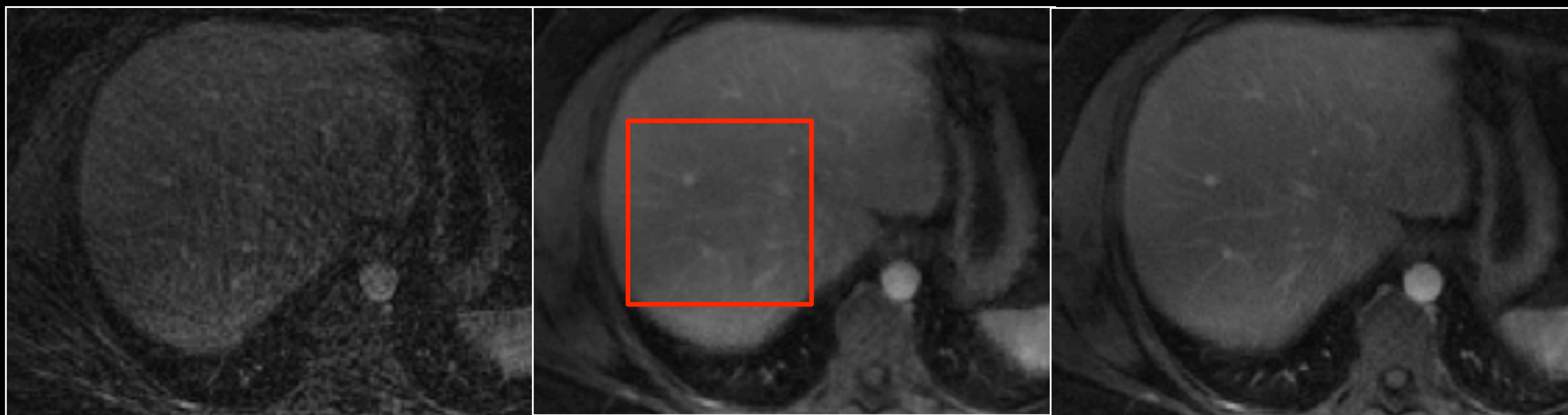
Free-breathing DCE-MRI of the liver



Siemens 3T scanner
Radial VIBE sequence
20 spokes/frame (12.8-fold acceleration)
T: temporal finite differences

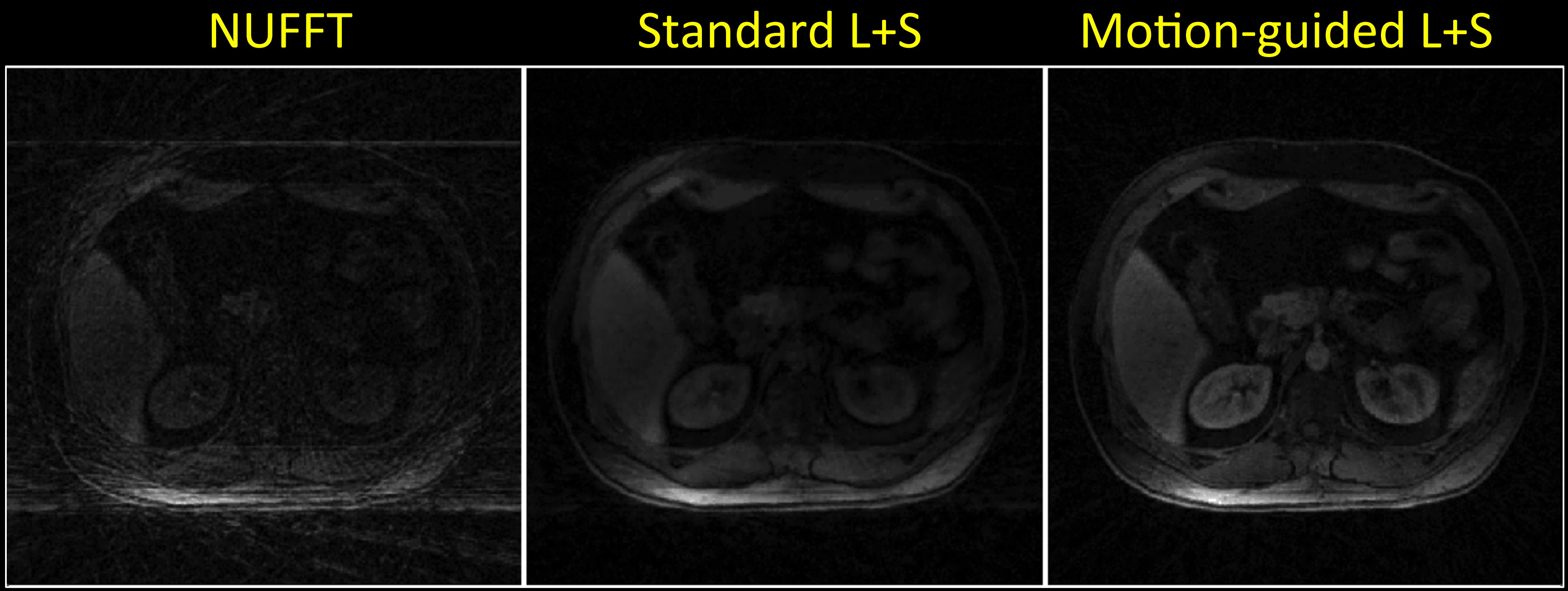
Free-breathing DCE-MRI of the liver

NUFFT | Standard L+S | Motion-guided L+S



Temporal blurring

Free-breathing DCE-MRI of the kidneys



Siemens 3T scanner
Radial VIBE sequence
20 spokes/frame (12.8-fold acceleration)
T: temporal finite differences

Free-breathing DCE-MRI of the kidneys

NUFFT | Standard L+S | Motion-guided L+S

Summary

- L+S reconstruction
  - Robust PCA for accelerated dynamic MRI
  - Higher performance than standard compressed sensing
  - Background/dynamic separation

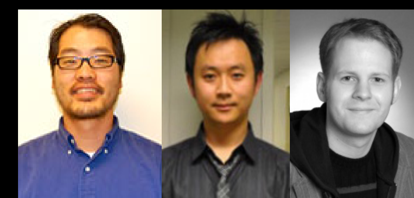
- Motion-guided L+S
  - Self-discovery of motion
  - Undersampled data only

- Lots of clinical applications

Acknowledgments

- Emmanuel Candes

- Daniel Sodickson
- Daniel Kim, Li Feng, Tobias Block – data acquisition



- Thomas Koesters – motion models

- Jack Poulson – parallel computing

- Leon Axel and Hersh Chandarana – clinical interpretation


NYU School of Medicine
Department of Radiology
Center for Biomedical Imaging
Thank you!