Super-resolution Optical Flow and Zooming Optical Flow: Variational Approaches

Yoshihiko Mochizuki, Atsushi Imiya, Yusuke Kameda, Tomoya Sakai
Chiba University, Japan

Setting R to be a pyramid-based downsampling operation, we deal with two problems. 1. Compute optical flow $\boldsymbol{u}(\boldsymbol{x},t)$ of an image sequence $f(\boldsymbol{x},t)$ from a low-resolution images $R^k f(\boldsymbol{x},t)$ for $k \geq 1$. 2. Compute optical flow $\boldsymbol{u}(\boldsymbol{x},t)$ of an image sequence $f(\boldsymbol{x},t)$ from a zooming image sequence $R^k f(\boldsymbol{x},t-k)$ $k=1,2,\cdots n$ for $n>1$.

Since the relations $RO \neq OR$, $EO \neq OE$, and $R^\dagger O \neq OR^\dagger$ for the generalized inverse of R are satisfied for the optical flow computation procedure O and an interpolation-based upsampling operation E which is the dual of R, any of $E\boldsymbol{v} = R^*\boldsymbol{v}$, $R^\dagger \boldsymbol{v} = \lim_{\varepsilon\to 0}(ER+\varepsilon I)^{-1}E\boldsymbol{v}$, and $E_{W\mu}\boldsymbol{v} = (ER+\mu W)^{-1}E\boldsymbol{v}$ for an appropriate operator W and a positive constant μ are not optical flow of f for the optical flow $\boldsymbol{v}$ of Rf. Therefore, for the first problem, we solve the variational problem

$$S(\boldsymbol{u}) = \int_{\mathbf{R}^2} \left[\left\{(R^k f - g)^2 + \kappa Q(f)\right\} + \left\{(\nabla f^\top \boldsymbol{u} + \partial_t f)^2 + \lambda P(\boldsymbol{u})\right\}\right] d\boldsymbol{x}$$

assuming that $P(\cdot)$ and $Q(\cdot)$ are convex priors. If $\lambda \gg \kappa > |\nabla f|$, we have four Euler-Lagrange equations, $Q_{f_x} - \frac{1}{\kappa}E^k(R^k f_x - \frac{1}{\sigma^k}g_x) = 0$, $Q_{f_y} - \frac{1}{\kappa}E^k(R^k f_y - \frac{1}{\sigma^k}g_y) = 0$, $Q_{f_t} - \frac{1}{\kappa}E^k(R^k f_t - g_t) = 0$, $P_{\boldsymbol{u}} - \frac{1}{\lambda}(\nabla f^\top \boldsymbol{u} + \partial_t f)\nabla f = 0$. This system of partial differential equations can be read that first recovering f_x, f_y, and f_t minimizing three independent variational forms, $I^x(\boldsymbol{u}) = \int_{\mathbf{R}^2}\{(\frac{1}{2^k}R^k f_x - g_x)^2 + \kappa Q(f_x)\}d\boldsymbol{x}$, $I^y(\boldsymbol{u}) = \int_{\mathbf{R}^2}\{(\frac{1}{2^k}R^k f_y - g_y)^2 + \kappa Q(f_y)\}d\boldsymbol{x}$, $I^t(\boldsymbol{u}) = \int_{\mathbf{R}^2}\{(R^k f_t - g_t)^2 + \kappa Q(f_t)\}d\boldsymbol{x}$, then we second compute the upsampled optical flow minimizing the variational form $J_S(\boldsymbol{u}) = \int_{\mathbf{R}^2}\left\{((\nabla f^\top \boldsymbol{u} + \partial_t f)^2 + \lambda P(\boldsymbol{u})\right\} d\boldsymbol{x}$, using three minimizers of the previous three variational forms.

Beyond engineering applications, the answer to the second problem clarify a relationship between motion cognition and focusing an a field of attention. For instance, humans see a moving object in a scene as a part of the environment around us. If we realize that a moving object is important for the cognition of the environment, we try to attend to the object, and start to watch it closer "by increasing the resolution locally." The series of minimization problems with a convex prior $P(\cdot)$

$$\begin{aligned} J(\boldsymbol{u}_{k-1}) \quad &= \iint_{\mathbf{R}^2} \left\{(f_{(k)}(\boldsymbol{x},t-k) - Rf_{(k-1)}(\boldsymbol{x} - E\boldsymbol{u}_k(t-k), t-(k-1)))^2\right. \\ &\qquad \left. + \lambda P(\boldsymbol{u}_k)\right\} d\boldsymbol{x}, \end{aligned}$$

where $f_{(k)} = R^k f$, generates a sequence $\{\boldsymbol{u}_k(\boldsymbol{x},t-k)\}_{k=1}^n$ which converges to $\boldsymbol{u}_0$, if $\boldsymbol{u}_k$ satisfies $|\boldsymbol{u}_k| \leq \alpha$, $|\frac{\partial \boldsymbol{u}_k}{\partial t}| \leq \beta$, and $|\nabla \boldsymbol{u}_k| \leq \gamma$ for a fixed resolution and $|\frac{\partial \boldsymbol{u}_k}{\partial k}| \leq \delta$ across resolution layers.