# Multimodal image registration by optimal control methods

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**Problem statement.** We present an optimal control approach to the registration of greyscale images $I_0(s)$, $I_1(s)\colon\ \Omega \to [\,0\,,\,1\,]$, $\Omega \subset \mathbb{R}^2$, with different modalities. Following [ 4 ] and [ 5 ], we determine an elastic deformation $x\colon\ \Omega \to \mathbb{R}^2$ as a solution of the following Dieudonné-Rashevsky type control problem:

$$(\mathrm{P})\colon\quad F(x) = \int_\Omega \Big( k_1\big(\, s - x(s)\,\big) - k_0(s) \Big)^2 \, ds \\ + \mu \cdot \int_\Omega r\big(\, Jx(s)\,\big)\, ds \;\longrightarrow\; \inf\!{}!\,;\ x \in W_0^{1,\infty}(\Omega, \mathbb{R}^2)\,;\ Jx(s) \in \mathrm{K} \subset \mathbb{R}^{2\times 2}\ \ (\forall)\, s \in \Omega\,.$$

The registration is based on edge sketches $k_0(s)$, $k_1(s)$, which will be derived form the image data $I_0$, $I_1$ by an optimal control method as well. Within (P), $\mathrm{K} \subset \mathbb{R}^{2\times 2}$ is a convex body with $\mathfrak{o} \in \mathrm{int}\,(\mathrm{K})$. We consider the linear-elastic or hyperelastic regularization terms

$$r\big(\, Jx(s)\,\big) \;=\; \sum_{i,j=1}^{2} \Big( \frac{\partial x_i(s)}{\partial s_j} \,+\, \frac{\partial x_j(s)}{\partial s_i} \Big)^2 \, ds\,,$$

$$r\big(\, Jx(s)\,\big) \;=\; \Big( c_1 \,\big\|\, E_2 - Jx(s) \,\big\|^{\,p} \,+\, c_2 \,\big(\, \mathrm{Det}\,(\, E_2 - Jx(s)\,)\,\big)^2 \Big)\, ds\,,$$

respectively. The introduction of constraints for the Jacobi matrix $Jx$ reflects the necessity to ensure the validity of the underlying elasticity models (the resulting shear stress must remain bounded).

**Numerical solution.** For the numerical solution of (P), an efficient direct method will be provided. We evaluate the necessary optimality conditions for the discretized problem and solve the resulting large-scale system of nonlinear equations by interior-point methods, cf. [ 1 ] and [ 5 ]. Selected numerical results will be presented and discussed.

**References.**

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