Abstract:

Solutions of certain PDEs and variational problems may be characterized by a "few significant features", and one may want to take advantage of this feature in order to design efficient numerical solutions.
For example, we can consider singular PDEs with solutions which may have a few discontinuities. We can also be interested to variational problems which impose sparse basis expansions of minimal solutions. Examples of such situations are ubiquitous: digital signal coding/decoding, compressed sensing, singular PDEs for image processing, crack modelling and free-discontinuity problems, viscosity solutions of Hamilton-Jacobi equations.
In the first part of the talk, we retrace the role of L1-minimization as a method for sparsifying solutions in several contexts.
Then we address particular applications and numerical methods. We present the analysis of a superlinear convergent algorithm for L1-minimization based on an iterative reweighted least squares. We show improved performances in compressed sensing.
A similar algorithm is then applied for the efficient solution of a system of singular PDEs for image recolorization in a relevant real-life problem of art restoration.
We conclude by presenting initial promising results in domain decomposition methods for singular PDEs, for which solutions may be discontinuous. The discontinuities may cross the interfaces of the domain decomposition patches. The crucial difficulty is the correct treatment of interfaces, with the preservation of crossing discontinuities and the correct matching where the solution is continuous instead. We discuss the convergence properties of the proposed method and several numerical examples both in 1D and 2D.