Exercise sheet 8

Exercise 1 [PSB update]

Prove Lemma 2.63 from the lecture.

Exercise 2 [SR1 update 1]

Let $d^k = x^{k+1} - x^k$ and $y^k = \nabla f(x^{k+1}) - \nabla f(x^k)$. We are looking for a updating formula of H_{k+1} such that

1. H_{k+1} is symmetric,
2. the equation $H_{k+1} d^k = y^k$ is satisfied,
3. $H_{k+1} = u_k + \gamma_k u^k (u^k)^\top$, $\gamma_k \in \mathbb{R}$ and $\|u^k\| = 1$ holds.

Find such γ_k and u_k and formulate assumption such that they exist. Is the update $\gamma_k u^k (u^k)^\top$ unique? Why can Theorem 2.59 not be applied in this case.

Exercise 3 [SR1 update 2]

We consider the inverse SR1 method described in lecture. Here $B_k = H_k^{-1}$ with H_k from the last exercise. We consider the case of a linear quadratic function:

$$f(x) = \langle b, x \rangle + \frac{1}{2} \langle Ax, x \rangle,$$

where A is symmetric positive definite. We assume that the condition $\langle d^k - B^k y^k, y^k \rangle \neq 0$ is satisfied for all k.

a) Find a relation between d^k and y^k.

b) Prove by induction that for all $k \geq 1$, for all $i = 0, ..., k-1$,

$$B_k y^i = d^i.$$

c) We assume that the vectors y^0,...,y^{n-1} are linearly independent. Prove that $B_n = A^{-1}$. What can we say about x^{n+1}?

Exercise 4 [Inverse BFGS update]

Formulate an update formula for B_{k+1}, where $B_k = H_k^{-1}$ and H_k is generated by the BFGS update formula from Lemma 2.65 in the lecture. Moreover formulate Assumptions such that B_{k+1} exists.