Hilbert Space Methods for Partial Differential Equations SS21, Exercise Sheet 14

See exercises 2.1 - 2.5 in Showalter's chapter 3.

1. Give the details of the construction of α, β on p. 79 in the lecture notes.
2. Verify the details of the examples on pp. 81 and 83 in the lectures notes.
3. With φ_u defined on p. 83 in the lectures notes, construct an $F \in (V/V_0)'$ satisfying $F(\hat{v}) = \varphi_u(v)$, $v \in V$, and check that it is continuous.
4. Show that $a(u, v) = \int_0^1 \partial u \partial v$, $V = \{u \in H^1(0, 1) : u(0) = 0\}$, and $f(v) = v(1/2)$ are admissible data in the theorem on p. 8ß in the lecture notes. Find a formula for the unique solution of the problem.
5. In the theorem on p. 80 in the lecture notes the continuous dependence of the solution u on the data b follows from the estimate made in the theorem. Consider the two abstract boundary value problems $\mathcal{A}_1 u_1 = b$ and $\mathcal{A}_2 u_2 = b$ where $b \in V'$, and $\mathcal{A}_1, \mathcal{A}_2 \in \mathcal{L}(V, V')$ are coercive with constants c_1, c_2, respectively. Show that the following estimates hold:

   $$\|u_1 - u_2\| \leq (1/c_1)\|(\mathcal{A}_2 - \mathcal{A}_1)u_2\|, \quad \|u_1 - u_2\| \leq (1/c_1 c_2)\|\mathcal{A}_2 - \mathcal{A}_1\|\|b\|.$$

   Explain how these estimates show that the solution u of $a(u, v) = b(v)$, $\forall v \in V$, depends continuously on the form $a(\cdot, \cdot)$ or operator A.