Hilbert Space Methods for Partial Differential Equations SS21, Exercise Sheet 8

See exercises 1.1 - 1.6 in Showalter's chapter 2.

1. Evaluate $(\partial - \lambda)(H(x)e^{\lambda x})$ and $(\partial^2 + \lambda^2)(\lambda^{-1}H(x)sin(\lambda x))$ for $\lambda \neq 0$.
2. Find all distributions of the form $F(t) = H(t)f(t)$ where $f \in C^2(\mathbb{R})$ such that $(\partial^2 + 4)F = c_1\delta + c_2\partial\delta$.
3. Let K be the square in $\mathbb{R}^2$ with corners at $(1, 1)$, $(2, 0)$, $(3, 1)$, $(2, 2)$, and let T_K be the function equal to 1 on K and 0 elsewhere. Evaluate $(\partial_1^2 - \partial_2^2)T_K$.
4. Obtain the results at the end of page 48 in the script using the constructions at the beginning of that page.
5. Evaluate $\Delta(1/|x|^{n-2})$.
6. Let G be given as on p. 48 in the script and show the following identities.
   (a) Show that for each function $f \in C^2(\overline{G})$ the identity
   $$\int_G \partial_j f(x)dx = \int_{\partial G} f(s)\nu_j(s)ds, \quad 1 \leq j \leq n,$$
   follows from the fundamental theorem of calculus.
   (b) Show that Green's first identity
   $$\int_G (\nabla u \cdot \nabla v + v\Delta u)dx = \int_{\partial G} \frac{\partial u}{\partial \nu} v ds$$
   follows from above for $u \in C^2(\overline{G})$ and $v \in C^2(\overline{G})$. Hint: Take $f_j = (\partial_j u)v$ and add.
   (c) Obtain Green's second identity from above.