Exercise Sheet 9 for Optimization 1
Winter Semester 2011/12

1. (Implementation of inexact Netwon) The same thing as every time: Implement the inexact Newton method and test it with the examples from the steepest descent problem of sheet 6. Which order of convergence do we get? Make similar experiments for the generalised Rosenbrock function

$$f(x) = \sum_{k=1}^{n-1} (1 - x_k)^2 + 100(x_{k+1} - x_k^2)^2,$$

and various n.

2. (A globalized Newton method) Consider the following algorithm:

> Choose $x_0 \in R^n$, $\beta \in (0,1)$ and $\sigma \in (0, 0.5)$, $\varepsilon \geq 0$.
> $k = 0$
> while $\|\nabla f(x_k)\| > \varepsilon$
>   Find solution $d_k \in \mathbb{R}^n$ of the Newton equation
> $\nabla^2 f(x_k) d = -\nabla f(x_k)$,
>   Determine $\alpha_k := \max_{k \in \mathbb{N}}(\beta^k)$ such that
> $f(x_k + \alpha_k d_k) \leq f(x_k) + \sigma \alpha_k \langle \nabla f(x_k), d_k \rangle$.
> $x_{k+1} := x_k + \alpha_k d_k$.
> $k = k + 1$
> endwhile

Now let be $f \in C^2(\mathbb{R}^n, \mathbb{R})$ uniformly convex. Show that

(a) the algorithm is well defined,

(b) the sequence $(x_k)_{k \in \mathbb{N}}$ converges for every x_0 to the unique minimizer x^*,

(c) for k sufficiently large $\alpha_k = 1$,

(d) if $\nabla^2 f$ is locally Lipschitz continuous, then the convergence is q-quadratic.

3. (Optimal control of a temperature source, and the Fourier method) We look at the following optimal control problem:

$$\min_{(y,u)} \frac{1}{2} \int_0^\pi (y(x) - z(x))^2 dx + \frac{\alpha}{2} \int_0^\pi u(x)^2 dx,$$

under the constraints $-\frac{d^2 y}{dx^2} = u$ and $y(0) = y(\pi) = 0$, where z and $\alpha > 0$ are given. Let z be a sine polynomial of degree n, namely $z(x) = \sum_{k=1}^n z_k \sin(kx)$. Now find sine polynomials y, u which are solutions of the minimization problem above. **Bonus:** Is there a connection to Fourier analysis? Can this model be applied to problems that arise at a Glühweinstandl?