Exercise Sheet 6 for Optimization 1
Winter Semester 2011/12

1. (q-convergence rates) Determine the q-convergence rates of

   (a) $x_k = 1/k$,

   (b) $x_k = 1 + 2^{-2^k}$,

   (c) $x_k = \left(\frac{1}{k}\right)^k$.

2. (Convergence of exact linesearch methods) Let $f \in C^1(\mathbb{R}^n, \mathbb{R})$. Consider the steepest descent exact line-search algorithm, i.e.,

$$x_{k+1} = x_k + \alpha_k d_k,$$

   where

$$d_k = -\frac{\nabla f(x_k)}{\|\nabla f(x_k)\|},$$

   and

$$\alpha_k = \text{arg} \min_{\alpha \in [0,1]} f(x_k + \alpha d_k).$$

   Show that every accumulation point of the sequence $\{x_k\}_{k \in \mathbb{N}_0}$ generated by the linesearch algorithm, is also a stationary point of f.

3. (Implementation of the steepest descent algorithm) Implement the steepest descent algorithm above, where the stepsize α_k is determined by

   (a) Exact linesearch (with Golden-section, Fibonacci, Newton or Bisection),

   (b) Backtracking (with Armijo or Armijo Goldstein), or

   (c) Wolfe-Powell.

   Test them on the Rosenbrock function with starting points $x_0 = (1.2, 1.2)$ and $x_0 = (-1.2, 1)$. Test them also on the Himmelblau function with starting points $x_0 = (-4, -4)$, $(2, -2)$, $(2, 2)$ and $(-4, 4)$. Experiment with the parameters to reveal their effects. Plot the number of iterations needed, and the stepsize in each iteration. **Bonus:** Demonstrate the searches graphically together with a contour plot of the function to be minimized.

4. (CG method and singular matrices) Let A be symmetric and positive semi-definite. Show that if $b \in \mathrm{R}(A)$ then the CG algorithm also finds a solution of the equation $Ax = b$ for an inital vector x_0. Is the solution unique (in some sense)?

5. (Implementation of the CG algorithm) Implement the CG algorithm. Investigate the theoretical results from the previous problem with an example.