Exercise Sheet 5 for Optimization 1
Winter Semester 2011/12

1. (Zoutendijk criterion) For $f \in C^1(\mathbb{R}^n, \mathbb{R})$ let $(x_k)_{k\in\mathbb{N}_0}$ be a sequence generated from a linesearch algorithm which satisfies the uniform descent criterion, i.e., there exists a positive constant ϑ independent of k such that

$$f(x_{k+1}) = f(x_k + \alpha_k d_k) \leq f(x_k) - \vartheta \left(\frac{\langle \nabla f(x_k), d_k \rangle}{\|d_k\|} \right)^2, \quad \forall k.$$

   Assume that the directions d_k also satisfy the *Zoutendijk criterion*, namely

$$\sum_{k=0}^{\infty} \left(\frac{\langle \nabla f(x_k), d_k \rangle}{\|d_k\| \|\nabla f(x_k)\|} \right)^2 = +\infty.$$

   Show that:

   (a) If f is bounded from below, then $\liminf_{k\to\infty} \|\nabla f(x_k)\| = 0$.

   (b) If f is uniformly convex, and the levelset $L(x_0)$ is convex, then the sequence converges to the unique minimum x^* of f.

2. (Armijo-Goldstein) Implement the backtracking algorithm with

   (a) the Armijo rule,

   (b) the Armijo-Goldstein rule,

   where the input are a function φ, the initial values $\varphi(0)$ and $\varphi'(0)$ and the parameters σ and ρ.

3. (Wolfe-Powell) Implement the Wolfe-Powell Algorithm for the minimization of the following functions:

$$f_1(x) = x^2, \text{ on } [-1, 1]$$
$$f_2(x) = (x + 10^{-3}) \sin(1/(x + 10^{-3})), \text{ on } [0, 1]$$
$$f_3(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0, \\ 0 & x = 0. \end{cases} \text{ on } [0, 1]$$

4. (Strict Wolfe-Powell) Fix $f \in C^1(\mathbb{R}^n, \mathbb{R})$, $x \in \mathbb{R}^n$ and a descent direction $d \in \mathbb{R}^n$. Further fix $\sigma \in (0, 1/2)$ and $\rho \in [\sigma, 1)$. A positive number α fullfills *the strict Wolfe-Powell rule* iff

$$f(x + \alpha d) \leq f(x) + \sigma\alpha \langle \nabla f(x), d \rangle$$
$$|\langle \nabla f(x + \alpha d), d \rangle| \leq -\rho \langle \nabla f(x), d \rangle$$

   is satisfied. Show that there exists such an α when f is bounded from below.