Numerische Mathematik II Übungen
Blatt 3 – Bearbeitung bis zum 05.12.2014

1. (VO) Write a Matlab program which will compute the highest order multistep method possible for a specified number of steps k and some of the α, β coefficients given. Your program should do the following:

   (a) Return the order p and the coefficient c_{p+1} of the leading order term in the local truncation error $c_{p+1}h^{p+1}y^{(p+1)}(t_n) + \mathcal{O}(h^{p+2})$.

   (b) Specify whether the method is 0-stable (strongly or weakly).

   (c) Plot the region of absolute stability.

2. (PS) Verify your code by computing the coefficients for the Adams and BDF methods. Give the coefficients, error term, and stability region for BDF5. What is the highest order 2-step method possible? What stability properties does it have?

3. (VO) Consider the two-step method

$$y_n - y_{n-1} = \frac{h}{16}(9f_n + 6f_{n-1} + f_{n-2})$$

   Investigate the properties of this method as compared with CrankNicolson and Adams Moulton 3. Does this method have any advantage? Hint: What happens to the test equation when λ tends toward $-\infty$?

4. (VO) Consider the family of two-step methods

$$y_{n+2} + (\theta - 2)y_{n+1} + (1-\theta)y_n = \frac{h}{4}[(6+\theta)f_{n+2} + 3(\theta - 2)f_n]$$

   in which θ is a free parameter. Determine the order of the method and the leading coefficient of the error. Show that neither depend on θ. For what values of θ does the method converge? i.e. for which θ are the method consistent and 0-stable? For which value of θ is the method A_0-stable?

5. (VO) Use AdamsBashforth 2 as a predictor for the CrankNicolson method.

$$\begin{cases} y^0_{n+2} = y_{n+1} + \frac{h}{2}[3f(t_{n+1}, y_{n+1} - f(t_n, y_n)] \\ y^{j+1}_{n+2} = y_{n+1} + \frac{h}{2}[f(t_{n+1}, y_{n+1}) + f(t_{n+2}, y^j_{n+2})] \end{cases}$$

   Plot the stability regions for the predictorcorrector pair for the cases PECE and P(EC)^{2}E.

6. 4. (PS) Consider the IVP

$$\begin{cases} \dot{x}(t) = -2y^3(t), & x(0) = 1 \\ \dot{y}(t) = 2x(t) - y^3(t), & y(0) = 1 \end{cases}$$

   Apply the forward and backward Euler methods as a predictor corrector pair. Compute a step size h such that the fixed point iteration converges and the largest possible eigenvalues of the linearized system lie within the stability region of the PECE method. Compare your numerical solution to the output of `ode45`. You may use the output of `ode45` to estimate a Lipschitz constant for $f(t, x(t), y(t))$.