

Übungsblatt 7

1. Für $n \in \mathbb{N}$ seien die *Lagrange* Polynome $\{L_{n,i}(x; \mathbf{x})\}_{i=1}^n$ in Abhängigkeit der Stützstellen $\mathbf{x} \in \mathbb{R}^n$ wie auf Seite 139 im Skriptum definiert. Gegeben seien die *Legendre* (λ), *Tchebychev* (τ), gleichmäßigen (γ) und zufälligen (ζ) Stützstellen,

$$\lambda = \left\{ \pm \sqrt{\frac{1}{35}(15 \pm 2\sqrt{30})} \right\}, \quad \tau = \left\{ \pm \frac{1}{2} \sqrt{2 \pm \sqrt{2}} \right\},$$

$$\gamma = \{-1, -1/3, +1/3, +1\}, \quad \zeta = 2 \cdot \text{rand}(4, 1) - 1.$$

Kreuzen Sie bei den wahren Behauptung an.

(a) Zu 4 signifikanten Ziffern gilt $\max_{i=1, \dots, 4} \max_{x \in [-1, +1]} |L_{4,i}(x; \lambda)| = 1.011.$

(b) Zu 4 signifikanten Ziffern gilt $\max_{i=1, \dots, 4} \max_{x \in [-1, +1]} |L_{4,i}(x; \tau)| = 1.004.$

(c) Zu 4 signifikanten Ziffern gilt $\max_{i=1, \dots, 4} \max_{x \in [-1, +1]} |L_{4,i}(x; \gamma)| = 1.056.$

(d) Es gilt $\sum_{i=1}^4 |L_{4,i}(x; \zeta)| = 1, \forall x \in [-1, +1].$

$n = 4 \rightarrow 4$ Stützstellen

$$L_{4,1} = \prod_{i,j=1}^4 \frac{x - x_j}{x_i - x_j} = \frac{x - x_2}{x_1 - x_2} \cdot \frac{x - x_3}{x_1 - x_3} \cdot \frac{x - x_4}{x_1 - x_4}$$

$$L_{4,2} = \frac{x - x_1}{x_2 - x_1} \cdot \frac{x - x_3}{x_2 - x_3} \cdot \frac{x - x_4}{x_2 - x_4}$$

$$L_{4,3} = \frac{x - x_1}{x_3 - x_1} \cdot \frac{x - x_2}{x_3 - x_2} \cdot \frac{x - x_4}{x_3 - x_4}$$

$$L_{4,4} = \frac{x - x_1}{x_4 - x_1} \cdot \frac{x - x_2}{x_4 - x_2} \cdot \frac{x - x_3}{x_4 - x_3}$$

$$= \frac{(x^2 - x x_3 - x x_2 + x_2 x_3)(x - x_4)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$$

$$= \frac{x^3 - x^2 x_4 - x^2 x_3 + x x_3 x_4 - x^2 x_2 + x x_2 x_4 + x x_2 x_3 - x_2 x_3 x_4}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$$

$$L_{4,1} = \frac{x^3 - x^2(x_2 + x_3 + x_4) + x(x_2 x_3 + x_2 x_4 + x_3 x_4) - x_2 x_3 x_4}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$$

Maximum bestimmen:

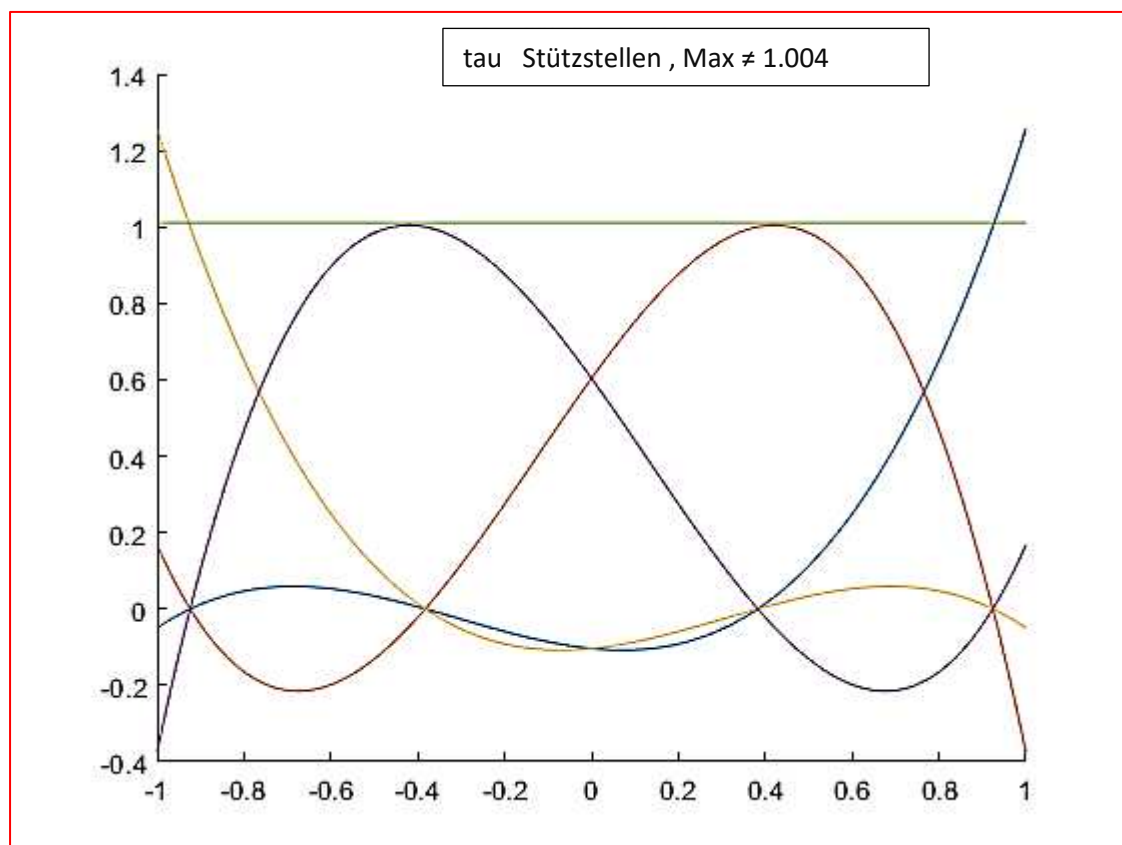
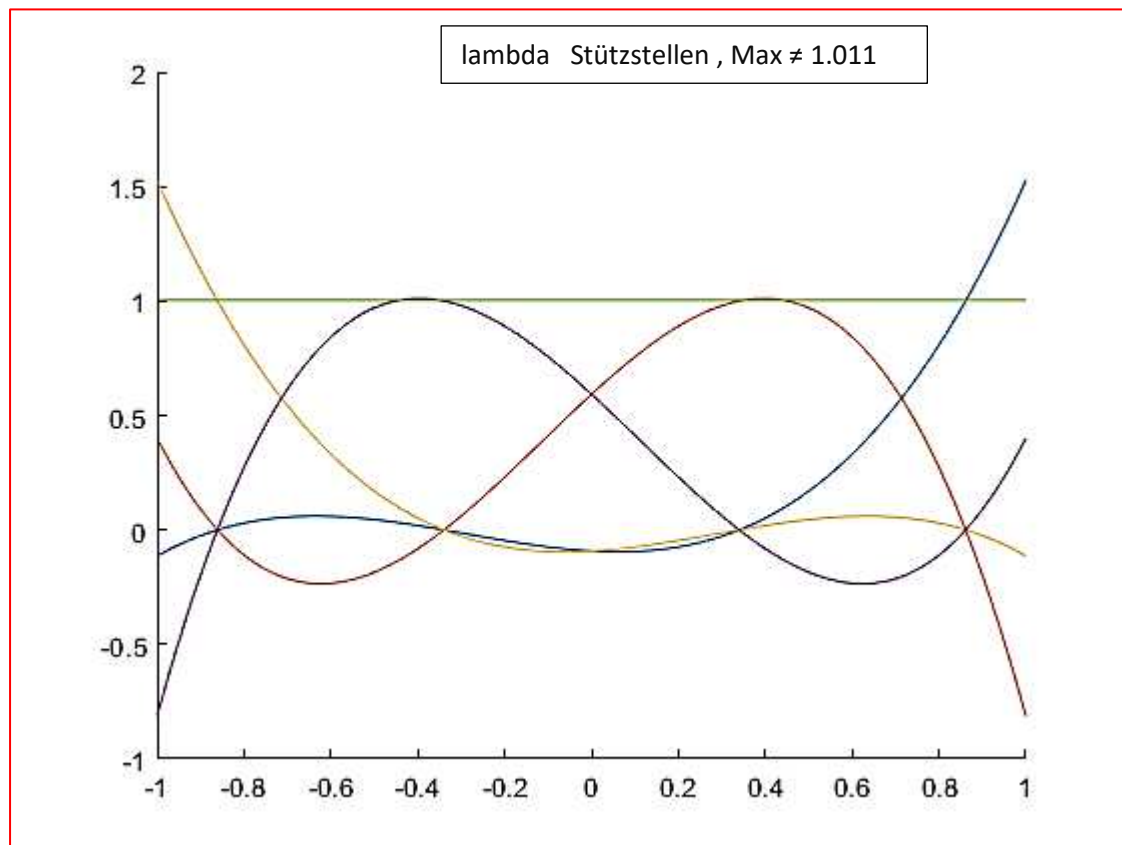
$$\frac{\partial L_{4,1}}{\partial x} = \frac{3x^2 - 2x(x_2 + x_3 + x_4) + (x_2 x_3 + x_2 x_4 + x_3 x_4)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$$

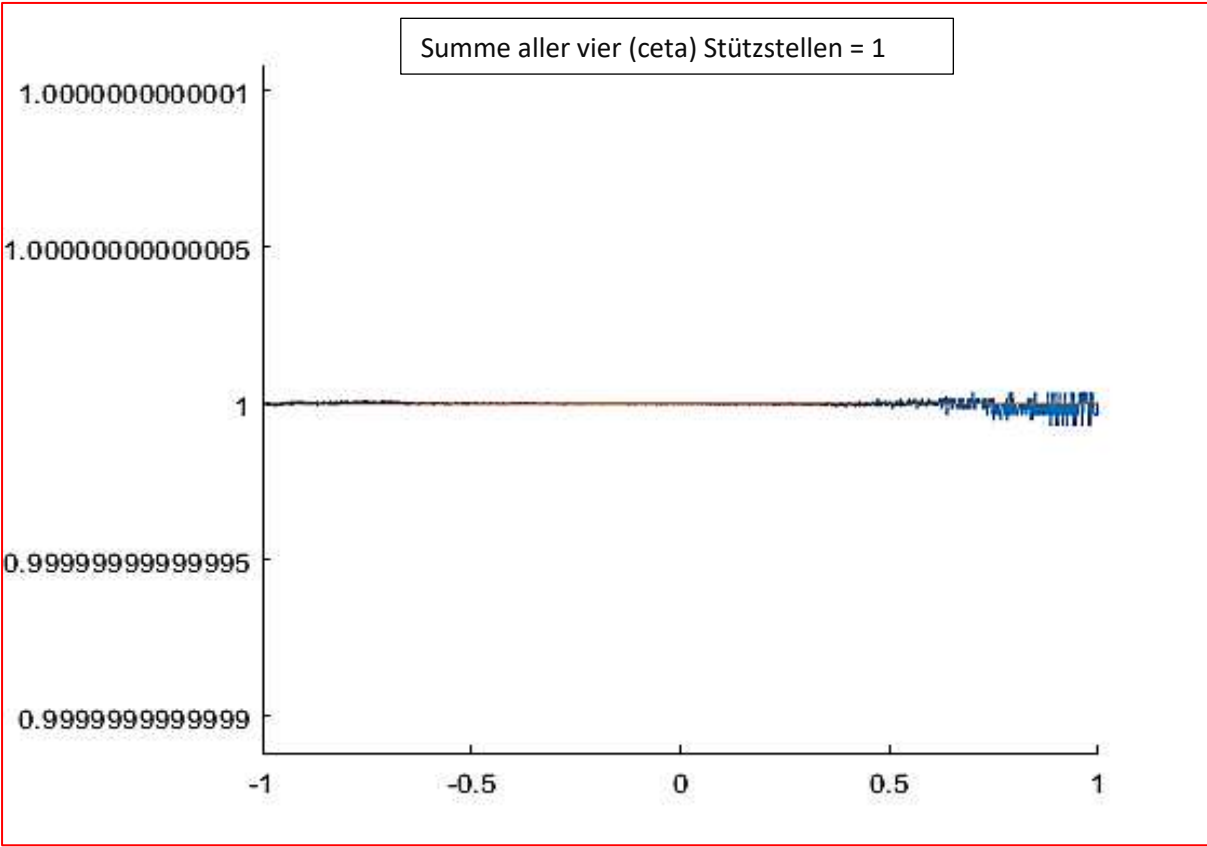
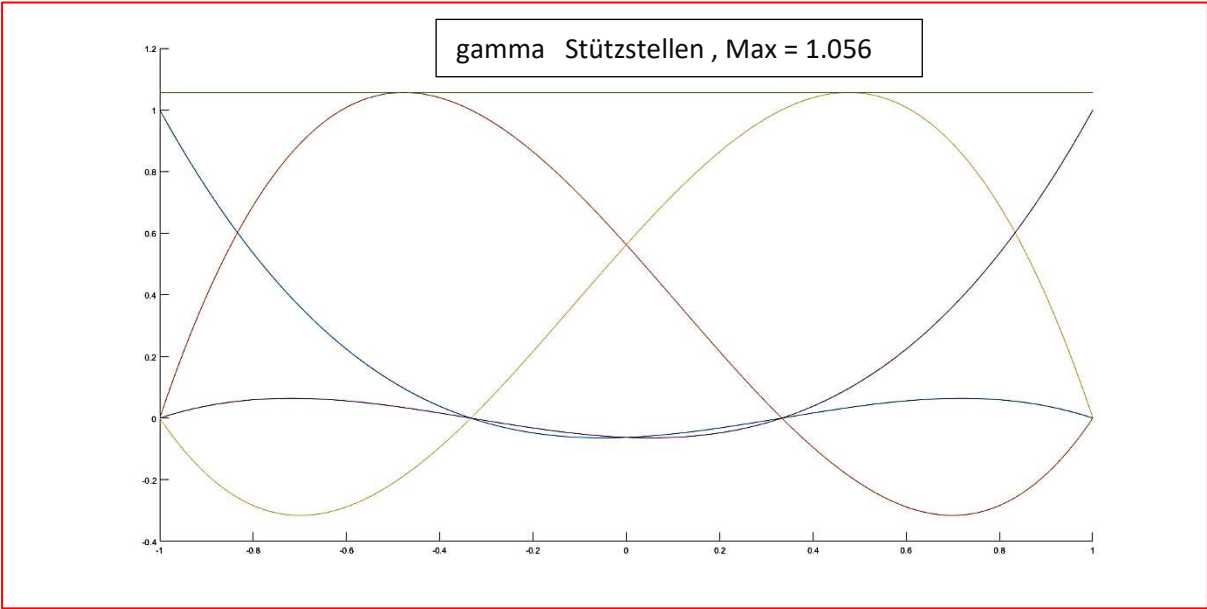
$$\frac{\partial^2 L_{4,1}}{\partial x^2} = \frac{6x - 2(x_2 + x_3 + x_4)}{(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$$

$$\rightarrow \frac{\partial L_{4,1}}{\partial x} \stackrel{!}{=} 0 \quad \text{und} \quad \frac{\partial^2 L_{4,1}}{\partial x^2}(x^*) < 0$$

x_1^*, x_2^*

analog für alle andere





```

clear all
close all
clc

lambda=[sqrt(1/35*(15+2*sqrt(30))),sqrt(1/35*(15-2*sqrt(30))),...
        -sqrt(1/35*(15+2*sqrt(30))),-sqrt(1/35*(15-2*sqrt(30)))];
tau=[0.5*sqrt(2+sqrt(2)),0.5*sqrt(2-sqrt(2)),...
     -0.5*sqrt(2+sqrt(2)),-0.5*sqrt(2-sqrt(2))];
gamma=[-1,-1/3,1/3,1];
%gamma=[-1,-1/3,1/3,1];
ceta=2*rand(4,1)-1;

%L41 erzeugen als Polynom
L41_lambda=[1, -lambda(2)-lambda(3)-lambda(4),...
            lambda(2)*lambda(3)+lambda(2)*lambda(4)+lambda(3)*lambda(4), -
            lambda(2)*lambda(3)*lambda(4) ]./...
            ((lambda(1)-lambda(2))*(lambda(1)-lambda(3))*(lambda(1)-lambda(4)));
L42_lambda=[1, -lambda(1)-lambda(3)-lambda(4),...
            lambda(1)*lambda(3)+lambda(1)*lambda(4)+lambda(3)*lambda(4), -
            lambda(1)*lambda(3)*lambda(4) ]./...
            ((lambda(2)-lambda(1))*(lambda(2)-lambda(3))*(lambda(2)-lambda(4)));
L43_lambda=[1, -lambda(2)-lambda(1)-lambda(4),...
            lambda(2)*lambda(1)+lambda(2)*lambda(4)+lambda(1)*lambda(4), -
            lambda(2)*lambda(1)*lambda(4) ]./...
            ((lambda(3)-lambda(1))*(lambda(3)-lambda(2))*(lambda(3)-lambda(4)));
L44_lambda=[1, -lambda(2)-lambda(3)-lambda(1),...
            lambda(2)*lambda(3)+lambda(2)*lambda(1)+lambda(3)*lambda(1), -
            lambda(2)*lambda(3)*lambda(1) ]./...
            ((lambda(4)-lambda(1))*(lambda(4)-lambda(2))*(lambda(4)-lambda(3)));

% % Maximum bestimmen:
% dL41_lambda_dx=[3*L41_lambda(1),2*L41_lambda(2),L41_lambda(3)];
% dL42_lambda_dx=[3*L42_lambda(1),2*L42_lambda(2),L42_lambda(3)];
% dL43_lambda_dx=[3*L43_lambda(1),2*L43_lambda(2),L43_lambda(3)];
% dL44_lambda_dx=[3*L44_lambda(1),2*L44_lambda(2),L44_lambda(3)];
%
% ddL41_lambda_ddx=[6*L41_lambda(1),2*L41_lambda(2)];
% ddL42_lambda_ddx=[6*L42_lambda(1),2*L42_lambda(2)];
% ddL43_lambda_ddx=[6*L43_lambda(1),2*L43_lambda(2)];
% ddL44_lambda_ddx=[6*L44_lambda(1),2*L44_lambda(2)];
%
% moegliche_Maxima=roots(dL41_lambda_dx);
% maxima(1)=moegliche_Maxima(polyval(ddL41_lambda_ddx,moegliche_Maxima)<0)
% val_max(1)=polyval(L41_lambda,maxima(1))
%
% moegliche_Maxima=roots(dL42_lambda_dx);
% maxima(2)=moegliche_Maxima(polyval(ddL42_lambda_ddx,moegliche_Maxima)<0)
% val_max(2)=polyval(L42_lambda,maxima(1))
%
% moegliche_Maxima=roots(dL43_lambda_dx);
% maxima(3)=moegliche_Maxima(polyval(ddL43_lambda_ddx,moegliche_Maxima)<0)
% val_max(3)=polyval(L43_lambda,maxima(1))
%
% moegliche_Maxima=roots(dL44_lambda_dx);
% maxima(4)=moegliche_Maxima(polyval(ddL44_lambda_ddx,moegliche_Maxima)<0)
% val_max(4)=polyval(L44_lambda,maxima(1))

```

```
% Plotten
t=-1:0.001:1;
figure;
hold on
plot(t,polyval(L41_lambda,t));
plot(t,polyval(L42_lambda,t));
plot(t,polyval(L43_lambda,t));
plot(t,polyval(L44_lambda,t));
plot(t,ones(size(t))*1.011);
title('Lambda Stuetzstellen')
```

```
% fuer tau
L41_tau=[1, -tau(2)-tau(3)-tau(4),...
tau(2)*tau(3)+tau(2)*tau(4)+tau(3)*tau(4), -tau(2)*tau(3)*tau(4) ]./...
((tau(1)-tau(2))*(tau(1)-tau(3))*(tau(1)-tau(4)))];
```

```
L42_tau=[1, -tau(1)-tau(3)-tau(4),...
tau(1)*tau(3)+tau(1)*tau(4)+tau(3)*tau(4), -tau(1)*tau(3)*tau(4) ]./...
((tau(2)-tau(1))*(tau(2)-tau(3))*(tau(2)-tau(4)))];
L43_tau=[1, -tau(2)-tau(1)-tau(4),...
tau(2)*tau(1)+tau(2)*tau(4)+tau(1)*tau(4), -tau(2)*tau(1)*tau(4) ]./...
((tau(3)-tau(1))*(tau(3)-tau(2))*(tau(3)-tau(4)))];
L44_tau=[1, -tau(2)-tau(3)-tau(1),...
tau(2)*tau(3)+tau(2)*tau(1)+tau(3)*tau(1), -tau(2)*tau(3)*tau(1) ]./...
((tau(4)-tau(1))*(tau(4)-tau(2))*(tau(4)-tau(3)))];
```

```
% Plotten
t=-1:0.001:1;
figure;
hold on
plot(t,polyval(L41_tau,t));
plot(t,polyval(L42_tau,t));
plot(t,polyval(L43_tau,t));
plot(t,polyval(L44_tau,t));
plot(t,ones(size(t))*1.004);
title('tau Stuetzstellen')
```

```
% fuer gamma
L41_gamma=[1, -gamma(2)-gamma(3)-gamma(4),...
gamma(2)*gamma(3)+gamma(2)*gamma(4)+gamma(3)*gamma(4), -gamma(2)*gamma(3)*gamma(4)
]./...
((gamma(1)-gamma(2))*(gamma(1)-gamma(3))*(gamma(1)-gamma(4)))];
L42_gamma=[1, -gamma(1)-gamma(3)-gamma(4),...
gamma(1)*gamma(3)+gamma(1)*gamma(4)+gamma(3)*gamma(4), -gamma(1)*gamma(3)*gamma(4)
]./...
((gamma(2)-gamma(1))*(gamma(2)-gamma(3))*(gamma(2)-gamma(4)))];
L43_gamma=[1, -gamma(2)-gamma(1)-gamma(4),...
gamma(2)*gamma(1)+gamma(2)*gamma(4)+gamma(1)*gamma(4), -gamma(2)*gamma(1)*gamma(4)
]./...
((gamma(3)-gamma(1))*(gamma(3)-gamma(2))*(gamma(3)-gamma(4)))];
L44_gamma=[1, -gamma(2)-gamma(3)-gamma(1),...
gamma(2)*gamma(3)+gamma(2)*gamma(1)+gamma(3)*gamma(1), -gamma(2)*gamma(3)*gamma(1)
]./...
((gamma(4)-gamma(1))*(gamma(4)-gamma(2))*(gamma(4)-gamma(3)))];
```

```
% Plotten
```

```

t=-1:0.001:1;
figure;
hold on
plot(t,polyval(L41_gamma,t));
plot(t,polyval(L42_gamma,t));
plot(t,polyval(L43_gamma,t));
plot(t,polyval(L44_gamma,t));
plot(t,ones(size(t))*1.056);
title('gamma Stuetzstellen')

```

```

% Aussage c stimmt, da x_max = 1.056

```

```

% fuer ceta

```

```

L41_ceta=[1, -ceta(2)-ceta(3)-ceta(4),...
ceta(2)*ceta(3)+ceta(2)*ceta(4)+ceta(3)*ceta(4), -ceta(2)*ceta(3)*ceta(4) ]./...
((ceta(1)-ceta(2))*(ceta(1)-ceta(3))*(ceta(1)-ceta(4))));

```

```

L42_ceta=[1, -ceta(1)-ceta(3)-ceta(4),...
ceta(1)*ceta(3)+ceta(1)*ceta(4)+ceta(3)*ceta(4), -ceta(1)*ceta(3)*ceta(4) ]./...
((ceta(2)-ceta(1))*(ceta(2)-ceta(3))*(ceta(2)-ceta(4))));

```

```

L43_ceta=[1, -ceta(2)-ceta(1)-ceta(4),...
ceta(2)*ceta(1)+ceta(2)*ceta(4)+ceta(1)*ceta(4), -ceta(2)*ceta(1)*ceta(4) ]./...
((ceta(3)-ceta(1))*(ceta(3)-ceta(2))*(ceta(3)-ceta(4))));

```

```

L44_ceta=[1, -ceta(2)-ceta(3)-ceta(1),...
ceta(2)*ceta(3)+ceta(2)*ceta(1)+ceta(3)*ceta(1), -ceta(2)*ceta(3)*ceta(1) ]./...
((ceta(4)-ceta(1))*(ceta(4)-ceta(2))*(ceta(4)-ceta(3))));

```

```

% Plotten

```

```

t=-1:0.001:1;
figure;
hold on
plot(t,polyval(L41_ceta,t)+polyval(L42_ceta,t)+polyval(L43_ceta,t)+polyval(L44_cet
a,t));
plot(t,ones(size(t))*1);

```

```

title('Summe aller 4 mit ceta Stuetzstellen')

```

```

% Aussage d stimmt auch

```

```

clc;clear;
%Angabe:
for i = 0:10
    t = cos(pi*(10-i+1/2)/11);
    tau(i+1) = t;
    f(i+1) = t^2/(1+20*t^2);
end

%Aufgabe a.)
for x = -1:0.1:1
    Q(:,1) = f;
    for i = 2:11
        for j = 2:i
            Q(i,j) = ((x - tau(i-j+1))*Q(i,j-1) - (x - tau(i))*Q(i-1,j-1))/(tau(i)-tau(i-j+1));
        end
    end
    disp(Q(5,5) - Q(8,8)) %4-7 weil Matlab beginnt bei 1 und nicht bei 0!
end

%Bemerkung zu a.) die zwei Polynome können aufgrund ihrer Grade nicht an 8
%Stellen übereinstimmen!

%Aufgabe b.) und c.)
x = 0.5;
Q(:,1) = f;
for i = 2:11
    for j = 2:i
        Q(i,j) = ((x-tau(i-j+1))*Q(i,j-1) - (x-tau(i))*Q(i-1,j-1))/(tau(i)-tau(i-j+1));
    end
end
(Q(5,5))
(Q(8,8))

%--> b und c richtig!

%Aufgabe d.)

V = [1,1,1,1,1,1,1,1,1,1,1;tau.^1;tau.^2;tau.^3;tau.^4;tau.^5;tau.^6;tau.^7;tau.^8;tau.^9;tau.^10];
c = inv(V).*f;
for x = 0.5
    P = c.*[1,x,x^2,x^3,x^4,x^5,x^6,x^7,x^8,x^9,x^10];
end
%disp(Q(11,11))
%disp(P)
disp(abs(P-Q(11,11)))
% ein Wert ist bei 0.0007
% --> also falsch, Gegenbeispiel gefunden!

%Bemerkung zu d.) wie im Skriptum auf Folie 146! Seien PL, PD ∈ Pn
% zwei durch Lagrange bzw. Dividierte Differenzen gegebenen Polynome,
% die die Daten interpolieren. Dann gilt PL(x) ≡ PD(x), weil Q = PL - PD ∈ P
% und Q die n + 1 Nullstellen hat.
% Also ist d.) falsch weil es Null ist und nicht größer gleich 0.001.

```

```

clear, clc;

lambda1 = sqrt((1/35)*(15+2*sqrt(30)));
lambda2 = -sqrt((1/35)*(15+2*sqrt(30)));
lambda3 = sqrt((1/35)*(15-2*sqrt(30)));
lambda4 = -sqrt((1/35)*(15-2*sqrt(30)));

Lambda = [lambda1, lambda2, lambda3, lambda4];

tau1 = (1/2)* sqrt(2 + sqrt(2));
tau2 = -(1/2)* sqrt(2 + sqrt(2));
tau3 = (1/2)* sqrt(2 - sqrt(2));
tau4 = -(1/2)* sqrt(2 - sqrt(2));

Tau = [tau1, tau2, tau3, tau4];

syms x;
%A = Lambda;
A = Tau;
%Berechnung der Lagrange Polynome für n = 4
L1 = 1;
for j = [2 3 4]
    L1 = L1*((x - A(j))/(A(1)- A(j)));
end
L1(x) = L1;
% L1(x) = ((x-A(2))*(x-A(3))*(x-A(4)))/((A(1)-A(2))*(A(1)-A(3))*(A(1)-A(4)));
L2 = 1;
for j = [1 3 4]
    L2 = L2*((x - A(j))/(A(2)- A(j)));
end
L2(x) = L2;
% L2(x) = ((x-A(1))*(x-A(3))*(x-A(4)))/((A(2)-A(1))*(A(2)-A(3))*(A(2)-A(4)));
L3 = 1;
for j = [1 2 4]
    L3 = L3*((x - A(j))/(A(3)- A(j)));
end
L3(x) = L3;
% L3(x) = ((x-A(1))*(x-A(2))*(x-A(4)))/((A(3)-A(1))*(A(3)-A(2))*(A(3)-A(4)));
L4 = 1;
for j = [1 2 3]
    L4 = L4*((x - A(j))/(A(4)- A(j)));
end
L4(x) = L4;
% L4(x) = ((x-A(1))*(x-A(2))*(x-A(3)))/((A(4)-A(1))*(A(4)-A(2))*(A(4)-A(3)));

%Ableitung der Lagrange Polynome
dL1(x) = diff(L1(x));
dL2(x) = diff(L2(x));
dL3(x) = diff(L3(x));
dL4(x) = diff(L4(x));

%Berechnung der Hermiteischen Polynome
H1(x) = (1-2*(x-A(1))*dL1(A(1)))*(L1(x)^2);
H2(x) = (1-2*(x-A(2))*dL2(A(2)))*(L2(x)^2);
H3(x) = (1-2*(x-A(3))*dL3(A(3)))*(L3(x)^2);
H4(x) = (1-2*(x-A(4))*dL4(A(4)))*(L4(x)^2);

%Berechnung von H-Dach
Hh1(x) = (x - A(1))* (L1(x)^2);
Hh2(x) = (x - A(2))* (L2(x)^2);
Hh3(x) = (x - A(3))* (L3(x)^2);
Hh4(x) = (x - A(4))* (L4(x)^2);

%Ableitung der Hermiteischen Polynome
dH1(x) = diff(H1(x));
dH2(x) = diff(H2(x));
dH3(x) = diff(H3(x));
dH4(x) = diff(H4(x));

%Berechnung von Maximum der Beträge der Ableitungen der Hermiteischen
%Polynome für x zwischen -1 und 1
max1 = 0;

```

```
for a = -1:1/100:1
    G = abs(dH1(a));
    if G > max1
        max1 = G;
    end
end
max2 = 0;
for a = -1:1/100:1
    G = abs(dH2(a));
    if G > max2
        max2 = G;
    end
end
max3 = 0;
for a = -1:1/100:1
    G = abs(dH3(a));
    if G > max3
        max3 = G;
    end
end
max4 = 0;
for a = -1:1/100:1
    G = abs(dH4(a));
    if G > max4
        max4 = G;
    end
end
u = vpa(max([max1, max2, max3, max4]));
maxdH = vpa(u,4)

%Berechnung von Maximum der Beträge von H-Dach
maxh1 = 0;
for b = -1:1/100:1
    F = abs(Hh1(b));
    if F > maxh1
        maxh1 = F;
    end
end

maxh2 = 0;
for b = -1:1/100:1
    F = abs(Hh2(b));
    if F > maxh2
        maxh2 = F;
    end
end

maxh3 = 0;
for b = -1:1/100:1
    F = abs(Hh3(b));
    if F > maxh3
        maxh3 = F;
    end
end

maxh4 = 0;
for b = -1:1/100:1
    F = abs(Hh4(b));
    if F > maxh4
        maxh4 = F;
    end
end
v = vpa(max([maxh1, maxh2, maxh3, maxh4]));
maxHh = vpa(v,4)
```

```

clear, clc;

lambda1 = sqrt((1/35)*(15+2*sqrt(30)));
lambda2 = -sqrt((1/35)*(15+2*sqrt(30)));
lambda3 = sqrt((1/35)*(15-2*sqrt(30)));
lambda4 = -sqrt((1/35)*(15-2*sqrt(30)));

Lambda = [lambda1, lambda2, lambda3, lambda4];

tau1 = (1/2)* sqrt(2 + sqrt(2));
tau2 = -(1/2)* sqrt(2 + sqrt(2));
tau3 = (1/2)* sqrt(2 - sqrt(2));
tau4 = -(1/2)* sqrt(2 - sqrt(2));

Tau = [tau1, tau2, tau3, tau4];

syms x;
%A = Lambda;
A = Tau;
%Berechnung der Lagrange Polynome für n = 4
L1 = 1;
for j = [2 3 4]
    L1 = L1*((x - A(j))/(A(1)- A(j)));
end
L1(x) = L1;
% L1(x) = ((x-A(2))*(x-A(3))*(x-A(4)))/((A(1)-A(2))*(A(1)-A(3))*(A(1)-A(4)));
L2 = 1;
for j = [1 3 4]
    L2 = L2*((x - A(j))/(A(2)- A(j)));
end
L2(x) = L2;
% L2(x) = ((x-A(1))*(x-A(3))*(x-A(4)))/((A(2)-A(1))*(A(2)-A(3))*(A(2)-A(4)));
L3 = 1;
for j = [1 2 4]
    L3 = L3*((x - A(j))/(A(3)- A(j)));
end
L3(x) = L3;
% L3(x) = ((x-A(1))*(x-A(2))*(x-A(4)))/((A(3)-A(1))*(A(3)-A(2))*(A(3)-A(4)));
L4 = 1;
for j = [1 2 3]
    L4 = L4*((x - A(j))/(A(4)- A(j)));
end
L4(x) = L4;
% L4(x) = ((x-A(1))*(x-A(2))*(x-A(3)))/((A(4)-A(1))*(A(4)-A(2))*(A(4)-A(3)));

%Ableitung der Lagrange Polynome
dL1(x) = diff(L1(x));
dL2(x) = diff(L2(x));
dL3(x) = diff(L3(x));
dL4(x) = diff(L4(x));

%Berechnung der Hermiteischen Polynome
H1(x) = (1-2*(x-A(1))*dL1(A(1)))*(L1(x)^2);
H2(x) = (1-2*(x-A(2))*dL2(A(2)))*(L2(x)^2);
H3(x) = (1-2*(x-A(3))*dL3(A(3)))*(L3(x)^2);
H4(x) = (1-2*(x-A(4))*dL4(A(4)))*(L4(x)^2);

%Berechnung von H-Dach
Hh1(x) = (x - A(1))* (L1(x)^2);
Hh2(x) = (x - A(2))* (L2(x)^2);
Hh3(x) = (x - A(3))* (L3(x)^2);
Hh4(x) = (x - A(4))* (L4(x)^2);

%Ableitung der Hermiteischen Polynome
dH1(x) = diff(H1(x));
dH2(x) = diff(H2(x));
dH3(x) = diff(H3(x));
dH4(x) = diff(H4(x));

%Berechnung von Maximum der Beträge der Ableitungen der Hermiteischen
%Polynome für x zwischen -1 und 1
max1 = 0;

```

```
for a = -1:1/100:1
    G = abs(dH1(a));
    if G > max1
        max1 = G;
    end
end
max2 = 0;
for a = -1:1/100:1
    G = abs(dH2(a));
    if G > max2
        max2 = G;
    end
end
max3 = 0;
for a = -1:1/100:1
    G = abs(dH3(a));
    if G > max3
        max3 = G;
    end
end
max4 = 0;
for a = -1:1/100:1
    G = abs(dH4(a));
    if G > max4
        max4 = G;
    end
end
u = vpa(max([max1, max2, max3, max4]));
maxdH = vpa(u,4)

%Berechnung von Maximum der Beträge von H-Dach
maxh1 = 0;
for b = -1:1/100:1
    F = abs(Hh1(b));
    if F > maxh1
        maxh1 = F;
    end
end

maxh2 = 0;
for b = -1:1/100:1
    F = abs(Hh2(b));
    if F > maxh2
        maxh2 = F;
    end
end

maxh3 = 0;
for b = -1:1/100:1
    F = abs(Hh3(b));
    if F > maxh3
        maxh3 = F;
    end
end

maxh4 = 0;
for b = -1:1/100:1
    F = abs(Hh4(b));
    if F > maxh4
        maxh4 = F;
    end
end
v = vpa(max([maxh1, maxh2, maxh3, maxh4]));
maxHh = vpa(v,4)
```