Mathematical Modelling in the Natural Sciences
SS21, Exercise Sheet 4

1. Show for $N = 2$ and $M = 1$ that the SIR model with diffusion on page 144 of the lecture notes has a locally asymptotically stable equilibrium $(S_{i,j}, I_{i,j}, R_{i,j}) = (S_1^*, I_1^*, R_1^*)$ if $I_2^\star < 0$ and a locally asymptotically stable equilibrium $(S_{i,j}, I_{i,j}, R_{i,j}) = (S_2^*, I_2^*, R_2^*)$ if $I_2^\star > 0$. Show this result for arbitrary N and M.

2. As on pages 143 − 149 of the lecture notes, implement an SIR model with two-dimensional diffusion and include any relevant effects, even such as loss of immunity, vaccination, etc. Use your model to develop an effective quarantine strategy.

3. Implement the model of morphogenesis,

$$\left\{\begin{array}{rcll} u_t &=& d_1\Delta u + p - uv^2, & \Omega\times(0,T) \\ v_t &=& d_2\Delta v + q - v + uv^2, & \Omega\times(0,T) \\ && u(0,y)=u(1,y), \quad u(x,0)=u(x,1), & \partial\Omega\times(0,T) \\ && v(0,y)=v(1,y), \quad v(x,0)=v(x,1), & \partial\Omega\times(0,T) \\ u &=& u_0, & \Omega\times\{0\} \\ v &=& v_0, & \Omega\times\{0\} \end{array}\right.$$

   where $\Omega = (0,1)^2$, $T > 0$, $p, q, d_1, d_2 \in \mathbb{R}$, $p > q > 0$, $(p+q)^3 > (p-q)$, $0 < d_2 \ll d_1$ and u_0 and v_0 are (random) perturbations of the equilbrium state in the absence of diffusion. Demonstrate pattern formation with this model.

4. Implement the logistic variant of the predator-prey SIR model on page 147 of the lecture notes and demonstrate pattern formation for an appropriate choice of parameters and initial conditions.

5. Based upon the work with pattern formation implement a model of swarm intelligence.