Mathematical Modelling in the Natural Sciences
SS19, Exercises, Sheet 6

Solutions to be presented on 6. May 2019

1. Show for $N = 2$ and $M = 1$ that the SIR model with diffusion on page 144 of the lecture notes has a locally asymptotically stable equilibrium $(S_{i,j}, I_{i,j}, R_{i,j}) = (S_1^*, I_1^*, R_1^*)$ if $I_2^\star < 0$ and a locally asymptotically stable equilibrium $(S_{i,j}, I_{i,j}, R_{i,j}) = (S_2^*, I_2^*, R_2^*)$ if $I_2^\star > 0$. Bonus: Show this result for arbitrary N and M. Solved by Mr. Haberl.

2. As on pages 143 – 149 of the lecture notes, implement an SIR model with two-dimensional diffusion and include any relevant effects, even such as loss of immunity, vaccination, etc. Use your model to develop an effective quarantine strategy. Solved by Mr. Stadler.

3. Implement the model of morphogenesis, $\Omega = (0, 1)^2$

$$\begin{cases} u_t &= d_1 \Delta u + p - uv^2, & \Omega \times (0, T) \\ v_t &= d_2 \Delta v + q - v + uv^2, & \Omega \times (0, T) \\ & u(0, y) = u(1, y), \quad u(x, 0) = u(x, 1), & \partial\Omega \times (0, T) \\ & v(0, y) = v(1, y), \quad v(x, 0) = v(x, 1), & \partial\Omega \times (0, T) \\ u &= u_0, & \Omega \times \{0\} \\ v &= v_0, & \Omega \times \{0\} \end{cases}$$

   where $p, q, d_1, d_2 \in \mathbb{R}$, $p > q > 0$, $(p + q)^3 > (p - q)$, $0 < d_2 \ll d_1$ and u_0 and v_0 are (random) perturbations of the equilbrium state in the absence of diffusion. Demonstrate pattern formation with this model. Solved by Mr. Habring.

4. Bonus: Implement the logistic variant of the predator-prey SIR model on page 147 of the lecture notes and demonstrate pattern formation for an appropriate choice of parameters and initial conditions. Solved by Mr. Habring.