Mathematical Modelling in the Natural Sciences
SS19, Exercises, Sheet 1

Solutions to be presented by 11. March 2019

1. For the *Supersize Me!* problem suppose that the following data have been measured:

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   n = 30;
   t = linspace(0,30,n);
   p = 1.0;
   m = 231.48 + (84-231.48)exp(-t/361.11) + p*randn(1,n);
   ```

   Formulate a method for estimating the parameter ϕ in the model

   $$m'(t) = \epsilon/\kappa - m(t)\phi/\kappa, \quad m(0) = 84, \quad \epsilon = 5000, \quad \kappa = 7800$$

   and implement this method using Matlab. For different values of the noise level p, compare your result with the value of $\phi = 21.6$ used in the lecture notes. Hint: Consider using the ℓ_2- or the ℓ_1-norm of the differences between simulated and measured data. Solved by Mr. Haberl.

2. Develop a refinement of the *Supersize Me!* model which is intended to be more suitable for (a) very small or (b) very large values of m, and argue why your model is more suitable for these extreme cases. Hint: Consider the behavior of the steady state with respect to other parameters. Solved by Mr. Stadler and Mr. Cardenas.

3. For the system of differential equations modelling the discovery of treasures by a random walk

   $$\begin{array}{rcl} p_0'(t) & = & -\beta p_0(t) \\ p_n'(t) & = & -\beta p_n(t) + \beta p_{n-1}(t), \quad n = 1, 2, \dots, N-1 \quad\quad 0 \le t \le T \\ p_N'(t) & = & \beta p_{N-1}(t) \end{array}$$

   determine the initial conditions which correspond to not having discovered any treasures at all at time $t = 0$. Choose values $\beta, T > 0$ and $N \in \mathbb{N}$ and solve the above system using Matlab plotting $\{p_n(t)\}_{n=0}^N$ together for $t \in [0, T]$. Given the values $\{p_n(t) : t \in [0, T]\}_{n=0}^N$ determine $E_N(t)$ the expected number of treasures discovered up to time t. Plot also $E_N(t)$ together with $E(t)$ presented in the lecture notes, and compare the two plots for ever larger N. Solved by Mr. Haberl.