Mathematical Modelling in the Natural Sciences
SS18, Exercises, Sheet 8

Solutions to be presented on 8. June 2018

1. Write an approximation $\tilde{J}$ to the potential J (on page 172 of the lecture notes) where the necessary optimality condition on a displacement u which minimizes $\tilde{J}$ is given by the Poisson boundary value problem
$$\begin{cases} -\nabla \cdot (T\nabla u) &= f, \quad \text{in} \quad \Omega \\ u &= 0, \quad \text{on} \quad \partial\Omega \end{cases}$$
Here, it is assumed that the membrane with displacement u, is clamped at the boundary $\partial\Omega$ of $\Omega = (0,1)^2$. For the chosen potential $\tilde{J}$, show by an explicit derivation that the necessary optimality condition is given by the IBVP above.

2. Write a Matlab code to solve the non-linear boundary value problem
$$\begin{cases} -\nabla \cdot \left(\dfrac{T\nabla u}{\sqrt{1+|\nabla u|^2}} \right) &= f, \quad \text{in} \quad \Omega \\ u &= 0, \quad \text{on} \quad \partial\Omega \end{cases}$$
using a Picard iteration in order to compute the membrane displacement.

3. Write a Matlab code to solve the initial and boundary value problem for the nonlinear wave equation
$$\begin{cases} u_{tt} &= \nabla \cdot \left(\dfrac{T\nabla u}{\sqrt{1+|\nabla u|^2}} \right), \quad \text{in} \quad \Omega \times (0,T] \\ u &= 0, \quad \text{on} \quad \partial\Omega \times [0,T] \\ u &= u_0, \quad \text{on} \quad \Omega \times \{0\} \\ u_t &= u_1, \quad \text{on} \quad \Omega \times \{0\} \end{cases}$$

4. After being well acquainted with the program in the lecture notes for solving the nonlinear wave equation to simulate the motion of a bungee cord, derive the optimality conditions for the membrane counterpart and investigate the steady state anomaly in the following Matlab implementation for the membrane:

`http://imsc.uni-graz.at/keeling/numpde_ss16/sheet.m`