Mathematical Modelling in the Natural Sciences
SS18, Exercises, Sheet 6

Solutions to be presented on 18. May 2018

1. Show for $N = 2$ and $M = 1$ that the SIR model with diffusion on page 144 of the lecture notes has a locally asymptotically stable equilibrium $(S_{i,j}, I_{i,j}, R_{i,j}) = (S_1^*, I_1^*, R_1^*)$ if $I_2^\star < 0$ and a locally asymptotically stable equilibrium $(S_{i,j}, I_{i,j}, R_{i,j}) = (S_2^*, I_2^*, R_2^*)$ if $I_2^\star > 0$. Bonus: Show this result for arbitrary N and M.

2. As on pages 143 – 149 of the lecture notes, implement an SIR model with two-dimensional diffusion and include any relevant effects, even such as loss of immunity, vaccination, etc. Use your model to develop an effective quarantine strategy.

3. Implement the model of morphogenesis, $\Omega = (0,1)^2$

$$\left\{\begin{array}{rcll} u_t &=& d_1\Delta u + p - uv^2, & \Omega\times(0,T) \\ v_t &=& d_2\Delta v + q - v + uv^2, & \Omega\times(0,T) \\ && u(0,y) = u(1,y), \quad u(x,0) = u(x,1), & \partial\Omega\times(0,T) \\ && v(0,y) = v(1,y), \quad v(x,0) = v(x,1), & \partial\Omega\times(0,T) \\ u &=& u_0, & \Omega\times\{0\} \\ v &=& v_0, & \Omega\times\{0\} \end{array}\right.$$

   where $p, q, d_1, d_2 \in \mathbb{R}$, $p > q > 0$, $(p+q)^3 > (p-q)$, $0 < d_2 \ll d_1$ and u_0 and v_0 are (random) perturbations of the equilibrium state in the absence of diffusion. Demonstrate pattern formation with this model.

4. Bonus: Implement the logistic variant of the predator-prey SIR model on page 147 of the lecture notes and demonstrate pattern formation for an appropriate choice of parameters and initial conditions.