Mathematical Modelling in the Natural Sciences
SS17, Exercises, Sheet 5

Solutions to be presented on 2. Mai 2017

1. Given the infinite system of ODEs for $(\{s_n\}, \{i_n\}, \{r_n\})$ of pages 130 – 136 from the lecture notes, derive a 3×3 system of ODEs for $(\bar{S}, \bar{I}, \bar{R})$. Do the same for the modification with a positive probability of spontaneous infection.

2. Given the infinite system of ODEs for $(\{s_n\}, \{i_n\}, \{r_n\})$ of pages 130 – 136 from the lecture notes, show that $i_n(t) \to \delta_{n,0}$, $t \to \infty$, if $(\bar{S}(t), \bar{I}(t), \bar{R}(t)) = (S_2^*, I_2^*, R_2^*)$, $\forall t$. Implement the Modell for an ever larger number possible states starting always with an initial condition corresponding to the endemic equilibrium.

3. For $N = 2$, $M = 1$ and given $\beta, \gamma, \lambda, \mu$ (and possibly ϵ) of the cellular automaton on pages 137 – 142 of the lecture notes, write the transition probabilities of the cellular automaton as a stochastic matrix and find the equilibrium. Bonus: Develop a general method for larger N and M so that results can be compared with the Monte-Carlo simulations from the lecture.

4. Implement the Monte-Carle simulation of the cellular automaton on pages 137 – 142 of the lecture notes. Bonus: Prove that $\bar{I}(t) \to 0$, $t \to \infty$.

5. Bonus: Develop a continous time lumped parameter infection model, other than those shown in the lecture, which possesses periodic solutions and perhaps even period doubling transitioning to chaos. Feel free to include any relevant effects, even such as loss of immunity, vaccination, etc.

6. Bonus: Develop a discrete time lumped parameter infection model (particularly other than the failed attempt shown in the lecture) which possesses periodic solutions and perhaps even period doubling transitioning to chaos. Feel free to include any relevant effects, even such as loss of immunity, vaccination, etc.