

Mathematical Modelling in the Natural Sciences

SS17, Exercises, Sheet 1

Solutions to be presented on 9. March 2017

1. For the *Supersize Me!* problem suppose that the following data have been measured:

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n = 30;
t = linspace(0,30,n);
p = 1.0;
m = 231.48 + (84-231.48)exp(-t/361.11) + p*randn(1,n);
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Formulate a method for estimating the parameter ϕ in the model

$$m'(t) = \epsilon/\kappa - m(t)\phi/\kappa, \quad m(0) = 84, \quad \epsilon = 5000, \quad \kappa = 7800$$

and implement this method using Matlab. For different values of the noise level p , compare your result with the value of $\phi = 21.6$ used in the lecture notes. (Solved by Ms. Lenger)

2. Develop a refinement of the *Supersize Me!* model which is intended to be more suitable for (a) very small or (b) very large values of m , and argue why your model is more suitable for these extreme cases. (Solved by Ms. Mayr)
3. For the system of differential equations modelling the discovery of treasures by a random walk

$$\begin{aligned} p'_0(t) &= -\beta p_0(t) \\ p'_n(t) &= -\beta p_n(t) + \beta p_{n-1}(t), \quad n = 1, 2, \dots, N-1 \\ p'_N(t) &= \beta p_{N-1}(t) \end{aligned} \quad 0 \leq t \leq T$$

determine the initial conditions which correspond to not having discovered any treasures at all at time $t = 0$. Choose values $\beta, T > 0$ and $N \in \mathbb{N}$ and solve the above system using Matlab plotting $\{p_n(t)\}_{n=0}^N$ together for $t \in [0, T]$. Given the values $\{p_n(t) : t \in [0, T]\}_{n=0}^N$ determine $E_N(t)$ the expected number of treasures ($\leq N$ for the above system) discovered up to time t . Plot also $E_N(t)$ together with $E(t)$ presented in the lecture notes, and compare the two plots for ever larger N . (Solved by Ms. Schmidhofer)