

Mathematical Modelling in the Natural Sciences

SS16, Exercises, Sheet 9

Solutions to be presented on 20. May 2016

1. Write an approximation \tilde{J} to the potential J (on page 162 of the lecture notes) where the necessary optimality condition on a displacement u which minimizes \tilde{J} is given by the Poisson boundary value problem

$$\begin{cases} -\nabla \cdot (T\nabla u) = f, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

Here, it is assumed that the membrane with displacement u , is clamped at the boundary $\partial\Omega$ of $\Omega = (0, 1)^2$. For the chosen potential \tilde{J} , show by an explicit derivation that the necessary optimality condition is given by the IBVP above.

2. Implement the code on page 61 of the lecture notes

http://imsc.uni-graz.at/keeling/numpde_ss16/numpde.pdf

to solve the non-linear boundary value problem

$$\begin{cases} -\nabla \cdot \left(\frac{T\nabla u}{\sqrt{1 + |\nabla u|^2}} \right) = f, & \text{in } \Omega \\ u = 0, & \text{on } \partial\Omega \end{cases}$$

using a Picard iteration in order to compute the membrane displacement.

3. Adapt the code on page 96 of the lecture notes

http://imsc.uni-graz.at/keeling/numpde_ss16/numpde.pdf

to solve the initial and boundary value problem for the wave equation

$$\begin{cases} u_{tt} = \nabla \cdot (T\nabla u), & \text{in } \Omega \times (0, T] \\ u = 0, & \text{on } \partial\Omega \times [0, T] \\ u = u_0, & \text{on } \Omega \times \{0\} \\ u_t = u_1, & \text{on } \Omega \times \{0\} \end{cases}$$

Bonus: Adapt your code to solve the nonlinear IBVP for the wave equation:

$$\begin{cases} u_{tt} = \nabla \cdot \left(\frac{T\nabla u}{\sqrt{1 + |\nabla u|^2}} \right), & \text{in } \Omega \times (0, T] \\ u = 0, & \text{on } \partial\Omega \times [0, T] \\ u = u_0, & \text{on } \Omega \times \{0\} \\ u_t = u_1, & \text{on } \Omega \times \{0\} \end{cases}$$

being careful to consider how to rewrite this problem in first order form for an appropriate state, e.g., (u, u_t) or otherwise.