Mathematical Modelling in the Natural Sciences
SS16, Exercises, Sheet 5

Solutions to be presented on 21. April 2016

1. Given the infinite system of ODEs for $(\{s_n\}, \{i_n\}, \{r_n\})$ from the lecture, derive a 3×3 system of ODEs for $(\bar{S}, \bar{I}, \bar{R})$. Do the same for the modification with a positive probability of spontaneous infection. (Solved by Ornella Moro)

2. Given the infinite system of ODEs for $(\{s_n\}, \{i_n\}, \{r_n\})$ from the lecture, show that $i_n(t) \to \delta_{n,0}$, $t \to \infty$, if $(\bar{S}(t), \bar{I}(t), \bar{R}(t)) = (S_2^*, I_2^*, R_2^*)$, $\forall t$. (Solved by everyone)

3. For $N = 2$, $M = 1$ and given $\beta, \gamma, \lambda, \mu$, write the transition probabilities of the cellular automaton as a stochastic matrix and find the equilibrium. Also a general method for larger N and M so that results can be compared with the Monte-Carlo simulations from the lecture. (Solved by Andreas Holm and Florian Thaler)

4. BONUS! Develop a continous time lumped parameter infection model, other than those shown in the lecture, which possesses periodic solutions and perhaps even period doubling transitioning to chaos. Feel free to include any relevant effects, even such as loss of immunity, vaccination, etc.

5. BONUS! Develop a discrete time lumped parameter infection model (particularly other than the failed attempt shown in the lecture) which possesses periodic solutions and perhaps even period doubling transitioning to chaos. Feel free to include any relevant effects, even such as loss of immunity, vaccination, etc.