ABSTRACTS: WORKSHOP ON HOMOLOGICAL INTERACTIONS (VERSION OF DEC 11)

R. Abuaf
Categorical Resolutions of Singularities.

Categorical resolution of singularities may be thought as a categorification of the notion of resolution of singularities for algebraic varieties. For curves and surfaces, if the singularities are not too bad (and if we work over the complex numbers), we always know how to construct a resolution of singularities which is minimal in a very strong sense. However, as soon as the dimension is bigger than two, it is impossible to find such strongly minimal resolution at the geometric level.

The theory of categorical resolution provides with a very new point of view on minimality and let us hope that we can always (almost always?) find a minimal categorical resolution for varieties with "reasonable" singularities. In this talk, I will discuss some recent results on the existence of minimal categorical resolutions of singularities. I will define the notion of wonderful resolution of singularities and talk about their ubiquity in geometric representation theory. I will then explain why these wonderful resolutions are so interesting in the categorical setting. Finally, I will discuss some connetions with Van-den-Bergh's notion of non-commutative resolution of singularities.

P. Bergh
n-angulated categories.

In a paper appearing last year, Geiss, Keller and Oppermann introduced higher analogues of triangulated categories, called n-angulated categories. They appear for instance when one studies certain cluster tilting subcategories of triangulated categories. In this talk, we give an overview of some of the recent developments.

R. Bocklandt
From A to B with SYZ.

An important conjecture by Strominger-Zaslow and Yau, states that mirror manifolds should be connected by dual torus fibrations. We will explore this conjecture in the case of punctured Riemann surfaces using dimer models. On the A-side the punctured surface arises as a spectral curve, which degenerates to a tropical curve. This degeneration gives us a torus fibration. On the B-side the mirror is constructed as a moduli space of representations and the torus fibration comes from a moment map. We connect the parameter space of the torus degenerations to obtain "a space of SYZ-fibrations". We relate this space to the moduli space of marked Riemann-surface and discuss mutations and flops in this context.

R. Buchweitz
A McKay Correspondence for Reflection Groups
(joint work with Eleonore Faber and Colin Ingalls).

Let G be a finite subgroup of $\mathsf{GL}(n, K)$ for a field K whose characteristic does not divide the order of G. The group G then acts linearly on the polynomial ring S in n variables over K and one may form the corresponding twisted or skew group algebra $A = S * G$. With e in A the idempotent corresponding to the trivial representation, consider the algebra A/AeA. If G is a finite subgroup of $\mathsf{SL}(2, K)$, then A is Morita-equivalent to the preprojective algebra of an extended Dynkin diagram and A/AeA to the preprojective algebra of the Dynkin diagram itself. This can be seen as a formulation of the McKay correspondence for the Kleinian singularities.

We want to establish an analogous result when G is a group generated by (pseudo-)reflections. With D the coordinate ring of the discriminant of the group action on S, we show that A/AeA is maximal Cohen-Macaulay as a module over D and that it is of finite global dimension as a ring. In case G is generated by reflections and D is a domain, then A/AeA is the endomorphism ring of a maximal Cohen-Macaulay module of rank $|G|/2$ over the ring of the discriminant, namely of the direct image of the coordinate ring of the associated hyperplane arrangement.

In this way one obtains a noncommutative resolution of singularities of that discriminant, a hypersurface that is a free divisor, thus, singular in codimension one.

E. Faber
Noncommutative desingularizations of not necessarily normal commutative rings.

Motivated by algebraic geometry, one studies non-commutative analogs of resolutions of singularities. In short, non-commutative resolutions (NCRs) of commutative rings R are endomorphism rings of certain R-modules of finite global dimension.

In this talk we consider the question of existence of a noncommutative resolution, in particular for non-normal commutative rings. We will give some explicit examples of NCRs for rings coming from algebraic geometry. Moreover, it is not clear which values of finite global dimensions for NCRs are possible, even for rings R of low Krull-dimension. This leads us to consider the so-called global spectrum of a ring, that is $gs_{\mathrm{MCM}(R)}(R)$, the set of all possible global dimensions of endomorphism rings of Cohen-Macaulay-modules. We will address some questions connected with the global spectrum. This is joint work with H. Dao and C. Ingalls.

G. Jasso
Towards higher homological algebra.

I will introduce n-exact sequences and describe their basic properties. These are sequences with $n + 2$ terms which are higher analogs of kernel-cokernel pairs in additive categories.

M. Herschend
2-representation finite algebras and tilting for one dimensional hypersurfaces.

Auslander-Reiten theory provides a homological approach to representation theory of finite dimensional algebras. A few years ago Osamu Iyama introduced a higher dimensional version of this theory, in which one studies algebras whose module categories have subcategories with suitable homological properties. The simplest class of such algebras are called d-representation finite algebras. These are algebras of global dimension d having d-cluster tilting subcategories with finitely many indecomposables. Thus 1-representation finite algebras are hereditary representation finite algebras. Over an algebraically closed ground field these correspond to Dynkin quivers by Gabriel's Theorem. For 2-representation finite algebras no similar classification is known. I will present some results from joint work with Osamu Iyama that allows one to construct several infinite families of 2-representation finite algebras. Although these families are infinite, the underlying data has a discrete nature.

In ongoing joint work with Osamu Iyama, Ryo Takahashi and Kota Yamaura we have discovered a new family of 2-representation finite algebras depending on several parameters. They are endomorphism algebras of tilting objects in certain triangulated categories of geometric origin. More precisely, these are stable categories of graded Cohen-Macaulay modules over one dimensional hypersurface rings. I will present these tilting objects and compute their stable and non-stable endomorphism algebras using graded quivers with potential.

F. Marks
Silting modules and universal localisations.

Silting modules generalise simultaneously tilting modules over any ring and support τ-tilting modules over finite dimensional algebras. In this talk, we study ring epimorphisms and universal localisations, as defined by Cohn and Schofield, that arise from partial silting modules. These ring epimorphisms are described explicitly by an idempotent quotient of the endomorphism ring of a completion of the given partial silting. In the context of hereditary rings, the above construction yields a bijection between universal localisations and certain minimal silting modules. Moreover, it turns out that all universal localisations over any ring arise from partial silting modules. This talk contains ongoing work with Lidia Angeleri Hügel and Jorge Vitoria, and with Jan Stovicek.

V. Miemietz
Isomorphisms of affine Schur algebras.

We describe how an isomorphism between certain completions of affine Hecke algebras associated to GL_n and quiver Hecke algebras of type A lifts to an isomorphism between similar completions of Vigneras' affine Schur algebra for GL_n and the affine quiver Schur algebra defined by Stroppel and Webster. This is joint work with Catharina Stroppel.

D. Paukzstello
A discrete introduction.

In this talk, we will provide a survey of the structure of discrete derived categories. The notion of a discrete derived category was introduced by Vossieck, who classified the algebras with discrete derived categories, the so-called derived-discrete algebras. Bobinski, Geiss and Skowronski then determined a canonical form for the derived equivalence class of a derived-discrete algebra and described the structure of their Auslander-Reiten quivers explicitly. Their structure is simple enough to enable concrete calculations, but sufficiently non-trivial to manifest interesting phenomena. This makes them an ideal natural laboratory to study derived representation theory. This is a report on joint work with Nathan Broomhead and David Ploog. The pun in the title is due to Robert Marsh.

C. Psaroudakis
Gorenstein Algebras, Singular Equivalences and Recollements.

Let Λ be an Artin algebra and a an idempotent element. Our aim in this talk is to present a common context in order to compare the algebras Λ and $a\Lambda a$ with respect to: (α) Gorensteinness, and (β) singular equivalences. More precisely, under some conditions on the idempotent element a, we show that Λ is Gorenstein if and only if $a\Lambda a$ is Gorenstein, and that Λ and $a\Lambda a$ are singularly equivalent, i.e. their singularity categories are triangle equivalent. The right framework for proving the above results is the setting of recollement of abelian categories. This talk will start by providing motivation for this work. In particular, we will discuss the finite generation condition (Fg) for Hochschild cohomology, a meaningful condition for the theory of support varieties of finite dimensional algebras (Snashall-Solberg). Then we discuss recollements of abelian categories and we introduce the notion of eventually homological isomorphisms between abelian categories. We explain this new notion for recollements of module categories, and under this condition we show Gorensteinness for Λ and $a\Lambda a$. Finally, we will give necessary and sufficient conditions for the quotient functor $e\colon \mathcal{B} \to \mathcal{C}$, of a recollement of abelian categories $(\mathcal{A}, \mathcal{B}, \mathcal{C})$, to induce a triangle equivalence between the singularity categories of $\mathcal{B}$ and $\mathcal{C}$. As an application to quotients of path algebras, we explain how to choose an idempotent element a in $\Lambda = kQ/\langle\rho\rangle$, such that the algebras Λ and $a\Lambda a$ are singularly equivalent.

This talk is based on joint work with Øystein Skartsæterhagen and Øyvind Solberg (arXiv:1402.1588).

S. Schroll
Trivial extensions and Brauer graph algebras.

Special biserial algebras form an important class of tame algebras, containing, for example, gentle algebras, string algebras and Brauer graph algebras. Brauer graph algebras are symmetric special biserial algebras defined by a graph that is locally embedded in the plane. In this talk we show that every symmetric special biserial algebra is a Brauer graph algebra and we construct its Brauer graph. We further show how to associate to every gentle algebra A, a graph G and we show that the trivial extension of A is the Brauer graph algebra with Brauer graph G. We recall admissible cuts and we show that admissible cuts of Brauer graph algebras are a gentle algebras and that they provide an inverse operation to taking trivial extensions of gentle algebras. We apply this to gentle Jacobian algebras defined by triangulations of marked Riemann surfaces and we show that the trivial extension of a gentle Jacobian algebra is the Brauer graph algebra with Brauer graph given by the arcs of the triangulation.

E. Shinder
Orbifold derived categories.

Given a finite group G acting on a smooth variety M, I will talk about an attempt to incorporate the orbifold cohomology decomposition for M/G into the equivariant derived category of (M, G).

L. de Thanhoffer de Völcsey
The cluster structure of singularity categories.

In the first part of this talk, we give a broad overview of the theory of cluster categories. We start with cluster algebras and discuss how and when one can construct categories which reflect the combinatorics of these algebras. In the second part of the talk we describe how the singularity category of certain isolated singularities naturally provide examples of this phenomenon and discus some examples.

A. van Roosmalen
Serre functors and derived equivalences for hereditary algebras.

This talk is based on joint work with Donald Stanley where we study the role of the Serre functor in the theory of derived equivalences. Let A be a k-linear (k a field) abelian category such that the bounded derived category D^bA has a Serre functor, and let H be the heart of a t-structure $(D^{\leq 0}, D^{\geq 0})$ on D^bA. One can verify that if the realization functor $D^bH \to D^bA$ is a triangle equivalence, then (1) the t-structure is bounded and (2) $D^{\leq 0}$ is closed under the Serre functor. In this talk, we will consider some cases where the converse holds as well.

J. Vitoria
An introduction to silting theory.

The original concept of tilting has been expanded from small to large modules, from finite dimensional algebras to arbitrary rings and from modules to complexes. The class of silting complexes appears as a strict generalisation of tilting complexes. While tilting complexes induce derived equivalences, silting complexes are useful to study t-structures in the derived category. Recently, in joint work with Lidia Angeleri Hügel and Frederik Marks, we introduced the notion of silting modules, which captures simultaneously some features of (large) tilting modules and (2-term) silting complexes. In this talk we will survey some results on tilting and silting, modules and complexes, and discuss how these notions relate among themselves and with t-structures in the derived category.