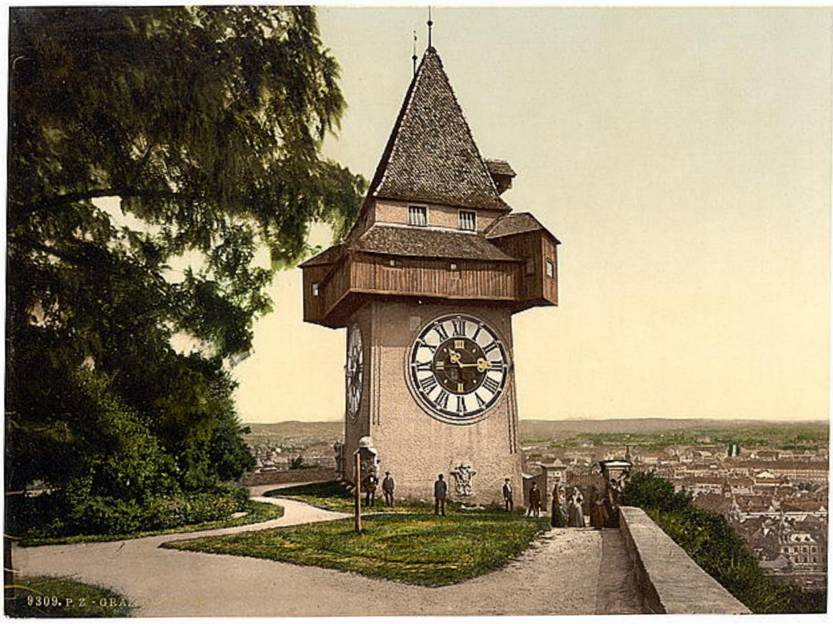
Arithmetic and Ideal Theory of Rings and Semigroups

With a one-day special session dedicated to **Franz Halter-Koch**
on the occasion of his 70^{th} birthday

September 22-26, 2014
Graz, Austria



CONFERENCE BOOKLET

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CONFERENCE INFORMATION

Date

September 22-26, 2014

Place of Event

University of Graz
Institute of Mathematics and Scientific Computing
Heinrichstraße 36
8010 Graz
Austria

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Conference Website

http://math.uni-graz.at/ideals2014

Plenary Speakers

- Evrim Akalan (Hacettepe University, Turkey), NAWI Graz Lecture:
  Multiplicative Ideal Theory in Non-commutative Rings
- Valentina Barucci (Sapienza University of Rome, Italy),
  NAWI Graz Lecture: From simple to less simple
- Jim Coykendall (Clemson University, USA):
  Atomicity, factorization, and graphs
- Marco Fontana (Roma Tre University, Italy):
  Franz Halter-Koch's contributions to ideal systems: a survey of some selected topics
- Pedro García-Sánchez (University of Granada, Spain):
  Computing nonunique factorization invariants
- K. Alan Loper (Ohio State University, USA):
  Chains of quadratics transforms of a regular local ring
- Hidetoshi Marubayashi (Tokushima Bunri University, Japan):
  Ore-Rees Rings which are Maximal Orders
- Jan Okniński (University of Warsaw, Poland):
  Noetherian semigroup algebras and beyond
- Bruce Olberding (New Mexico State University, USA):
  Geometric and topological applications to intersections of valuation rings
- Wolfgang A. Schmid (University Paris 8 & 13, LAGA, France):
  Sets of lengths: retrospects and prospects

List of participants

Evrim Akalan, Hacettepe University, Turkey
Phạm Ngọc Ánh, Hungarian Academy of Sciences, Hungary
Ivan Arzhantsev, Lomonosov Moscow State University, Russia
Başak Ay, Izmir Institute of Technology, Turkey
Ayman Badawi, American University of Sharjah, United Arab Emirates
Nicholas R. Baeth, University of Central Missouri, USA
Paul Baginski, Fairfield University, USA
Valentina Barucci, Sapienza Università di Roma 1, Italy
Karin Baur, University of Graz, Austria
Silvana Bazzoni, Università di Padova, Italy
Oz Ben-Shimol, Bar-Ilan University, Israel
Driss Bennis, Mohammed V-Agdal University, Morocco
Dusko Bogdanic, University of Graz, Austria
Geoffrey Booth, Nelson Mandela Metropolitan University, South Africa
Horst Brunotte
Paul-Jean Cahen, Université d'Aix Marseille, France
Jean-Luc Chabert, Université de Picardie, France
Gyu Whan Chang, Incheon National University, Korea
Scott Chapman, Sam Houston State University, USA
Jim Coykendall, Clemson University, USA
Kálmán Cziszter, Budapest Business School, Hungary
Marco D'Anna, Università di Catania, Italy
Mátyás Domokos, Hungarian Academy of Sciences, Hungary
Sabine Evrard, LAMFA University of Picardy, Amiens, France
Carmelo Antonio Finocchiaro, Università degli Studi Roma Tre, Italy
Marco Fontana, Università degli Studi Roma Tre, Italy
Stefania Gabelli, Università degli Studi Roma Tre, Italy
Pedro García-Sánchez, Universidad de Granada, Spain
Alfred Geroldinger, University of Graz, Austria
Franz Halter-Koch, University of Graz, Austria
Evan Houston, University of North Carolina at Charlotte, USA
I-Chiau Huang, Academia Sinica, Taiwan
Florian Kainrath, University of Graz, Austria

Abdul Nadim Khan, Aligarh Muslim University, India
Hwankoo Kim, Hoseo University, Korea
Ulrich Krause, University of Bremen, Germany
Dijana Kreso, Graz University of Technology, Austria
Łukasz Kubat, Polish Academy of Sciences, Poland
Günter Lettl, University of Graz, Austria
Alan Loper, Ohio State University, USA
Thomas Lucas, University of North Carolina Charlotte, USA
Waqas Mahmood, Quaid-I-Azam University Islamabad, Pakistan
László Márki, Rényi Institute, Hungarian Academy of Sciences, Hungary
Hidetoshi Marubayashi, Tokushima Bunri University, Japan
Ryûki Matsuda, Ibaraki University, Japan
María Ángeles Moreno Frías, Universidad de Cádiz, Spain
Reza Naghipour, University of Tabriz, Iran
Mara D. Neusel, Texas Tech University, USA
Christopher O'Neill, Texas A&M University, USA
Jan Okniński, University of Warsaw, Poland
Bruce Olberding, New Mexico State University, USA
Mi Hee Park, Chung-Ang University, Korea
Giulio Peruginelli, Università di Padova, Italy
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Daniel Smertnig, University of Graz, Austria
Fouad Taraza, Mohammed V-Agdal University, Morocco
Francesca Tartarone, Università degli Studi Roma Tre, Italy
Denise Torrão, Universidade de Évora, Portugal
Salvatore Tringali, École Polytechnique, France
Arunachalam Umamaheswaran, Periyar University, India
Alberto Vigneron-Tenorio, University of Cádiz, Spain
Hannah Vogel, University of Graz, Austria
Roger Wiegand, University of Nebraska-Lincoln, USA

Sylvia Wiegand, University of Nebraska-Lincoln, USA
Thomas Wunderer, University of Graz, Austria
Paolo Zanardo, University of Padova, Italy
Qinghai Zhong, University of Graz, Austria

Library

Location: Institute of Mathematics and Scientific Computing
Heinrichstraße 36
3rd floor
Opening hours: Monday - Friday: 9.00am-1.00pm

Internet access

Internet services are available for free. To access the Internet, turn on the Wireless LAN on your computer in the area of the conference premises and select the network called

"KFU-Tagung"

This network is not secured, so you do not require a password to access it, though depending on your OS you might receive a general security warning when accessing an unsecured network.

Organizer's telephone numbers (for emergencies)

+43 68181625821 (Alfred Geroldinger)
+43 6508713200 (Daniel Smertnig)

Social activities

Monday, September 22nd: Reception

Reception by the Mayor of Graz, S. Nagl, at 7:00pm at Rathausplatz 1.

Wednesday, September 24th: Excursion to South Styria

There will be a free afternoon on Wednesday. On this afternoon, we have organized a bus trip to the wine growing area in the south of Styria. The tour includes a wine tasting, a drive through the scenic landscape and ends in a rustic tavern (Buschenschank), where a simple dinner will be served.

The details of the trip are as follows: We meet next to the conference venue and leave at 1:00pm by bus. The bus will take us to Weinbauschule Silberberg in Leibnitz, a local school for wine-growing. There we will get a guided tour of the wine cellars and enjoy a wine tasting. The tour and wine tasting will take about three hours. Afterwards, there will be the possibility to either have a nice walk (about 45 minutes) through the scenic landscape, or to take the bus, to arrive at Buschenschank Kieslinger. At this rustic tavern a dinner buffet made from regional products will be served.

We plan to go back to Graz at 8:00pm, and expect to arrive at the conference venue again at 9:00pm.

Restaurants located near the conference venue

Bierbaron, Heinrichstraße 56
Bierfactory XXL, Halbärthgasse 14
Bistro Zeppelin, Goethestraße 21
Cafe Einstein Dino, Heinrichstraße 29
Cafeteria Resowi, Universitätsstraße 15
Café-Restaurant Liebig, Liebiggasse 2
Galliano, Harrachgasse 22
Gastwirtschaft zum Weissen Kreuz, Heinrichstraße 67
Lokal Müller, Villefortgasse 3
L'Originale Klöcherperle, Heinrichstraße 45
Uni Café, Heinrichstraße 36
Zu den 3 goldenen Kugeln, Heinrichstraße 18

Pharmacies located near the University

Glacis Apotheke
Glacisstraße 31, 8010 Graz, Geidorf

Apotheke & Drogerie Zu Maria Trost
Mariatrosterstraße 31, 8043 Graz, Mariatrost

Apotheke zur Göttlichen Vorsehung
Heinrichstraße 3, 8010 Graz, Geidorf

Emergency telephone numbers

Fire brigade: 122
Police: 133
Doctors' emergence service: 141
Ambulance: 144

CAFETERIA RESOWI

22 September 2014 to 26 September 2014

Opening hours: Monday - Friday: 7.30am-6.00pm

Menu available from 11.00am on

Each day, the soup and the dish of Menu I are vegetarian

Monday:	Vegetables cream soup	
Menu **I**	Baked gnocchi with zucchini, tomatoes and feta cheese with salad	€ 5,30
Menu **II**	Pork cutlet with glazed carrots and noodles, with salad	€ 5,50
Dish of the day	Grilled turkey stripes on salad	€ 4,90
Tuesday:	Celery cream soup	
Menu **I**	Mushroom goulash with dumplings napkins and salad	€ 5,30
Menu **II**	Turkey steak in onion sauce with potatoe dumplings and salad	€ 5,50
Dish of the day	Grilled meatloaf with rice and peas	€ 4,90
Wednesday:	Clear leek soup	
Menu **I**	Homemade fruit rice pudding with berry sauce and compote	€ 5,30
Menu **II**	Steamed root roast beef with noodles and salad	€ 5,50
Dish of the day	Spaghetti with sauce bolognaise	€ 4,90
Thursday:	Bound onion soup	
Menu **I**	Pumpkin and potatoe strudel with yoghurt sauce and salad	€ 5,30
Menu **II**	Baked chicken breasts with vegetable rice and salad	€ 5,50
Dish of the day	Chilli con carne with bread	€ 4,90
Friday:	Bouillon with insert	
Menu **I**	Tagliatelle with fine vegetables in österkronsauce, with salad	€ 5,30
Menu **II**	Chops grilled with baked potatoes, vegetables and dip, and salad	€ 5,50
Dish of the day	Bernese sausages with French fries	€ 4,90

ABSTRACTS

Multiplicative Ideal Theory in Non-commutative Rings

Evrim Akalan

The aim of this talk is to survey maximal orders, Asano orders and Dedekind prime rings in non-commutative setting, and introduce the classes of generalized Dedekind prime rings and pseudo-principal rings. Several characterizations and examples of these classes of rings will be provided.

Hacettepe University
Department of Mathematics, Beytepe Campus
Ankara
Turkey
E-mail address: eakalan@hacettepe.edu.tr

A generalization of Clifford's theorem

Phạm Ngọc Ánh

An ideal theory of arithmetical rings with one minimal prime ideal is, except one case, a lattice-theoretical factor of an ideal theory of Prüfer domains, i.e., of positive cones of lattice-ordered abelian groups although such rings are, not necessarily, factors of Prüfer domains, even in the case of valuation rings, by negative answer to Kaplansky's conjecture for valuation rings. Recall that an ideal theory of a ring is a monoid of finitely generated ideals under ideal multiplication partially ordered by reverse inclusion.

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences
Reáltanoda u. 13–15
Budapest
Hungary
E-mail address: anh.pham.ngoc@renyi.mta.hu

Cox rings and unique factorization

Ivan Arzhantsev

In the talk we will discuss total coordinate rings, or Cox rings, of algebraic varieties with emphasis on unique factorization properties. The Cox ring $R(X)$ of a normal variety X is a factorially graded ring, and it is a unique factorization domain provided the divisor class group $\mathrm{Cl}(X)$ is finitely generated and free. Relations to the divisor theory of an abstract semigroup will be established and recent algebraic characterizations of Cox rings will be presented.

References

[1] I.V. Arzhantsev, *On the factoriality of Cox rings*, Math. Notes **85** (2009), no. 5, 623–629.

[2] I.V. Arzhantsev, U. Derenthal, J. Hausen, A. Laface, *Cox rings.* Cambridge Studies in Advanced Mathematics, No. 144, Cambridge University Press, 2014, 472 pp.

[3] I.V. Arzhantsev, S.A. Gaifullin, *Cox rings, semigroups and automorphisms of affine algebraic varieties*, Sbornik Math. **201** (2010), no. 1, 1–21.

[4] B. Bechtold, *Factorially graded rings and Cox rings*, J. Algebra **369** (2012), no. 1, 351–359.

[5] F. Berchtold, J. Hausen, *Homogeneous coordinates for algebraic varieties*, J. Algebra **266** (2003), no. 2, 636–670.

[6] E.J. Elizondo, K. Kurano, K. Watanabe, *The total coordinate ring of a normal projective variety*, J. Algebra **276** (2004), no. 2, 625–637.

National Research University Higher School of Economics, School of Computer Science
Bolshoi Trekhsvyatitelskiy 3
Moscow 109028
Russia
and
Lomonosov Moscow State University, Faculty of Mechanics and Mathematics
Leninskie Gory 1
Moscow 119991
Russia
E-mail address: `arjantse@mccme.ru`

Locally Isomorphic Torsionless Modules

Başak Ay

Let R be an integral domain. An R-module G is torsionless if it is isomorphic to a submodule of a finitely generated free R-module. In 2002, P. Goeters and B. Olberding examine the relationship between local, stable and near isomorphisms of torsionless modules over h-local domains in a paper. In our talk, we investigate some important facts to explain the relationship among these "weak" isomorphisms over integral domains of finite character.

Izmir Institute of Technology
Izmir
Turkey
E-mail address: basakay@iyte.edu.tr

On the dot product graph of a commutative ring

Ayman Badawi

Let A be a commutative ring with nonzero identity, $1 \leq n < \infty$ be an integer, and $R = A \times A \times \cdots \times A$ (n times). The *total dot product graph* of R is the (undirected) graph $TD(R)$ with vertices $R^* = R \setminus \{(0, 0, ..., 0)\}$, and two distinct vertices x and y are adjacent if and only if $x \cdot y = 0 \in A$ (where $x \cdot y$ denote the normal dot product of x and y). Let $Z(R)$ denote the set of all zero-divisors of R. Then the *zero-divisor dot product graph* of R is the induced subgraph $ZD(R)$ of $TD(R)$ with vertices $Z(R)^* = Z(R) \setminus \{(0, 0, ..., 0)\}$. It follows that each edge (path) of the classical zero-divisor graph $\Gamma(R)$ is an edge (path) of $ZD(R)$. We observe that if $n = 1$, then $TD(R)$ is a disconnected graph and $ZD(R)$ is identical to the well-known zero-divisor graph of R in the sense of Beck-Anderson-Livingston, and hence it is connected. In this paper, we study both graphs $TD(R)$ and $ZD(R)$. For a commutative ring A and $n \geq 3$, we show that $TD(R)$ ($ZD(R)$) is connected with diameter two (at most three) and with girth three. Among other things, for $n \geq 2$, we show that $ZD(R)$ is identical to the zero-divisor graph of R if and only if either $n = 2$ and A is an integral domain or R is ring-isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$.

References

[1] D.F. Anderson, A. Badawi, *On the zero-divisor graph of a ring*, Comm. Algebra **36** (2008), 3073–3092.
[2] D.F. Anderson, P.S. Livingston, *The zero-divisor graph of a commutative ring*, J. Algebra **217** (1999), 434–447.
[3] D.F. Anderson, S.B. Mulay, *On the diameter and girth of a zero-divisor graph*, J. Pure Appl. Algebra **210** (2007), 543–550.
[4] A. Badawi, *On the dot product graph of a commutative ring*, to appear in Comm. Algebra (2015).
[5] A. Badawi, *On the annihilator graph of a commutative ring*, Comm. Algebra, **42** (2014), 108–121.
[6] I. Beck, *Coloring of commutative rings*, J. Algebra **116** (1988), 208–226.
[7] T.G. Lucas, *The diameter of a zero-divisor graph*, J. Algebra **301** (2006), 3533–3558.

American University of Sharjah
University City
Sharjah
United Arab Emirates
E-mail address: abadawi@aus.edu

Factorization theory in noncommutative settings

Nicholas Baeth

We give an example-based introduction to the study of factorizations of non zero-divisors in noncommutative rings and semigroups. In particular, we introduce several notions of factorizations, distances between factorizations, and arithmetical invariants which give a measure of how unique or nonunique factorizations are. We provide several examples to compare and contrast these invariants with their commutative counterparts.

University of Central Missouri
W.C. Morris 213
Warrensburg
U.S.A.
E-mail address: baeth@ucmo.edu

New Developments for the Plus-Minus Davenport Constant

Paul Baginski

Let G be a finite abelian group. The classical and extensively studied Davenport constant $D(G)$ is the least integer n such that any sequence $S = g_1, g_2, \dots, g_n$ of n elements of G has a nonempty, zero-sum subsequence. That is, there is a nonempty $I \subseteq \{1, \dots, n\}$ such that $\sum_{i \in I} g_i = 0$. An elementary argument shows $D(G) \leq |G|$ and a general construction shows that $D(G) \geq D^*(G)$, for a constant $D^*(G)$ expressed simply in terms of the parameters in the standard decomposition of G as a sum of cyclic groups. These bounds, however, are not sharp, though in most cases where the exact value of $D(G)$ is known, it equals the lower bound $D^*(G)$. These cases include groups of rank ≤ 2, p-groups, and certain other groups.

Recently, several authors have begun varying this zero-sum problem by allowing a fixed set $A \subset \mathbb{N}$ of weights for the sums. We shall focus on the weight set $A = \{1, -1\}$, which gives us the choice to add or subtract elements. We thus define the plus-minus Davenport constant $D_{\pm}(G)$ to be the least integer n such that any sequence $S = g_1, g_2, \dots, g_n$ of n elements of G has a nonempty, plus-minus zero-sum subsequence. That is, there exist $a_i \in \{-1, 0, 1\}$ not all zero such that $\sum_{i=1}^{n} a_i g_i = 0$. This constant can arise in factorization theory, like the classical Davenport constant, yet it also plays a role in Fourier-analytic arguments in combinatorics. Its values are known for far fewer groups, yet the known general upper and lower bounds are far closer than for the classical Davenport constant. This gives hope that this problem may be more tractable to solve. We will provide an overview of the efforts to date and recent progress by undergraduates from the Fairfield REU during summer 2014.

Fairfield University
1073 North Benson Rd
Fairfield, CT
USA
E-mail address: pbaginski@fairfield.edu

From simple to less simple

Valentina Barucci

Although in a commutative ring the additive structure is axiomatically stronger than the multiplicative structure, the last one seems to be more important in several cases. An exemplification of that is what happens in semigroup rings with coefficients in a field, where the semigroup has a big role in the study of the ring. In case of numerical semigroups and numerical semigroup rings this connection is particularly sharp. Anyway, easy and natural concepts coming from numerical semigroups have been recently generalized to ring context for rings of any Krull dimension, which seem to have nothing to do with semigroup rings. The talk concerns one of these concepts, the almost Gorenstein property, in connection with some constructions as the Nagata's idealization and the amalgamated duplication of a ring along an ideal. Given the Rees algebra of a ring R with respect to an ideal I, $R_+ = \oplus_{n\geq 0} I^n t^n$, both these constructions can be obtained as particular members of a family of quotients of R_+ mod an ideal, which is the contraction to R_+ of an ideal generated by a monic polynomial of degree two, $t^2 + at + b$ in $R[t]$, cf. [1]. Denoting by $R(I)_{a,b}$ such a ring, in case (R, m) is a one-dimensional local Cohen Macaulay ring, a joint result with Marco D'Anna and Francesco Strazzanti is that R is almost Gorenstein if and only if $R(m)_{a,b}$ is almost Gorenstein. A sketch of the proof of that will be given in the talk.

References

[1] V. Barucci, M. D'Anna, F. Strazzanti, *A family of quotients of the Rees algebra*, Comm. Algebra (in print).

Dipartimento di Matematica, Sapienza, Università di Roma 1
Piazzale A. Moro 2
Rome
Italy
E-mail address: barucci@mat.uniroma1.it

Homological epimorphisms of rings with weak global dimension one and the telescope conjecture

Silvana Bazzoni

We show that for a ring R of weak global dimension one there is a bijection between homological epimorphisms starting in R and smashing localizations of the derived category of R. If R is a valuation domain, we have a classification of all homological epimorphisms in terms of disjoint collections of intervals in the Zariski spectrum of R. As a consequence we prove that a commutative ring R of weak global dimension one satisfies the telescope conjecture if and only if every homological epoimorphism starting in R is flat.

This is a joint work with Jan Šťovíček (arXiv:1402.7294)

Università di Padova
Via Trieste 63
35121 Padova
Italy
E-mail address: bazzoni@math.unipd.it

On semi-trivial extensions

Driss Bennis

Let R be a commutative ring and M an R-module. Consider $\phi : M \otimes_R M \longrightarrow R$ an R-module homomorphism satisfying $\Phi(m \otimes m') = \Phi(m' \otimes m)$ and $\Phi(m \otimes m')m'' = m\Phi(m' \otimes m'')$. Then, the additive abelian group $R \oplus M$ becomes a commutative ring, if multiplication is defined as follows:

$$(r, m)(r', m') = (rr' + \Phi(m \otimes m'), rm' + r'm)$$

This ring is known by either semi-trivial extension or ϕ-trivial extension of R by M and it is denoted by $R \ltimes_\Phi M$.
In particular, if $\Phi = 0$, Φ-trivial extension of R by M is just the classical trivial extension of R by M.
In this talk, we present some recent results on semi-trivial extensions.

References

[1] I. Palmer, *The global homological dimension of semi-trivial extensions of rings*, Math. Scand. **37** (1975), 223–256.

[2] K. Sakano, *Some aspects of the ϕ-trivial extensions of rings*, Comm. Algebra **13** (1985), 2199–2210.

Mohammed V-Agdal University
B.P. 1014, Faculty of Science
Rabat
Morocco
E-mail address: d.bennis@fsr.ac.ma; driss_bennis@hotmail.com

On Primeness and Ideals in Near-Rings of Homogeneous Functions

Geoffrey L. Booth

Let R be a ring with unity. A function $f : R^2 \to R^2$ is said to be *homogeneous* if $f\left(\begin{bmatrix} x \\ y \end{bmatrix} r\right) = f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) r$ for all $x, y, r \in R$. If R is a topological ring, the set of all continuous, homogeneous self-maps of R^2 will be denoted $N_R(R^2)$. $N_R(R^2)$ is a (right) near-ring with the operations pointwise addition and composition of maps. In this talk we will discuss relationships between prime ideals of R, and certain kinds of prime ideals of $N_R(R^2)$. Much of this work comes from the doctoral thesis of Mogae [1] and is a continuation of work started by Veldsman [2].

References

[1] K. Mogae, *Primeness in near-rings of continuous maps*, Ph.D. thesis, Nelson Mandela Metropolitan University, 2014.

[2] S. Veldsman, *On equiprime near-rings*, Comm. Algebra **20** (1992), 2569–2587.

Nelson Mandela Metropolitan University
University Way
Port Elizabeth
South Africa
E-mail address: `geoff.booth@nmmu.ac.za`

About number fields with Pólya group of order ≤ 2

Jean-Luc Chabert

This is a joint work with David Adam.

Let K be a number field with ring of integers $\mathcal{O}_K$. There is a well known result of Carlitz [1] which says that the class group $\mathcal{C}l(K)$ is of order ≤ 2 if and only if, for every integer x of K, the lengths of all the decompositions of x in irreducible elements of $\mathcal{O}_K$ are equal. Following a suggestion of Jesse Elliott, we are interested in the number fields K whose Pólya group $\mathcal{P}o(K)$ is of order ≤ 2. Recall that $\mathcal{P}o(K)$ is the subgroup of $\mathcal{C}l(K)$ generated by the classes of Bhargava's factorial ideals of $\mathcal{O}_K$ (cf. [2, Chapter II])

Analogously to Carlitz' result, we study here the links between the order of the Pólya group $\mathcal{P}o(K)$ and the decompositions in irreducible elements of $\mathcal{O}_K$ of some rational numbers. Our main results concern quadratic fields where we prove equivalences between :

- $|\mathcal{P}o(K)| = 1$ and uniqueness of the decomposition of these rational numbers,
- $|\mathcal{P}o(K)| = 2$ and uniqueness of the length of their decompositions.

References

[1] L. Carlitz, *A characterization of algebraic number fields with class number two*, Proc. Amer. Math. Soc. **11** (1960), 391–392.

[2] P.-J. Cahen and J.-L. Chabert, *Integer-Valued Polynomials*, Amer. Math. Soc. Surveys and Monographs No 48, A.M.S., Providence, 1997. xix+322 pp.

Université de Picardie
33 rue Saint Leu
Amiens
France
E-mail address: jean-luc.chabert@u-picardie.fr

Locally GCD domains and the ring $D + XD_S[X]$

Gyu Whan Chang

Let D be an integral domain. We say that D is a *locally GCD domain* if D_M is a GCD domain for every maximal ideal M of D. We show that D is a locally GCD domain if and only if $aD \cap bD$ is locally principal for all $0 \neq a, b \in D$, and flat overrings of a locally GCD domain are locally GCD. We also show that the t-class group of a locally GCD domain is just its Picard group. Finally, we use the $D + XD_S[X]$ construction to give some interesting examples of locally GCD domains that are not GCD domains. This is a joint work with T. Dumitrescu and M. Zafrullah.

Department of Mathematics, Incheon National University
Incheon
Republic of Korea
E-mail address: whan@incheon.ac.kr

Atomicity, factorization, and graphs

Jim Coykendall

Since about 1990, the subject of factorization in integral domains has experienced an international renaissance. This interest in factorization has bled over into other mathematical structures (e.g. commutative rings with nontrivial zero divisors, monoids, etc.). In this expository talk we will give some history of the research done and some of the important directions taken in the past quarter century. Additionally, we will explore some new results and possibilities for the field. This talk will be designed mostly as a general interest overview and many examples will be given to hopefully augment intuition.

Clemson University
Department of Mathematical Sciences
Clemson, SC 29678
USA
E-mail address: jcoyken@clemson.edu

A new connection between additive number theory and invariant theory

Kálmán Csiszter

The generalized Davenport constant $D_k(A)$ was introduced by Halter-Koch in [4] for an abelian group A as the maximal length of a zero-sum sequence of elements in A that cannot be written as the concatenation of $k+1$ non-empty zero-sum subsequences. This quantity also has a meaning related to non-modular representations V of A over a field $\mathbb{F}$: it gives the highest possible degree of a homogeneous polynomial in the invariant ring $R := \mathbb{F}[V]^A$ not contained in the $k+1$-th power of the maximal homogeneous ideal R_+. This latter reformulation of the original concept naturally generalizes to any finite non-abelian group G too, thereby defininig its *generalized Noether number* $\beta_k(G)$. We have extended most of the known properties of D_k for β_k, as well, using different algebraic proofs in [1]. Similarly, the so called "inductive method" which is a powerful technique in the context of D_k (see chapter 5.7. in [3]) was extended in [2] to the case of some particular non-abelian groups. These findings point at the possibility of some futher connections and interactions between combinatorial number theory and invariant theory.

References

[1] K. Cziszter and M. Domokos, *On the generalized Davenport constant and the Noether number*, Central European Journal of Mathematics **11** (2013), no. 9, 1605–1615.

[2] K. Cziszter and M. Domokos, *Noether's bound for the groups with a cyclic subgroup of index two*, Journal of Algebra **399** (2014), 546–560.

[3] A. Geroldinger and F. Halter-Koch, *Non-unique factorizations. Algebraic, combinatorial and analytic theory*, Monographs and Textbooks in Pure and Applied Mathematics, Chapman & Hall/CRC, 2006.

[4] F. Halter-Koch, *A generalization of Davenport's constant and its arithmetical applications*, Colloquium Mathematicum **63** (1992), 203–210.

Budapest Business School
Alkotmány utca 9-11
Budapest
Hungary
E-mail address: cziszter.kalman@gmail.com

Ring and semigroup constructions

Marco D'Anna

In this talk I will present a survey on some ring constructions recently studied, such as duplication, amalgamation and a family of quotients of the Rees algebra; in case of algebroid branches, these constructions lead to analogous semigroup constructions, that, starting with a numerical semigroup produce either numerical semigroups or subsemigroups of $\mathbb{N}^2$. In particular, I will present results about the Gorenstein property for these ring constructions and about the symmetry for the analogous semigroup constructions.

Università di Catania
Viale A. Doria, 6
Catania
Italy
E-mail address: mdanna@dmi.unict.it

On the Noether number of finite groups

Mátyás Domokos

The *Noether number* of a finite group G is defined as $\beta(G) := \sup_V \beta(G, V)$, where V ranges over all finite dimensional complex vector spaces endowed with a linear G-action, and $\beta(G, V)$ is the minimal positive integer d such that the corresponding algebra $\mathbb{C}[V]^G$ of polynomial invariants is generated by its elements of degree at most d. A classical theorem of E. Noether [1] asserts that $\beta(G) \leq |G|$. It is easy to see that for a cyclic group G we have $\beta(G) = |G|$. On the other hand B. Schmid [2] showed that this equality holds only if G is a cyclic group, so $\beta(G)/|G| < 1$ for any non-cyclic G. In joint works [3], [4] with K. Cziszter we proved that

$$\limsup_{G \text{ non-cyclic}} \beta(G)/|G| = 1/2.$$

In the talk we shall overview the results leading to this statement. In particular, a crucial role in the proof is played by a generalization of the Noether number, which can be viewed as a non-commutative variant of the generalized Davenport constant introduced by F. Halter-Koch [5] (see also Chapter 6.1 in [6]).

References

[1] E. Noether, *Der Endlichkeitssatz der Invarianten endlicher Gruppen*, Math. Ann. **77** (1916), 89–92.

[2] B.J. Schmid, *Finite groups and invariant theory*, Topics in Invariant Theory, 35–66, Lect. Notes in Math. **1478**, 1991.

[3] K. Cziszter, M. Domokos, *Groups with large Noether bound*, Ann. Inst. Fourier (Grenoble), to appear.

[4] K. Cziszter, M. Domokos, *The Noether number for the groups with a cyclic subgroup of index two*, J. Algebra **399** (2014), 546–560.

[5] F. Halter-Koch, *A generalization of Davenport's constant and its arithmetical applications*, Colloquium Mathematicum **63** (1992), 203–210.

[6] A. Geroldinger, F. Halter-Koch, *Non-unique factorizations. Algebraic, Combinatorial and Analytic Theory*, Monographs and textbooks in pure and applied mathematics, Chapman & Hall/CRC, 2006.

Rényi Institute of Mathematics, Hungarian Academy of Sciences
1053 Reáltanoda utca 13-15
Budapest
Hungary
E-mail address: domokos.matyas@renyi.mta.hu

Topological considerations on semistar operations

Carmelo Antonio Finocchiaro

Let D be an integral domain, let K be the quotient field of D, and let $\mathrm{SStar}(D)$ denote the set of all semistar operations on D. We will define a new topology on $\mathrm{SStar}(D)$, called the *Zariski topology*, in such a way that the space $\mathrm{Over}(D)$ of all overrings of D, with the usual topology having the collection $\{\mathrm{Over}(D[x]) : x \in K\}$ as a subbasis of open sets, is canonically homeomorphic to a subspace of $\mathrm{SStar}(D)$. After discussing about the main properties of the Zariski topology of $\mathrm{SStar}(D)$, we will prove that the subspace $\mathrm{SStar}_f(D)$ of all the finite type semistar operations on D is a spectral space, in the sense of Hochster (see [1]), i.e., it is homeomorphic to the prime spectrum of a commutative ring. Moreover, we will relate the compactness of the subspaces of $\mathrm{SStar}_f(D)$ with the finite type property of their infimum (with respect to the natural partial order of $\mathrm{SStar}(D)$). By using also the inverse topology on $\mathrm{Spec}(D)$, we will give a topological description of the stable closure of a semistar operation. This is a joint paper with Dario Spirito.

References

[1] M. Hochster, *Prime ideal structure in commutative rings*, Trans. Amer. Math. Soc. **142** (1969), 43–60.

Dipartimento di Matematica e Fisica – Università degli Studi Roma Tre
Largo San Leonardo Murialdo 1
Rome
Italy
E-mail address: carmelo@mat.uniroma3.it

One-dimensional stable domains

Stefania Gabelli

This is part of a joint work with Moshe Roitman.
In 1972 J. Sally and W. Vasconcelos defined an ideal I of a ring R to be *stable* if I is projective over its endomorphism ring. When I is a nonzero ideal of a domain R, I is stable if and only if I is invertible in the overring $(I : I)$. A domain R is called *(finitely) stable* if each (finitely generated) nonzero ideal is stable.
Of course, when R is Noetherian, stability and finite stability coincide, but in general these two classes of domains are distinct. For example, any valuation domain is finitely stable while a valuation domain is stable if and only if it is strongly discrete.
Any stable Noetherian domain is one-dimensional, but B. Olberding constructed several examples of one-dimensional local domains that are stable and not Noetherian. All these domains must have zero conductor in the integral closure.
We show that a one-dimensional stable domain is Mori (i.e., it satisfies the ascending chain condition on divisorial ideals). Indeed, we prove that a domain is stable and one-dimensional if and only if it is Mori and finitely stable.
We also show that in the semilocal case a stable domain is Mori if and only if it is Archimedean (i.e., $\cap_{n\geq 0} r^n R = (0)$, for each nonunit $r \in R$). However, we give an example of a Prüfer domain satisfying the ascending chain condition on principal ideals that is stable of dimension two.

Dipartimento di Matematica e Fisica, Università Roma Tre
Largo S. Leonardo Murialdo, 1
00146 - Roma
Italy
E-mail address: gabelli@mat.uniroma3.it

Computing nonunique factorization invariants

Pedro A. García-Sánchez

We will review some of the existing algorithms for computing sets of lengths, Delta sets, elasticity, catenary and tame degrees and ω-primality. Some of them are particularized for numerical semigroups and are part of our `GAP` package `numericalsgps`.
The main issue when dealing with these invariants is that we are facing a linear integer programming problem, and thus the complexity is high if the number of atoms is not small. Some workarounds using graphs and Apéry sets associated to elements have appeared during the last decade.

Departamento de Álgebra, Universidad de Granada
Campus Fuentenueva s/n
18071 Granada
España
E-mail address: `pedro@ugr.es`

Star Regularity for Noetherian Domains

Evan Houston

Let R be a Noetherian domain, and let $\mathrm{Star}(R)$ denote the set of star operations on R. For several years, I (along with Mi Hee Park and Abdeslam Mimouni) have been interested in the question of when $\mathrm{Star}(R)$ is finite. (It is trivial that $|\mathrm{Star(R)}| = 1$ if and only if all nonzero ideals of R are divisorial, and this case has received a great deal of attention.) In this talk, we discuss the relationship between the star operations on R and those on overrings of R. Calling R *star regular* if $|\mathrm{Star}(R)| \geq |\mathrm{Star}(T)|$ for each overring T of R, we investigate the extent to which star regularity is a local property and prove that if R is a local Noetherian domain with infinite residue field for which $1 < |\mathrm{Star}(R)| < \infty$, then R is star regular. The proof relies on a characterization of local Noetherian domains R with infinite residue field that satisfy $|\mathrm{Star}(R)| < \infty$ [1].

References

[1] E. Houston and M.H. Park, *A characterization of local Noetherian domain which admit only finitely many star operations: The infinite residue field case*, J. Algebra **407** (2014), 105–134.

University of North Carolina at Charlotte
9201 University City Blvd.
Charlotte, NC 28223
USA
E-mail address: eghousto@uncc.edu

Extensions of Tangent Cones

I-Chiau Huang

Over a field, a numerical semigroup S defines a tangent cone $G(S)$ (the associated graded ring of the corresponding numerical semigroup ring). Given another numerical semigroup S' so that $G(S')$ is naturally a subalgebra of $G(S)$, we call $G(S)/G(S')$ an extension. Examples of extensions include $G(S)/G(\mathbb{N})$. We define Cohen-Macaulayness, Gorensteiness and complete intersection for extensions generalizing those for rings in the following sense: The extension $G(S)/G(\mathbb{N})$ is Cohen-Macaulay (Gorenstein or complete intersection) if and only if the ring $G(S)$ is Cohen-Macaulay (Gorenstien or complete intersection). A hierarchy of rings extends to extensions: A complete intersection extension is Gorenstein. A Gorenstein extension is Cohen-Macaulay. Extensions can be used to construct Cohen-Macaulay (Gorenstein or complete intersection) tangent cones using transitivity. More precisely, if $G(S)/G(S')$ and $G(S')/G(S'')$ are Cohen-Macaulay (Gorenstein or complete intersection) extensions, so is $G(S)/G(S'')$. Together with [4], we generalize the characterizations for a tangent cones to be Gorenstein [3] or complete intersection [1]. We will also examine tangent cones obtained by gluing [2, 5] from the viewpoint of extensions.

References

[1] M. D'Anna, V. Micale, A. Sammartano, *When the associated graded ring of a semigroup ring is complete intersection*, J. Pure Appl. Algebra **217** (2013), no. 6, 1007–1017.

[2] F. Arslan, P. Mete, and M. Şahin, *Gluing and Hilbert functions of monomial curves*, Proc. Amer. Math. Soc. **137** (2009), no. 7, 2225–2232.

[3] L. Bryant, *Goto numbers of a numerical semigroup ring and the Gorensteiness of associated graded rings*, Comm. Algebra **38** (2010), no. 6, 2092–2128.

[4] I-C. Huang, *Residual Complex on the Tangent Cone of a Numerical Semigroup Ring*, Preprint (2013).

[5] R. Jafari, S. Zarzuela Armengou, *On monomial curves obtained by gluing*, Semigroup Forum **88** (2014), no. 2, 397–416.

Institute of Mathematics, Academia Sinica
Astronomy-Mathematics Building, No. 1, Sec. 4, Roosevelt Road
Taipei
Taiwan
E-mail address: ichuang@math.sinica.edu.tw

An engel condition with generalized derivations in rings

Abdul Nadim Khan

Let R be a non-commutative prime ring with center $Z(R)$. Let U be the left Utumi quotient ring of R and C be the center of U. A map $d : R \to R$ is called a derivation of R if $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. In [Glasgow Math. J. 33 (1991), 89–93], Brašar introduced the notion of generalized derivation as follows: an additive mapping $F : R \to R$ is called a generalized derivation of R if there exist a derivation d of R such that $F(xy) = F(x)y+xd(y)$ for all $x, y \in R$. Later, Lee [Comm. Algebra 27 (1999), no. 8, 4057-4073] extended the definition of generalized derivation as follows: By a generalized derivation he means an additive mapping $F : J \to U$ such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in J$, where U is the left Utumi quotient ring of R, J is a dense right ideal of R and d is a derivation from J to U. He also showed that every generalized derivation can be uniquely extended to a generalized derivation of U. In fact, there exists $a \in U$ and a derivation d of U such that $F(x) = ax + d(x)$ for all $x \in U$.
In this talk, we shall continue the similar study and discuss the characterization of generalized derivations on prime rings.

Aligarh Muslim University
Aligarh
India
E-mail address: abdulnadimkhan@gmail.com

Piecewise w-Noetherian domains and their applications

Hwankoo Kim

In this talk, we present piecewise Noetherian (resp., piecewise w-Noetherian) properties in several settings including flat (resp., t-flat) overrings, Nagata rings, integral domains of finite character (resp., w-finite character), pullback of type ($\square$), polynomial rings, and $D + XK[X]$ constructions. As applications, over a piecewise w-Noetherian domain, we also present the existence of an associated prime ideal for any GV-torsionfree module and consider the smallest test set (for injectivity) of prime w-ideals.

Hoseo University
79th gil, Hoseo-ro, Baebang-up
Asan
Republic of Korea
E-mail address: hkkim@hoseo.edu

Atomic decay in domains and monoids

Ulrich Krause

By definition, an irreducible element cannot split - but powers of it can split into different irreducible elements. This decay of atoms is the root of non-unique factorization: Unique factorization holds if and only if no atom can decay into other ones. This applies in particular to the ring of integers of an algebraic number field as well as to certain Diophantine monoids. In case of non-unique factorization the talk analyses the various kinds of proper atomic decay and its decay rates.

University of Bremen, Department of Mathematics
Bibliothekstrasse 1
Bremen
Germany
E-mail address: krause@math.uni-bremen.de

Structure and representations of plactic algebras

Łukasz Kubat

This talk is about the ring-theoretical properties of the plactic algebras over a field and their consequences for the plactic monoids. In particular, I will focus on recently obtained results concerning irreducible representations, the primitive and minimal prime spectrum, Gröbner-Shirshov bases and semigroup identities.

Institute of Mathematics, Polish Academy of Sciences
Śniadeckich 8
00-656 Warsaw
Poland
E-mail address: kubat.lukasz@gmail.com

Chains of quadratic transforms of a regular local ring

K. Alan Loper

Let D be an n-dimensional regular local ring with maximal ideal m generated by regular parameters $\{x_1, x_2, \ldots, x_n\}$. Choose a particular x_i and extend D by adding the fractions of the form x_j/x_i where $j \neq i$. Then localize the resulting ring at a prime ideal which lies over m. We call this local ring a quadratic transform of D. The quadratic transform is again a regular local ring of dimension at most n. This process can be iterated to produce a quadratic transform tree. Anhyankar proved in 1956 that if D is two-dimensional then the union of every branch is a valuation domain. Shannon proved in 1973 that if D has dimension three then for some branches the union is not a valuation domain. Very little is known about either when the union is a valuation domain or what the structure of such a union is in case it is not a valuation domain. We give many results concerning both of these questions. Our discussion involves connections with mathematics outside of ring theory.

Ohio State University
1179 University Drive
Newark, Ohio
USA
E-mail address: loper.4@osu.edu

Lobal Properties of Integral Domains

Thomas Lucas

This article takes an alternate viewpoint with regard to discovering properties of a given integral domain. Essentially the goal is to determine global properties of an integral domain by "looking" locally without actually localizing. We say that a property **P** is lobal if it is possible to determine that a given domain R satisfies **P** by analyzing containment relations among the ideals and elements that are contained in a single maximal ideal. For example R is a PID if and only if there is a maximal ideal M such that for each ideal $I \subseteq M$, there is an element $x \in I$ such that each ideal $J \subseteq M$ that contains x also contains I. Other lobal properties include being a Krull domain, being a UFD and being h-local.

University of North Carolina Charlotte
9201 University City Blvd.
Charlotte, NC
USA
E-mail address: tglucas@uncc.edu

Commutative orders in semigroups

László Márki

We consider commutative orders, that is, commutative semigroups having a semigroup of fractions in a local sense defined as follows. An element $a \in S$ is *square-cancellable* if for all $x, y \in S^1$ we have that $xa^2 = ya^2$ implies $xa = ya$ and also $a^2x = a^2y$ implies $ax = ay$. It is clear that being square-cancellable is a necessary condition for an element to lie in a subgroup of an oversemigroup. In a commutative semigroup S, the square-cancellable elements constitute a subsemigroup $\mathcal{S}(S)$. Let S be a subsemigroup of a semigroup Q. Then S is a *left order* in Q and Q is a *semigroup of left fractions* of S if every $q \in Q$ can be written as $q = a^\sharp b$ where $a \in \mathcal{S}(S)$, $b \in S$ and $a^\sharp$ is the inverse of a in a subgroup of Q and if, in addition, every square-cancellable element of S lies in a subgroup of Q. *Right orders* and *semigroups of right fractions* are defined dually. If S is both a left order and a right order in Q, then S is an *order* in Q and Q is a *semigroup of fractions* of S. We remark that if a commutative semigroup is a left order in Q, then Q is commutative so that S is an order in Q. A given commutative order S may have more than one semigroup of fractions. The semigroups of fractions of S are pre-ordered by the relation $Q \geq P$ if and only if there exists an onto homomorphism $\phi : Q \to P$ which restricts to the identity on S. Such a ϕ is referred to as an *S-homomorphism*; the classes of the associated equivalence relation are the S-isomorphism classes of orders, giving us a partially ordered set $\mathcal{Q}(S)$. In the best case, $\mathcal{Q}(S)$ contains maximum and minimum elements. In a commutative order S, $\mathcal{S}(S)$ is also an order and has a maximum semigroup of fractions R, which is a Clifford semigroup. We investigate how much of the relation between $\mathcal{S}(S)$ and its semigroups of fractions can be lifted to S and its semigroups of fractions.

This is a joint work with P. N. Ánh, V. Gould, and P. A. Grillet.

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences
Reáltanoda u. 13-15
1053 Budapest
Hungary
E-mail address: marki.laszlo@renyi.mta.hu

Ore-Rees Rings which are Maximal Orders

Hidetoshi Marubayashi

Let R be a Noetherian prime ring with its quotient ring Q, σ be an automorphism of R and δ be a left σ-derivation, that is δ is an additive map from R to R and $\delta(ab) = \sigma(a)\delta(b) + \delta(a)b$ for all $a, b \in R$. The non-commutative polynomial ring $R[t; \sigma, \delta] = \{f(t) = a_n t^n + \cdots + a_0 | a_i \in R\}$ in an indeterminate t with multiplication : $ta = \sigma(a)t + \delta(a)$ for any $a \in R$, is called an *Ore extension* of R.
For a fixed invertible ideal X with $\sigma(X) = X$, we define an *Ore-Rees* ring $S = R[Xt; \sigma, \delta] = R \oplus Xt \oplus \cdots \oplus X^n t^n \oplus \ldots$ which is a subring of $R[t, \sigma, \delta]$.
First we show that if R is a maximal order, then so is S and the converse is not necessarily true. Here a ring R is a *maximal order* if and only if for each non-zero ideal $\mathfrak{a}$ of R $O_l(\mathfrak{a}) = R = O_r(\mathfrak{a})$, where $O_l(\mathfrak{a}) = \{ q \in Q | q\mathfrak{a} \subseteq \mathfrak{a}\}$ and $O_r(\mathfrak{a}) = \{q \in Q | \mathfrak{a}q \subseteq \mathfrak{a} \}$.
Note ; in case of commutative domains, the maximal orders are equivalent to the completely integrally closed domains.
In case $\sigma = 1$, we denote S by $R[Xt; \delta]$ and called it t*he differential Rees ring*. We define the concepts of $(\delta; X)$-stable ideals of R and $(\delta; X)$-maximal orders as follows:
An ideal $\mathfrak{a}$ is called $(\delta; X)$-*stable* if $X\mathfrak{a} = \mathfrak{a}X$ and $X\delta(\mathfrak{a}) \subseteq \mathfrak{a}$ and R is a $(\delta; X)$-*maximal order* if $O_l(\mathfrak{a}) = R = O_r(\mathfrak{a})$ for each $(\delta; X)$-stable ideal $\mathfrak{a}$.
In case $\delta = 0$, similar to the case $\sigma = 1$, we define the concepts of $(\sigma; X)$-invariant ideals of R and $(\sigma; X)$-maximal orders. We denote S by $R[Xt; \sigma]$ which is called a *skew Rees ring*. We have the following characterizations of the differential Rees rings and the skew Rees rings:
(1) $R[Xt; \delta]$ is a maximal order if and only if R is a $(\delta; X)$-maximal order.
(2) $R[Xt; \sigma]$ is a maximal order if and only if R is a $(\sigma; X)$-maximal order.
These results are obtained by describing all fractional divisorial ideals.
In fact, any divisorial $R[Xt; \delta]$-ideal A is of the form $A = w\mathfrak{a}[Xt; \delta]$, where $\mathfrak{a}$ is a divisorial $(\delta; X)$-stable R-ideal and w is a central element of $Q(Q[t; \delta])$, the quotient ring of $Q[t; \delta]$.
Any divisorial $R[Xt; \sigma]$-ideal A is of the form $A = t^n w\mathfrak{a}[Xt; \sigma]$, where $\mathfrak{a}$ is a divisorial $(\sigma; X)$-invariant R-ideal, w is a central element of $Q(Q[t; \sigma])$ and n is an integer.
We provide some examples of $(\delta; X)$-maximal orders and $(\sigma; X)$-maximal orders but not maximal orders, respectively by using special hereditary Noetherian prime rings.
Finally we would like to mention of open questions which are related to arithmetical ideal theory in non-commutative rings.

Tokushima Bunri University
Shido
Sanuki, Kagawa
Japan
E-mail address: marubaya@naruto-u.ac.jp

The construction of all the star operations and all the semistar operations on 1-dimensional Prüfer domains

Ryûki Matsuda

Let $\Sigma(D)$ (resp., $\Sigma'(D)$) be the set of star (resp., semistar) operations on a domain D. E. Houston gave necessary and sufficient conditions for an integrally closed domain D to have $|\Sigma(D)| < \infty$. Moreover, under those conditions, he gave the cardinality $|\Sigma(D)|$ (The Booklet of Abstracts of The Conference "Commutative Rings and their Modules, 2012, Italy). We proved that an integrally closed domain D has $|\Sigma'(D)| < \infty$ if and only if it is a finite dimensional Prüfer domain with finitely many maximal ideals. Also we gave conditions for a pseudo-valuation domain (resp., an almost pseudo-valuation domain) D has $|\Sigma'(D)| < \infty$. In this paper, we construct all the star operations and all the semistar operations on a 1-dimensional Prüfer domain D. We introduce sigma operation, and show that every semistar operation on D is decomposed uniquely as $s \times \sigma$, where s is a star operation and σ is a sigma operation on D.

Ibaraki University, Department of Mathematical Sciences
Mito
Japan
E-mail address: rmazda@adagio.ocn.ne.jp

On the quintasymptotic prime ideals

Reza Naghipour

Let R denote a commutative Noetherian ring, I an ideal of R, and let S be a multiplicatively closed subset of R. L.J. Ratliff showed that the sequence of sets $\mathrm{Ass}_R R/\overline{I} \subseteq \mathrm{Ass}_R R/\overline{I^2} \subseteq \mathrm{Ass}_R R/\overline{I^3} \subseteq \dots$ increases and eventually stabilizes to a set denoted $\overline{A^*}(I)$. S. McAdam gave an interesting description of $\overline{A^*}(I)$ by making use of $R[It, t^{-1}]$, the Rees ring of I. In this paper, we give a second description of $\overline{A^*}(I)$ by making use of the Rees valuation rings of I. We also reprove a result concerning when $\overline{I^n}R_S \cap R = \overline{I^n}$ for all integers $n > 0$.

University of Tabriz and IPM
Tabriz
Iran
E-mail address: naghipour@ipm.ir

Vector Invariants and Hilbert Ideals

Mara D. Neusel

Let $\rho : G \hookrightarrow GL(n, \mathbb{F})$ be a faithful representations of a finite group G over a finite field of characteristic p. This induces an action of the group G on the vector space $V = \mathbb{F}^n$, hence on the dual space and thus on the full symmetric algebra on the dual, denoted by $\mathbb{F}[V]$. The G-invariants $\mathbb{F}[V]^G$ form a subring in $\mathbb{F}[V]$. The Hilbert ideal $\mathcal{H}(G) \subseteq \mathbb{F}[V]$ is the ideal generated by all invariants of positive degree. In this talk we discuss recent results on the Hilbert ideal of vector invariants of the Sylow p-subgroup of $GL(n, \mathbb{F})$. This is joint work with Chris Monico (TTU).

Texas Tech University
Department of Mathematics and Statistics, MS 1042
Lubbock TX 79409
USA
E-mail address: `mara.d.neusel@ttu.edu`

How do you measure primality?

Christopher O'Neill

In commutative monoids, the ω-value measures how far an element is from being prime. This invariant, which is important in understanding the factorization theory of monoids, has been the focus of much recent study. This talk discusses several results for the ω-value in the setting of numerical monoids, including eventual quasilinearity (i.e. periodic linearity). Some related open problems will also be given.

Texas A&M University
Mailstop 3368
College Station, TX
USA
E-mail address: musicman@math.duke.edu

Noetherian semigroup algebras and beyond

Jan Okniński

The main structural and combinatorial results on (non-commutative, in general) Noetherian semigroup algebras are presented. Important motivating classes of such algebras are discussed. These include certain algebras defined in terms of a finite presentation (via generators and defining relations) and certain algebras defined in terms of the structure of the underlying semigroups. Certain recent results on non-noetherian orders in prime Noetherian algebras are also presented.

Institute of Mathematics, Warsaw University
Banacha 2
02-791 Warsaw
Poland
E-mail address: okninski@mimuw.edu.pl

Geometric and topological applications to intersections of valuation rings

Bruce Olberding

The Zariski-Riemann space of a field F is the collection of all valuation rings having quotient field F endowed with the Zariski topology. While this topology has proved very useful since its introduction by Zariski in the first half of the twentieth century, it is not in general strong enough to detect whether a collection of valuation overrings is a Prüfer domain, an irredundant collection, or satisfies various approximation theorems. For these applications, one needs to take into account a sheaf structure that makes the Zariski-Riemann space a locally ringed space. We discuss this point of view and recent results in this direction.

New Mexico State University
Department of Mathematical Sciences
Las Cruces, NM
USA
E-mail address: olberdin@nmsu.edu

Fixed divisor of polynomials over matrices

Giulio Peruginelli

Let R be a commutative ring. The fixed divisor of a polynomial $g(X)$ in $R[X]$ is defined as the ideal of R generated by the values of $g(X)$ over R. We generalize this classical notion by evaluating $g(X)$ over matrices over R. Let $M_n(R)$ be the R-algebra of $n \times n$ matrices over R. Given a prime ideal P of R and $g \in R[X]$ we look for the highest power of P such that $g(M)$ is in $M_n(P^k)$, for each $M \in M_n(R)$ (that is, all the entries of these polynomial matrices are in P^k). In order to determine the fixed divisor of $g(X)$ over matrices, we show that it is sufficient to consider companion matrices. Moreover, if R is a Dedekind domain with finite residue fields, we show that we can consider companion matrices of primary polynomials. Applications of these notion are related to integer-valued polynomials over matrices and polynomially dense subsets of matrices.

Department of Mathematics, University of Padova
Via Trieste 63
Padova
Italy
E-mail address: gperugin@math.unipd.it

Semigroup subsets defined by factorization-related properties

Maciej Radziejewski

We study some combinatorial properties of subsets of semigroups with divisor theory defined by factorial properties, motivated by the investigation of oscillations of the counting functions of such subsets. Compared to the study of the main term of the counting function, the study of oscillations requires, in general, more information on the structure of the set in question. In addition to the results related to the class of subsets called Ω-sets by the author (cf. [1]) we will attempt to tackle some subsets outside of this class, defined by more subtle, type-dependent properties.

References

[1] M. Radziejewski, *Structural results for semigroup subsets defined by factorization properties dependent on Ω functions*, Semigroup Forum (2014), DOI: 10.1007/s00233-014-9593-0

Adam Mickiewicz University
ul. Umultowska 87, PL-61-614
Poznań
Poland
E-mail address: maciejr@amu.edu.pl

Null ideal of a matrix

Roswitha Rissner

Given a square matrix A with entries in a commutative ring R, the ideal of $R[X]$ consisting of polynomials f with $f(A) = 0$ is called the null ideal of A in $R[X]$. If R is a domain, then the null ideal of every square matrix over R is principal if and only if R is integrally closed. Very little is known about null ideals of matrices over general commutative rings. Better understanding of the null ideals of matrices over $R = \mathbb{Z}/p^n\mathbb{Z}$ (and finite rings in general) would have applications in the theory of integer-valued polynomials and in the theory of polynomial mappings on non-commutative rings. We will present preliminary results and open questions in this area.

References

[1] W.C. Brown, *Null Ideals and spanning ranks of matrices*, Comm. Algebra **26** (1998), no. 8, 2401–2417.

[2] W.C. Brown, *Null Ideals of matrices*, Comm. Algebra **33** (2005), no. 12, 4491–4504.

[3] S. Frisch, *Integrally closed domains, minimal polynomials, and null ideals of matrices*, Comm. Algebra **32** (2004), no. 5, 2015–2017.

Graz University of Technology - Math A
Steyrergasse 30
Graz
Austria
E-mail address: roswitha.rissner@tugraz.at

Products of elementary and idempotent matrices over integral domains

Luigi Salce

A ring R such that invertible matrices over R are products of elementary matrices is called (after Cohn) generalized Euclidean. Characterizations of generalized Euclidean commutative domains, extending those proved by Ruitenburg for Bézout domains, are illustrated. These results connect generalized Euclidean domains with those domains R such that singular matrices over R are products of idempotent matrices. This latter property is investigated, focusing on 2x2 matrices, which is not restrictive in the context of Bézout domains. The relationship with the existence of a weak Euclidean algorithm, a notion introduced by O'Meara for Dedekind domains, is also described (joint work with Paolo Zanardo).

University of Padova
Via Trieste 63
35121 Padova
Italy
E-mail address: salce@math.unipd.it

Sets of lengths: retrospects and prospects

Wolfgang A. Schmid

For an element a of a commutative and cancelative monoid H, we say that n is a length of a if there exist irreducible elements $u_1, \dots, u_n$ such that $a = u_1 \dots u_n$. The set of lengths $\mathsf{L}(a)$ of a is the set of all n such that n is a length of a.
An attempt will be made to sketch the development of research on sets of lengths and to discuss some potential future directions of research. The main focus will be on the case that H is a Krull monoid with finite class group and on recent developments.

Université Paris 13, Sorbonne Paris Cité, LAGA, CNRS, UMR 7539, Université Paris 8
99, avenue Jean-Baptiste Clément
93430 Villetaneuse
France
E-mail address: schmid@math.univ-paris13.fr

Skinny and finiteness properties of extension functors

Monireh Sedghi

Let I denote an ideal in a regular local (Noetherian) ring and let N be a finitely generated R-module with support in $V(I)$. The aim of this paper is to show that the R-modules $\mathrm{Ext}^j_R(N, H^i_I(R))$ are skinny, for all $i, j \geq 0$, in the following cases: (i) $\dim R \leq 4$; (ii) $\dim R/I \leq 3$ and R containing a field. In addition, we show that if $\dim R = 5$ and R containing a field, then the associated primes of $\mathrm{Ext}^j_R(N, H^i_I(R))$ are finite. These generalize the main results of Marley and Vassilev.

Azarbaijan Shahid Madani University
Tabriz
Iran
E-mail address: m_sedghi@tabrizu.ac.ir

On an extension of the classical zero divisor graph

Fouad Taraza

This is a joint work with D. Bennis and J. Mikram.
In this talk we present a new approach of studying the relation between zero divisors. In our case two zero divisors of a commutative ring R are adjacent whenever there exist two non negative integers n and m such that $x^n y^m = 0$ with $x^n \neq 0$ and $y^m \neq 0$. This yield an extension of the classical zero divisor graph $\Gamma(R)$ of R which will be denoted by $\overline{\Gamma}(R)$. At first, we distinguish when $\overline{\Gamma}(R)$ and $\Gamma(R)$ coincide. We also show that when $\overline{\Gamma}(R)$ and $\Gamma(R)$ are different, then $\overline{\Gamma}(R)$ contains necessarily a cycle and if, moreover, $\overline{\Gamma}(R)$ is complemented then the total ring of R is 0-dimensional. Diameter and girth of $\overline{\Gamma}(R)$ are also studied.

References

[1] D.F. Anderson and P.S. Livingston, *The zero-divisor graph of a commutative ring*, J. Algebra **69** (1990), 434–447.

[2] D.F. Anderson, R. Levy and J. Shapiro, *Zero-divisor graphs, von Neumann regular rings, and Boolean algebras*, J. Pure Appl. Algebra **180** (2003), 221–241.

[3] D.F. Anderson and A. Badawi, *On the zero-divisor graph of a ring*, J. Algebra **36** (2008), 3073–3092.

[4] R. Levy and J. Shapiro, *The zero-divisor graph of von Neumann regular rings*, Comm. Algebra **30** (2002), 745–750.

[5] D.F. Anderson and A. Badawi, *Divisibility conditions in commutative rings with zero-divisors*, Comm. Algebra **30** (2002), 4031–4047.

Faculty of Sciences, Mohammed V-Agdal University
Av. Ibn Batouta, B.P 1014
Rabat
Morocco
E-mail address: ftaraza@hotmail.com

When Bhargava rings are PvMD

Francesca Tartarone

Let D be a domain with quotient field K and $x \in D$. The Bhargava ring $\mathbb{B}_x(D) = \{f(X) \in K[X]; f(xX + a) \in D[X], \forall\, a \in D\} = \bigcap_{a\in D} D[\frac{X-a}{x}]$ is a polynomial ring in between $D[X]$ and $K[X]$ such that $\bigcup_{x\in D} \mathbb{B}_x(D) = \mathrm{Int}(D)$, the integer-valued polynomial ring over D. Recent literature on Bhargava rings concerned in describing their prime spectrum, which is still in general an open problem (see for instance [1], [3]). It is easy to check that Bhargava rings are never Prüfer domains unless they coincide with $K[X]$ (as instead it may happen for $\mathrm{Int}(D)$). A complete characterization of domains D such that $\mathrm{Int}(D)$ is a Prüfer v-multiplication domain (PvMD) is given in [2]. In a joint work with Mi Hee Park we investigate conditions on D and on $x \in D$ in order to have that $\mathbb{B}_x(D)$ is a PvMD.

References

[1] I. Alrasasi, L. Izelgue, *On the prime ideal structure of Bhargava rings*, Comm. Algebra **38** (2010), no. 4, 1385–1400.

[2] P.-J. Cahen, K.A. Loper, F. Tartarone, *Integer-valued polynomials and Prüfer v-multiplication domains*, J. Algebra **226** (2000), no. 2, 765–787.

[3] J. Yeramian, *Anneaux de Bhargava*, Comm. Algebra **32** (2004), no. 8, 3043–3069.

Dipartimento di Matematica e Fisica, Università degli Studi Roma Tre
Largo San Leonardo Murialdo 1
Roma
Italy
E-mail address: tfrance@mat.uniroma3.it

Sets of positive integers closed under product and the number of decimal digits

Denise Torrão

A digital semigroup D is a subsemigroup of $(\mathbb{N} \setminus \{0\}, \cdot)$ such that if $d \in D$ then $\{x \in \mathbb{N} \setminus \{0\} \mid l(x) = l(d)\} \subseteq D$ with $l(n)$ the number of digits of n written in decimal expansion. In this note, we compute the smallest digital semigroup containing a set of positive integers. For this, we establish a connection between the digital semigroups and a class of numerical semigroups called LD-semigroups.

References

[1] M. Bras-Amorós and Klara Stokes, *The semigroup of combinatorial configurations*, Semigroup Forum **84** (2012), 91–96.

[2] M. Bras-Amorós, P.A. García-Sánchez, *Nonhomogeneous patterns on numerical semigroups*, Int. J. Alg. Computation **23** (2013), 1469–1483.

[3] Klara Stokes and M. Bras-Amóros, *Linear nonhomogeneous, symmetric patterns and prime power generators in numerical semigroups associated to combinatorial configurations*, Semigroup Forum **88** (2014), 11–20.

[4] J.L. Ramírez Alfonsín, *The diophantine Frobenius problem*, Oxford Univ. Press, New York, 2005.

[5] J.C. Rosales, *Families of numerical semigroups closed under finite intersections and for the Frobenius number*, Houston J. Math. **34** (2008), 339–348.

Universidade de Évora
Colégio Luís Verney
Évora
Portugal
E-mail address: denisetorrao@hotmail.com

Seminormed structures and factorization systems

Salvatore Tringali

We let a seminormed semigroup be an ordered pair $\mathcal{S}$ consisting of a semigroup $\mathbb{S} = (S, \cdot)$, either commutative or not, and a function $\|\cdot\| : S \to [0, \infty]$ such that

$$\|xy\| \le \|x\| + \|y\| \text{ for all } x, y \in S.$$

Next, we define a factorization system as a 4-tuple composed of a semigroup $\mathbb{S} = (S, \cdot)$, a set of "primitive factors" $\mathfrak{P} \subseteq S$, a set of "negligible factors" $\Theta \subseteq S$ disjoint from the subsemigroup of $\mathbb{S}$ generated by $\mathfrak{P}$, and an equivalence $\mathcal{E}$ on the Kleene star of S.
Finally, we take an η-system to be a 4-tuple $\Phi = (\mathcal{S}, \mathfrak{P}, \Theta, \mathcal{E})$ for which $\mathcal{S} = (\mathbb{S}, \|\cdot\|)$ is a seminormed semigroup and $(\mathbb{S}, \mathfrak{P}, \Theta, \mathcal{E})$ is a factorization system.
In this talk, we will discuss basic algebraic, topological, and combinatorial aspects of these and related objects (such as seminormed categories), of which we give several examples.
In particular, we will show how to generalize sets of lengths, distortion elements, Davenport constants, and some other fundamental notions from factorization theory, geometric group theory, and zero-sum theory to the context of η-systems, and we will shortly address elementary properties of these generalizations.

Centre de Mathématiques Laurent Schwartz, École Polytechnique
91128 Palaiseau Cedex
France
E-mail address: salvo.tringali@gmail.com

C-Gorenstein $\mathcal{X}$-projective, $\mathcal{X}$-injective and $\mathcal{X}$-flat Modules

A. Umamaheswaran† and C. Selvaraj†

In this paper, we introduce and study some properties of C-Gorenstein $\mathcal{X}$-projective, $\mathcal{X}$-injective and $\mathcal{X}$-flat R-modules and discuss some connections between C-Gorenstein $\mathcal{X}$-projective module and C-Gorenstein $\mathcal{X}$-flat Modules. We also investigate some connections between C-Gorenstein $\mathcal{X}$-projective module and C-Gorenstein $\mathcal{X}$-flat Modules of change of rings.

† Department of Mathematics
Periyar University
Salem - 636 011, Tamil Nadu
India
E-mail address: ruthreswaran@gmail.com

Computing some families of Cohen-Macaulay, Gorenstein and Buchsbaum semigroups

ALBERTO VIGNERON-TENORIO

Given $F \subset \mathbb{R}^r_{\geq}$ a non-empty convex body, we call convex body semigroup to the semigroup $\mathcal{F} = \bigcup_{i=0}^{\infty} F_i \cap \mathbb{N}^r$ where $F_i = i \cdot F$ with $i \in \mathbb{N}$. They generalize to arbitrary dimension the concept of proportionally modular numerical semigroup of [4]. An interesting computational property of the convex body semigroups is one can check whether an element belongs to $\mathcal{F}$ easily.

In general, these semigroups are not finitely generated. We study the necessary and sufficient conditions for a convex body semigroup associated to convex polygon or a circle in $\mathbb{R}^2$ to be finitely generated. These conditions are related to the slopes of the extremal rays of the minimal cone which includes to $\mathcal{F}$ (see [1]). We give effective methods to obtain their minimal system of generators.

Furthermore, Cohen-Macaulay, Gorenstein and Buchsbaum circle and convex polygonal semigroups can be characterized using basic geometry. Thus convex body semigroups and the theory developed to check their properties allow us to generate instances of Cohen-Macaulay, Gorenstein and Buchsbaum semigroup rings ([2] and [3]).

REFERENCES

[1] J.I. García-García, M.A. Moreno-Frías, A. Sánchez-R.-Navarro, A. Vigneron-Tenorio, *Affine convex body semigroups*, Semigroup Forum **87** (2) (2013), 331–350.

[2] J.I. García-García, A. Sánchez-R.-Navarro, A. Vigneron-Tenorio, *Affine convex body semigroups and Buchsbaum rings*, arXiv:1402.2597 [math.AC].

[3] J.I. García-García, A. Vigneron-Tenorio, *Computing families of Cohen-Macaulay and Gorenstein rings*, to appear in Semigroup Forum, arXiv:1304.5089 [math.AC].

[4] J.C. Rosales, P.A. García-Sánchez, J.I. García-García, J.M. Urbano-Blanco, *Proportionally modular Diophantine inequalities*, J. Number Theory **103** (2003), 281–294.

UNIVERSITY OF CÁDIZ
AVDA. DE LA UNIVERSIDAD, NO. 4
11406 JEREZ DE LA FRONTERA (CÁDIZ)
SPAIN
E-mail address: `alberto.vigneron@uca.es`

Direct-sum decompositions of modules over one-dimensional local rings

Roger Wiegand

Let R be a one-dimensional Noetherian local domain, and let Σ be the semigroup of isomorphism classes of finitely generated R-modules (with semigroup structure induced by the direct sum). I will discuss the structure of Σ and also the structure of the smaller, but less tractable, semigroup of isomorphism classes of finitely generated torsion-free R-modules.

References

[1] N. Baeth, R. Wiegand, *Factorization theory and decompositions of modules*, Amer. Math. Monthly **120** (2013), 3–34.

Department of Mathematics, University of Nebraska
203 Avery Hall
Lincoln, NE 68588-0130
USA
E-mail address: rwiegand@math.unl.edu

Prime ideals in quotients of mixed polynomial-power series rings

Sylvia Wiegand

We consider the partially ordered set of prime ideals in homomorphic images of three-dimensional mixed polynomial-power series rings; that is, in $R[y][\![x]\!]/Q$ or $R[\![x]\!][y]/Q$, where R is a one-dimensional Noetherian domain, x and y are indeterminates, Q is a height-one prime ideal of the appropriate ring, and $x \notin Q$. We characterize the prime spectra that occur if the coefficient ring R is a countable unique factorization domain with infinitely many maximal ideals. The characterization involves a finite subset of dimension one that determines the entire partially ordered set. We show that every one-dimensional finite partially set occurs for an appropriate height-one prime ideal Q if $R = \mathbb{Z}$.

References

[1] E. Celikbas, C. Eubanks-Turner, S. Wiegand, *Prime ideals in quotients of mixed polynomial-power series rings*, in preparation.

Department of Mathematics, University of Nebraska
203 Avery Hall
Lincoln, NE 68588-0130
USA
E-mail address: swiegand@math.unl.edu

Fully inert subgroups of Abelian groups

Paolo Zanardo

A subgroup H of an Abelian group G is called *fully inert* if, given any endomorphism ϕ of G, the group $(\phi H + H)/H$ is finite. Dikranjan, Giordano Bruno, Salce and Virili were led to introduce this natural concept when they defined the intrinsic algebraic entropy.
Two subgroups A, B of G are said to be *commensurable* if both $(A+B)/A$ and $(A+B)/B$ are finite. Commensurable subgroups may be considered “close”. Let the subgroup H of G be commensurable with a fully invariant subgroup K; then one shows that H is automatically fully inert. Since the fully invariant subgroups are well understood for certain classes of Abelian groups, it is natural to ask whether the reverse implication holds.
If G is either free or a direct sum of cyclic p-groups, we prove that every fully inert subgroup of G is commensurable with a fully invariant subgroup. We remark that the proofs of the two cases use deeply different arguments and techniques. A similar result holds for fully inert submodules of complete modules over J_p, the completion of $\mathbb{Z}$ in the p-adic topology. Using realization theorems of algebras as endomorphism rings of Abelian groups, we show that, in general, fully inert subgroups are not commensurable with fully invariants.
This research was made in collaboration with D. Dikranjan, B. Goldsmith and L. Salce.

Università di Padova
Via Trieste 63
35121 Padova
Italy
E-mail address: pzanardo@math.unipd.it

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