Joint Reconstruction in Multi-Spectral Electron Tomography

Richard Huber[†], Kristian Bredies[†], Georg Haberfehlner[*], Martin Holler[†], Gerald Kothleitner[*]

[†]Institute for Mathematics and Scientific Computing, University of Graz, Austria

[*]Institute of Electron Microscopy and Nanoanalysis (FELMI) and Graz Centre for Electron Microscopy (ZFE), Graz University of Technology, Austria

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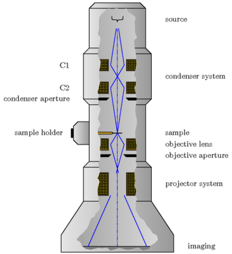
In a nutshell



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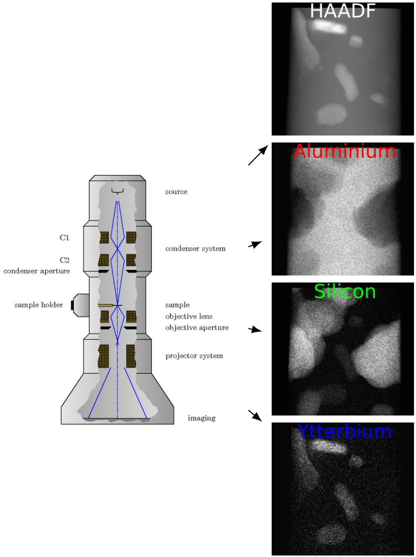

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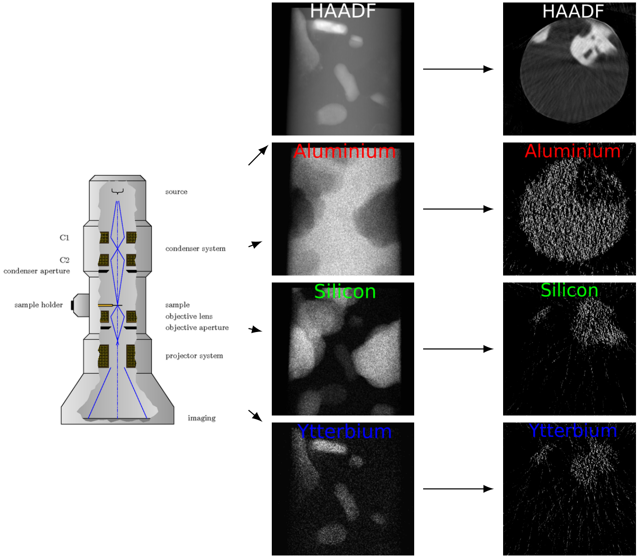

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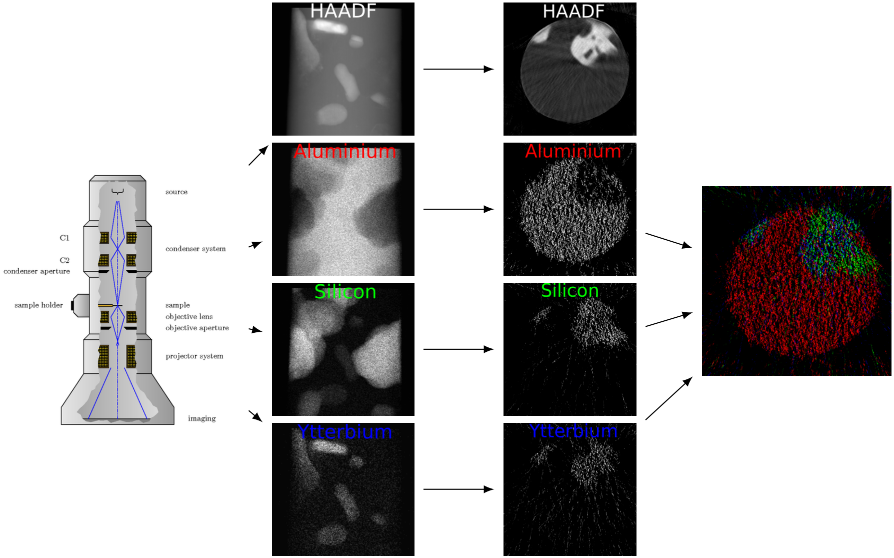

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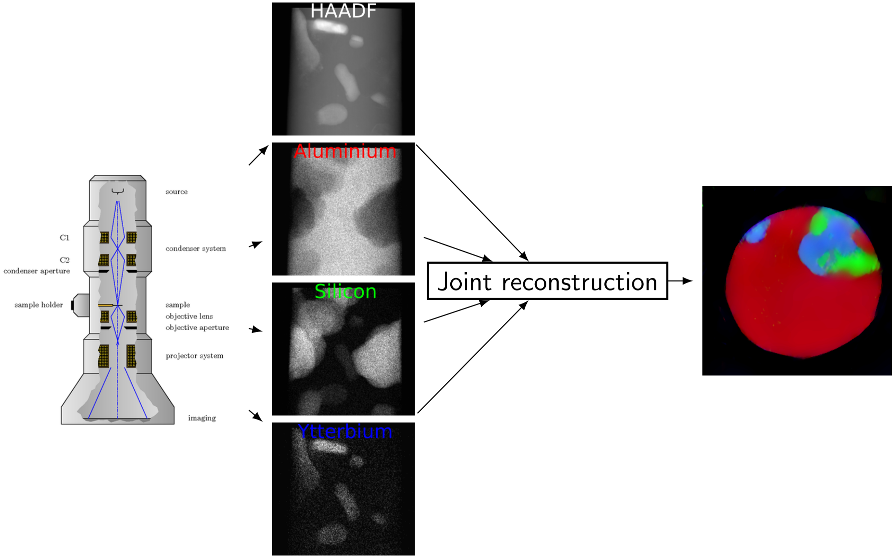

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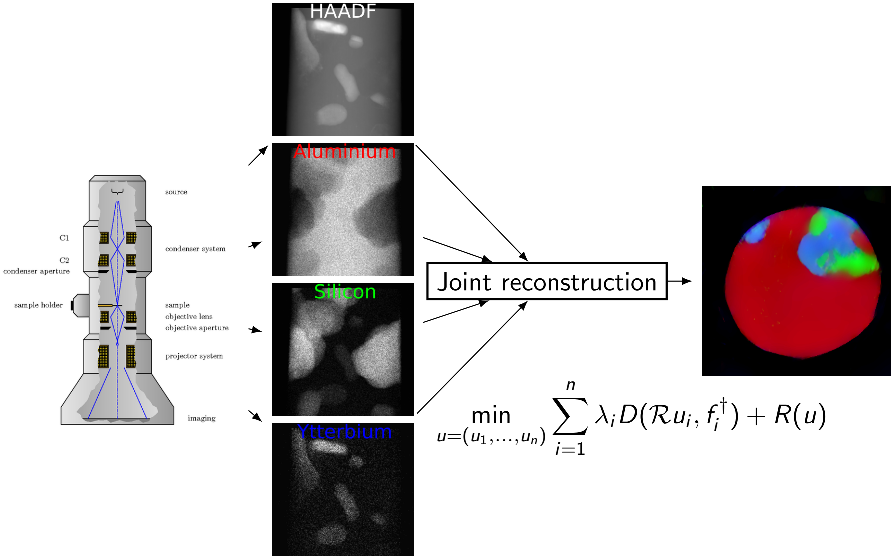

$$\min_{u=(u_1,\dots,u_n)} \sum_{i=1}^{n} \lambda_i D(\mathcal{R}u_i, f_i^{\dagger}) + R(u)$$

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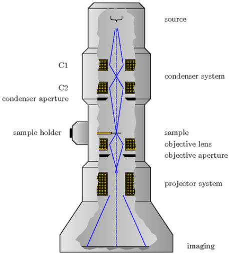

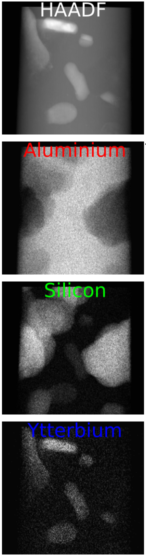

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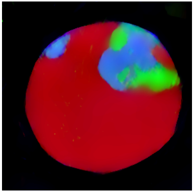

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- u multi spectral image

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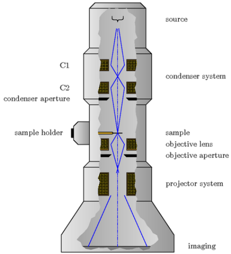

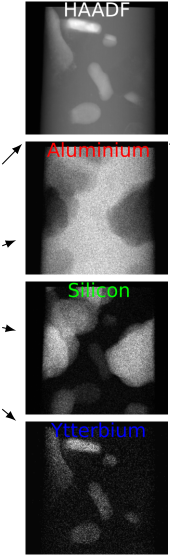

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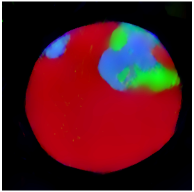

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- $D(\mathcal{R}u_i, f_i^\dagger)$: Data fidelity, $f_i^\dagger$ sinogram data of ith channel

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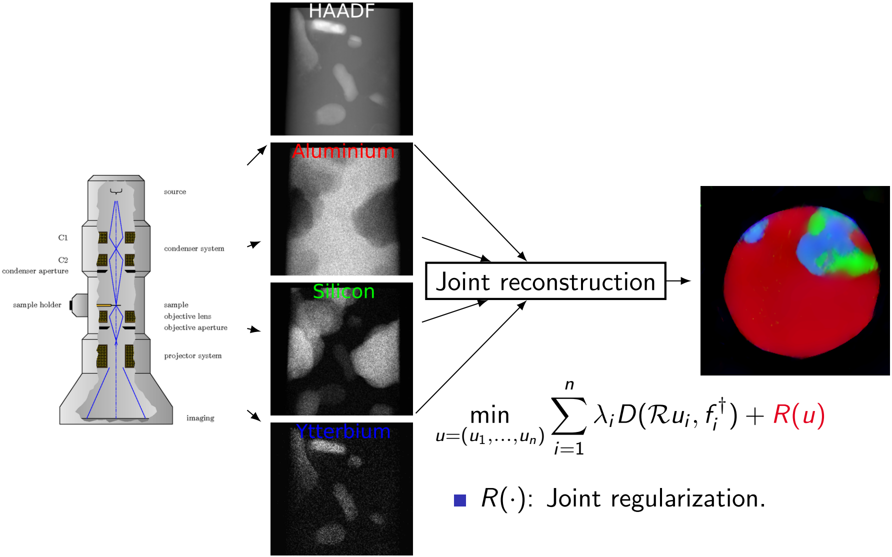

$$\min_{u=(u_1,\ldots,u_n)} \sum_{i=1}^{n} \lambda_i D(\mathcal{R}u_i, f_i^\dagger) + R(u)$$

- $R(\cdot)$: Joint regularization.

Outline

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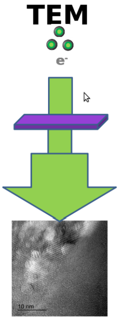
Electron tomography

- Scanning Transmission Electron Microscopy
  - Creates 2D projections of thin samples.
  - Depicts density distribution.
  - Atomic resolution, magnification to resolution of 0.1 nm.
  - Analyse small samples in various fields.



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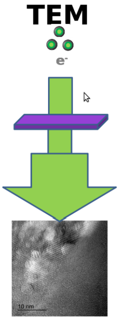

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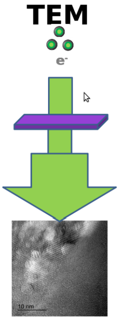

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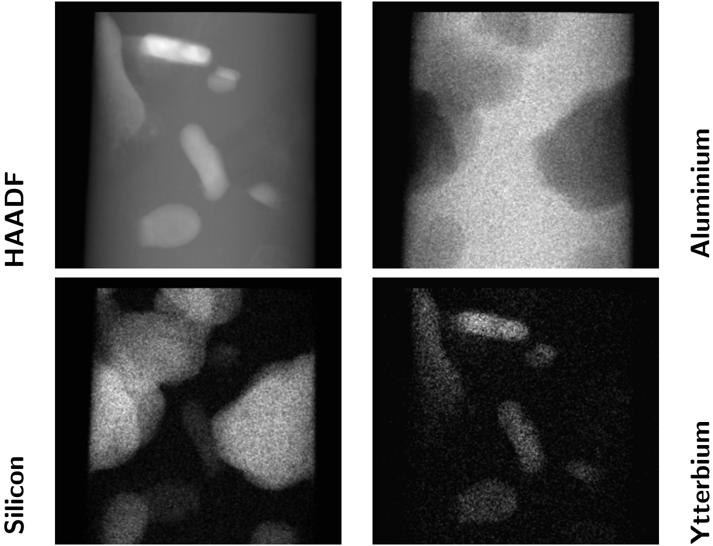
- Tomography reconstruction
  - 3D distributions allows more insight.
  - Projections from various angles.
  - Tomography reconstruction required.

Sinogram of EDS-Data



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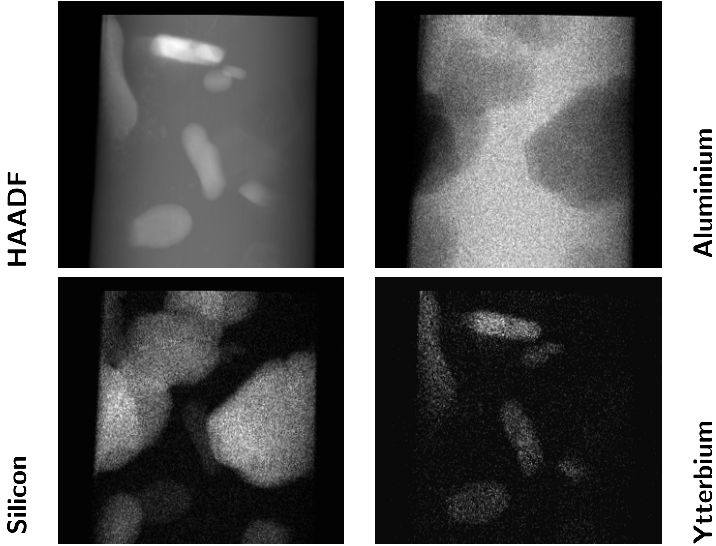

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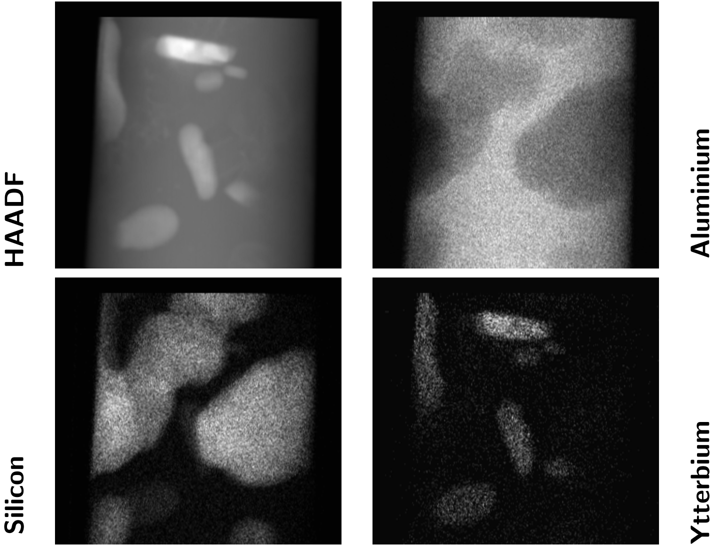

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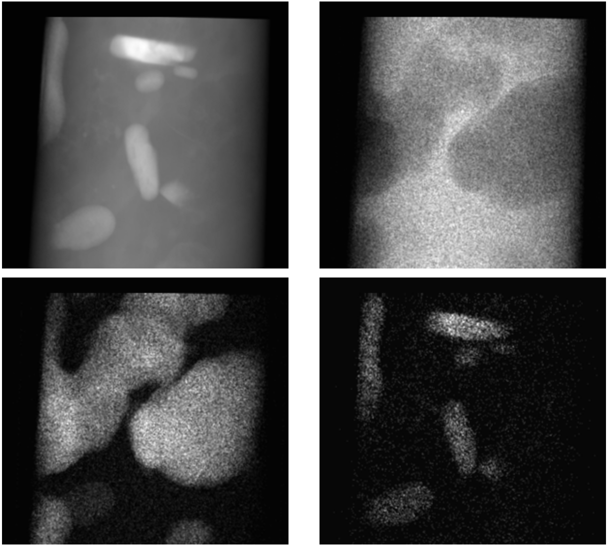


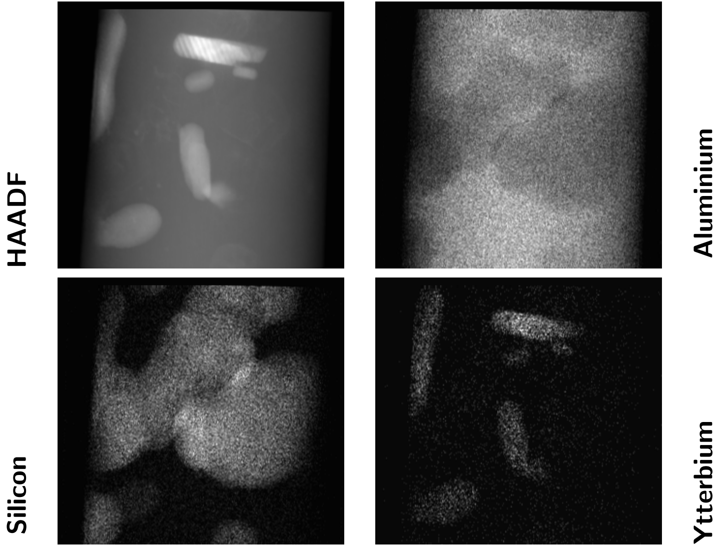
HAADF

Aluminium

Silicon

Ytterbium

Sinogram of EDS-Data



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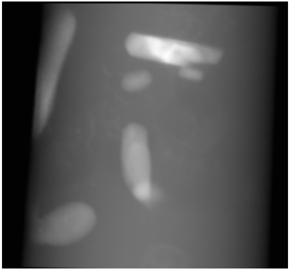

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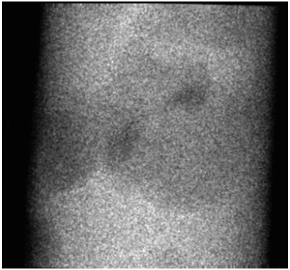
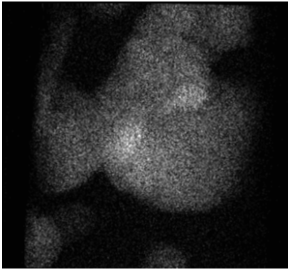
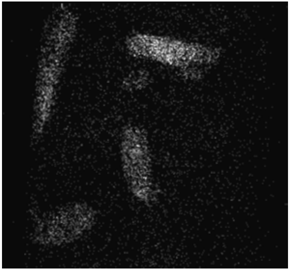


Aluminium



Silicon



Ytterbium

Sinogram of EDS-Data

HAADF

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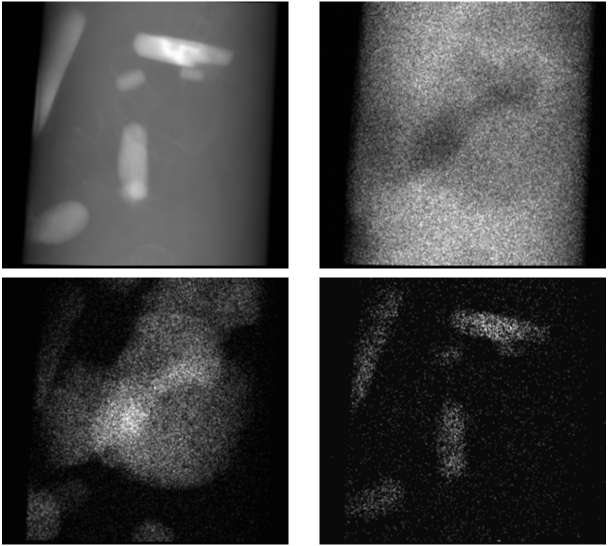

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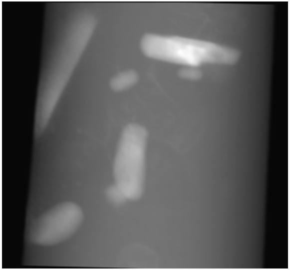

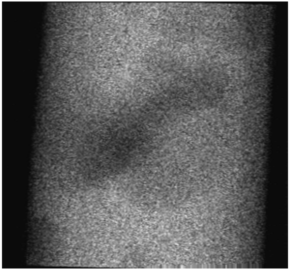

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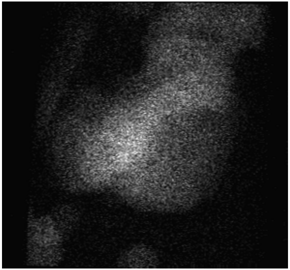

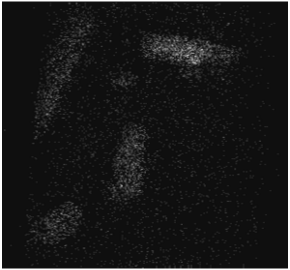

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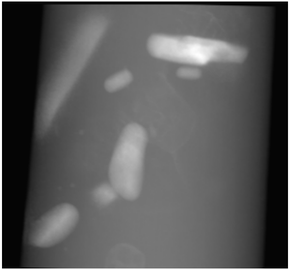

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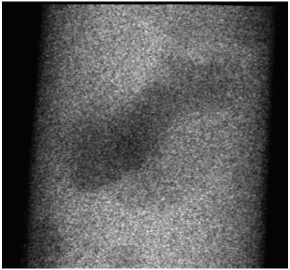
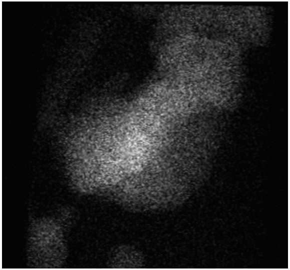


Silicon



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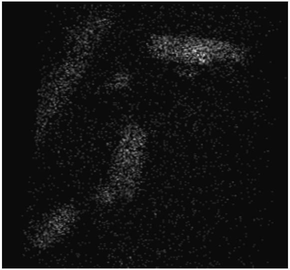

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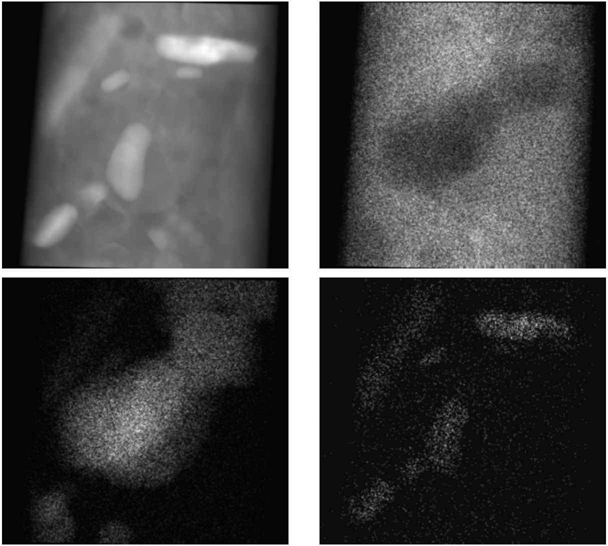

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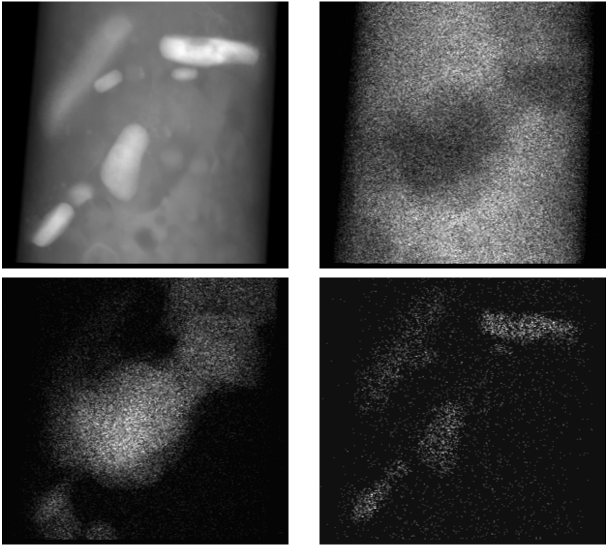

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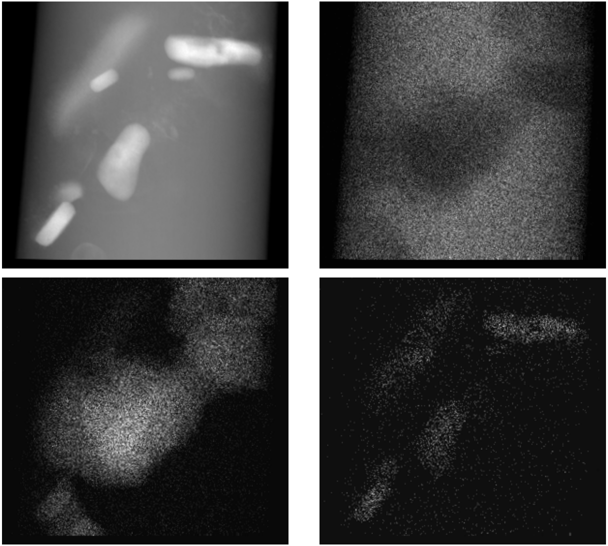

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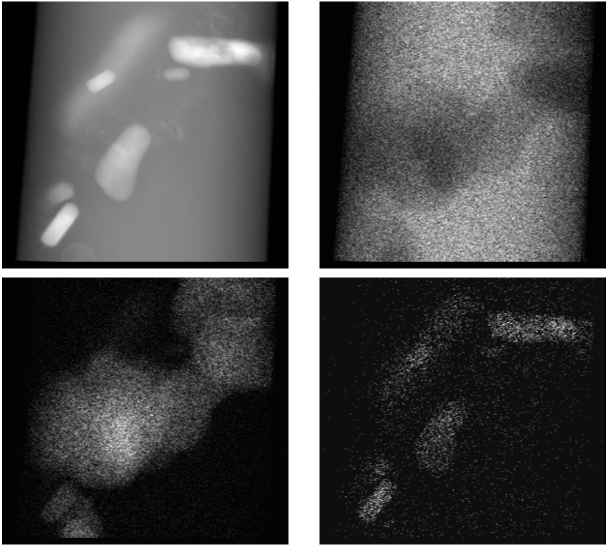

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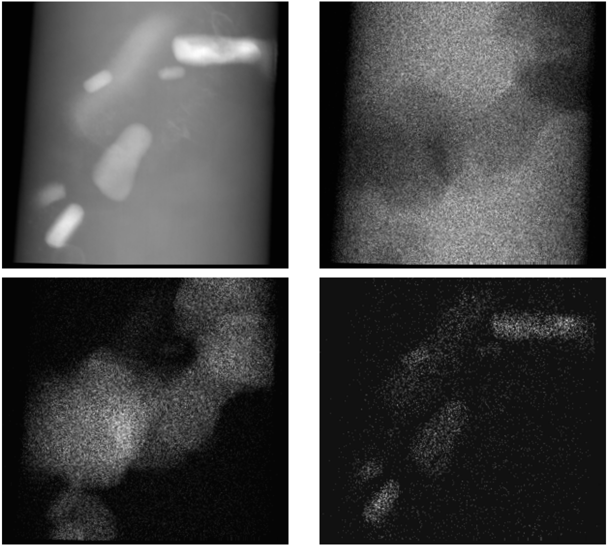

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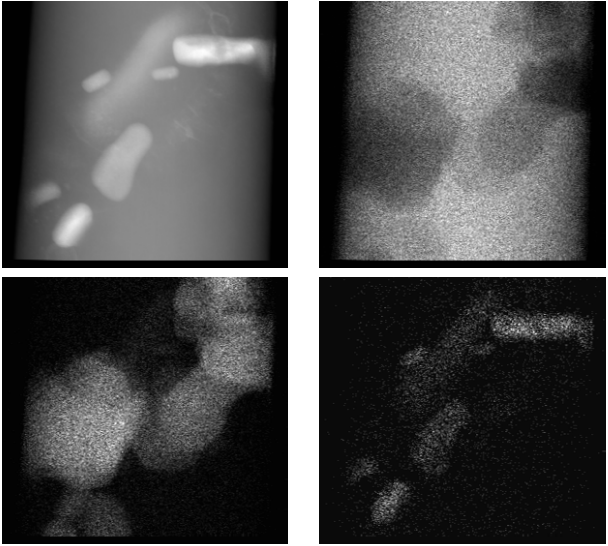

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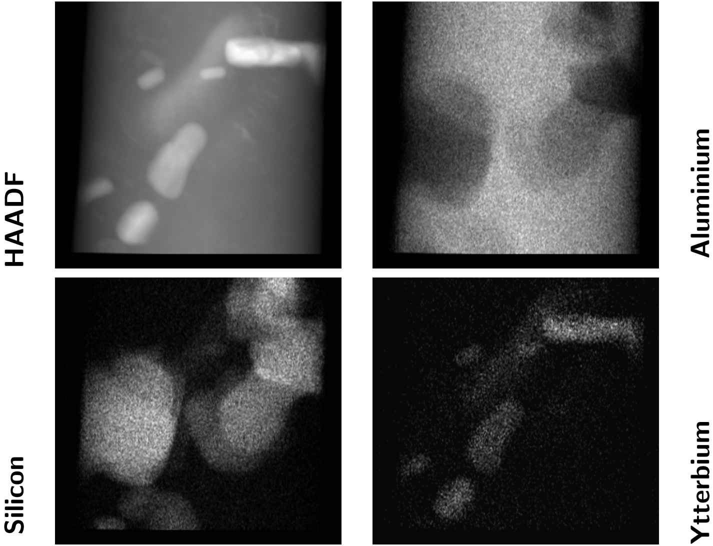

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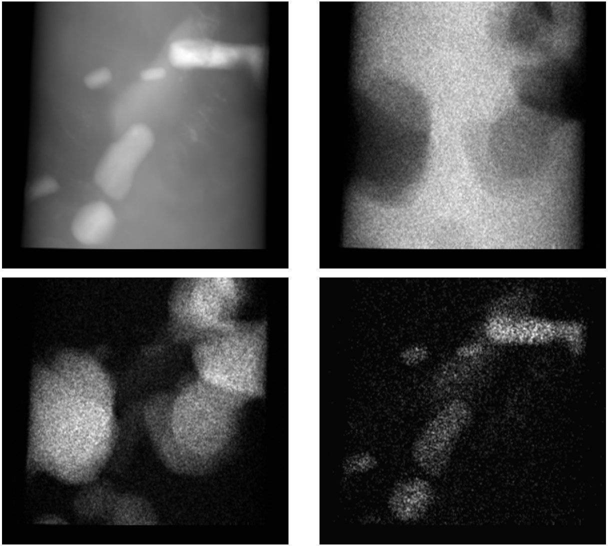

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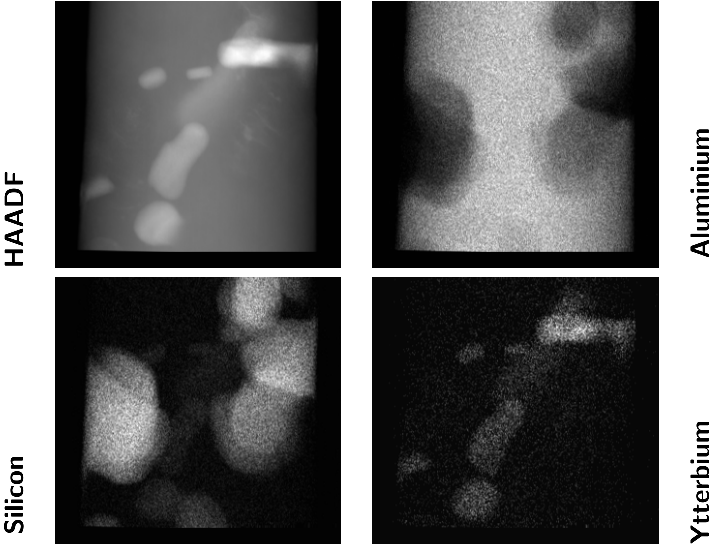
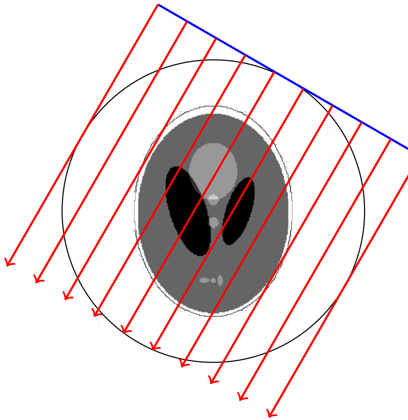


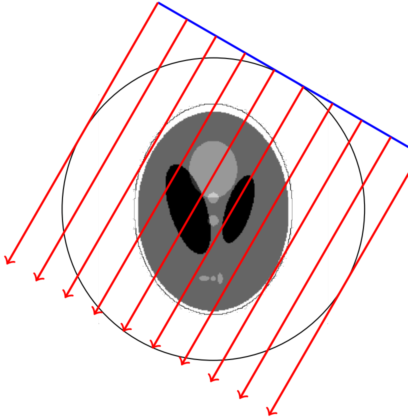
Forward model

- All possible line integrals.



Forward model

- All possible line integrals.
- Radon transform
  $\mathcal{R}\colon L^1(\Omega_R) \to L^1(\Omega_S)$.



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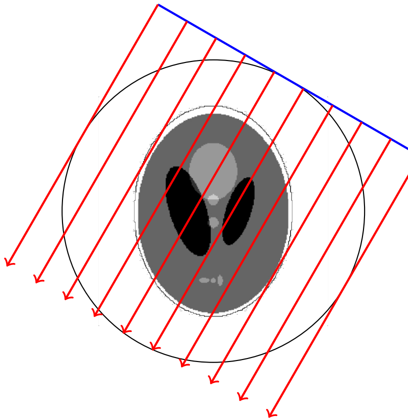
$$\mathcal{R}u_1 = f_1^\dagger, \quad \dots \quad \mathcal{R}u_n = f_n^\dagger.$$



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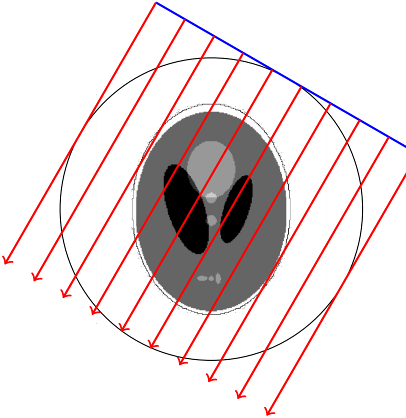
- $\mathcal{R}u = f$ is ill-posed problem.



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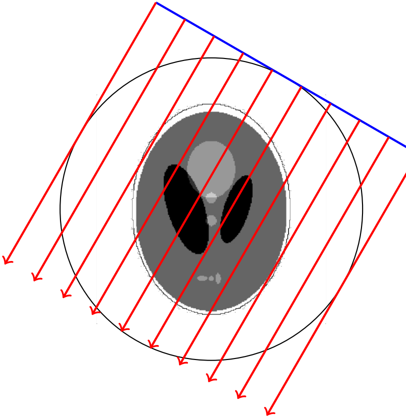

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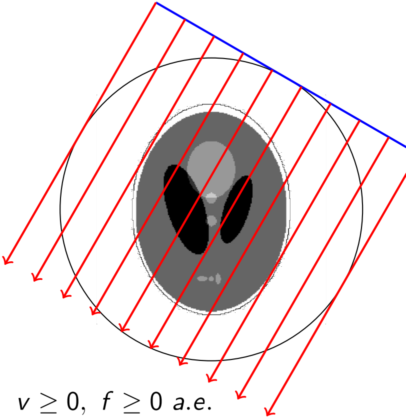
- Kullback-Leibler divergence
  $D_{KL}\colon L^1(\Omega_S) \times L^1(\Omega_S) \to [0, \infty]$:



$$D_{KL}(v, f) = \begin{cases} \int_{\Omega_S} v - f - f \ln\left(\frac{v}{f}\right) \mathrm{d}\,x & v \geq 0,\ f \geq 0\ a.e. \\ \infty & \text{otherwise} \end{cases}$$

Outline

Regularization

$$\min_{u=(u_1,\ldots,u_n)} R(u) + \sum_{i=1}^{n} \lambda_i D_{KL}(\mathcal{R}u_i, f_i^\dagger)$$

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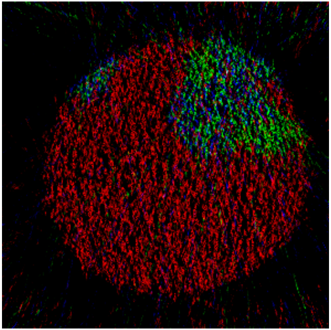

Sirt reconstruction of a slice

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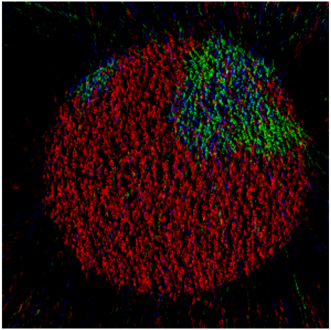

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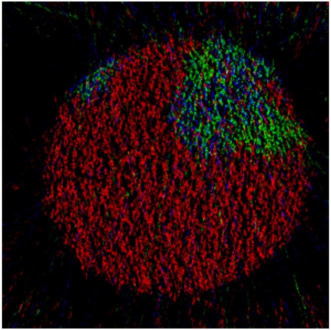

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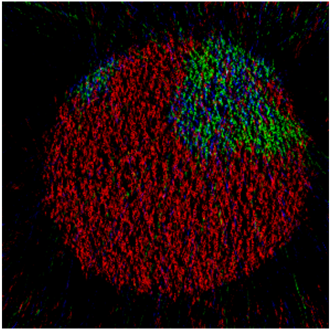

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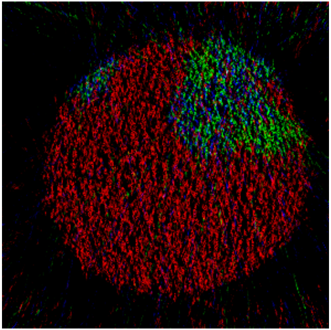
- Complementing information:
  - Common features,
  - Diffusion and mixing.



Sirt reconstruction of a slice

TGV for scalar valued images

For $u \in L^1(\Omega, \mathbb{R})$,

$$\mathrm{TV}(u) = \sup \left\{ \int_\Omega u \operatorname{div} \phi \;\middle|\; \phi \in \mathcal{C}_c^\infty(\Omega,\ \mathbb{R}^d), \right.$$

$$\left. \| |\phi|_2 \|_\infty \leq 1 \right\}.$$

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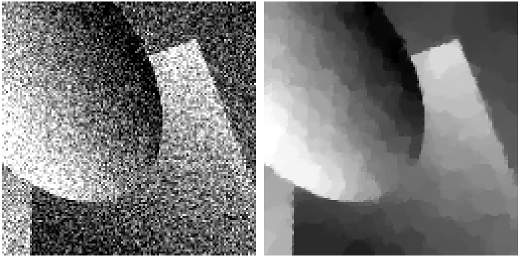

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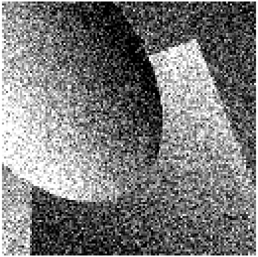

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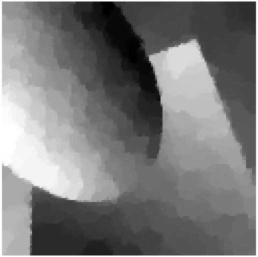

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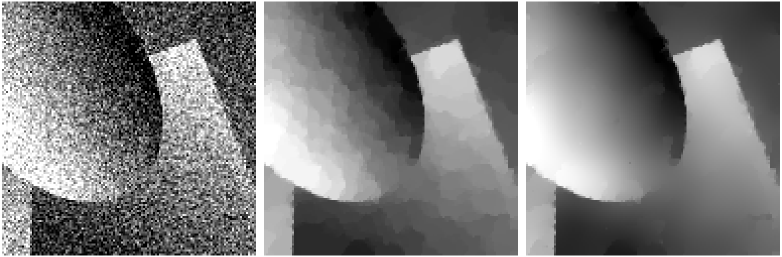

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M. Holler, R. Huber, F. Knoll: "Coupled regularization with multiple data discrepancies", Inverse Problems

Outline

Implementation

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- Time consumption: in average 5 seconds per slice. (300 $\times$ 300 with 4 channels)

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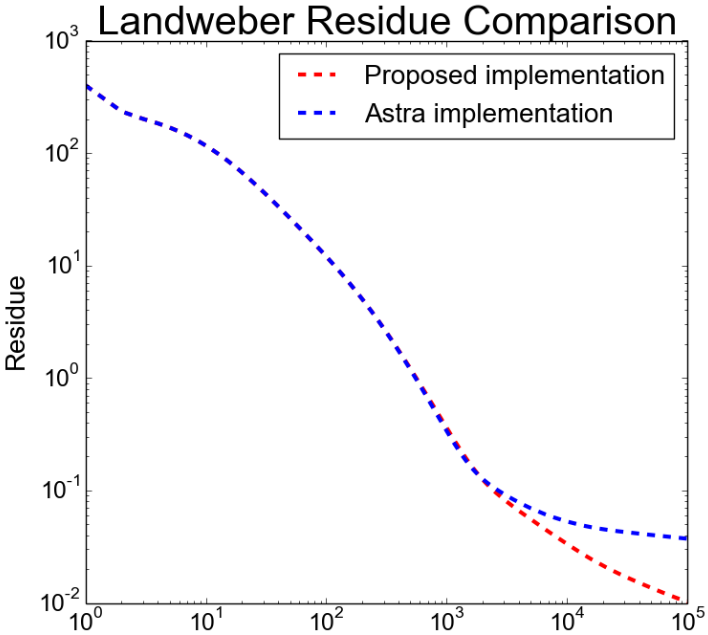
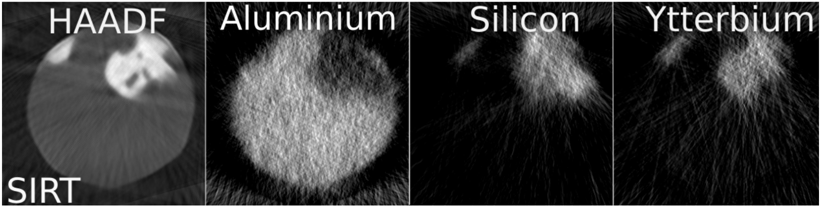

Landweber Residue Comparison
Proposed implementation
Astra implementation
Residue
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^0
10^1
10^2
10^3
10^4
10^5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SIRT
Aluminium
Silicon
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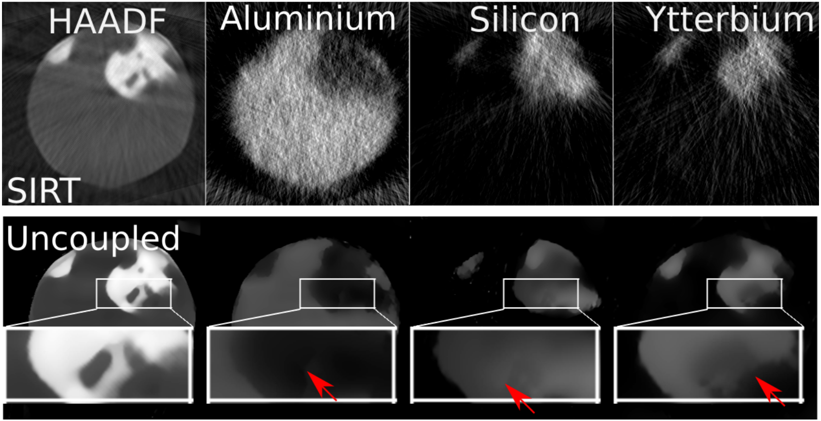
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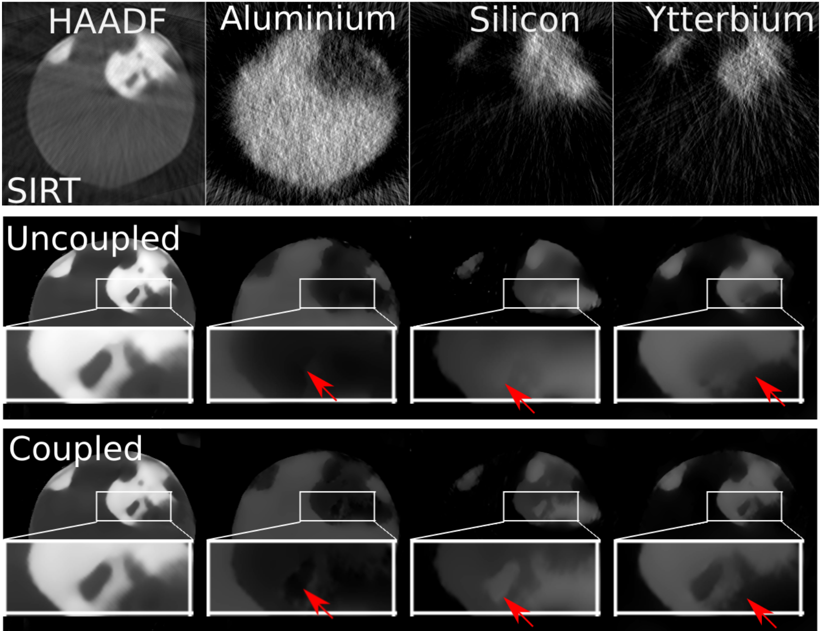
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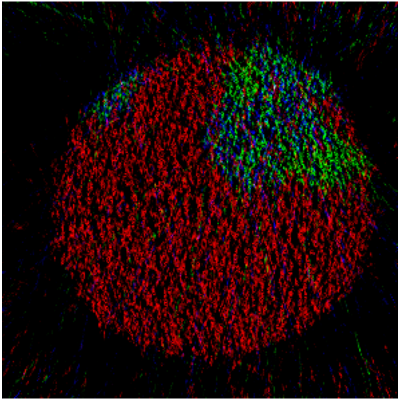
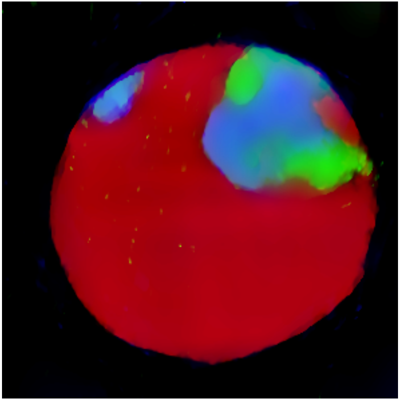
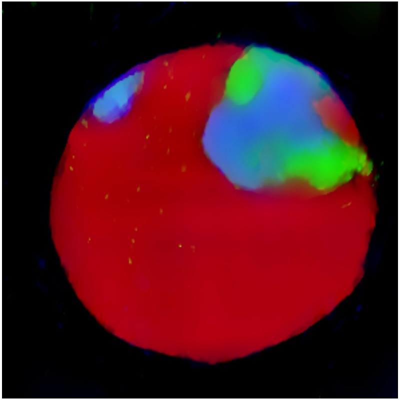


Sirt reconstruction



Decoupled TGV reconstruction



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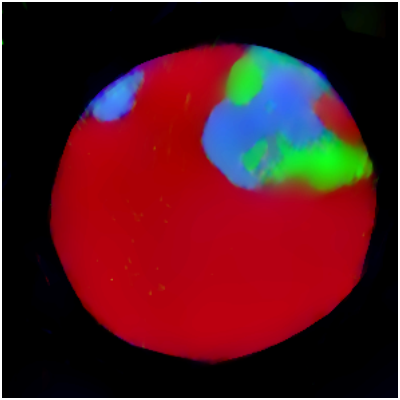

Coupled TGV reconstruction

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  - Superior reconstructions.

Acknowledgement

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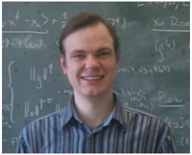
Institute of Electron Microscopy and Nanoanalysis (FELMI) and Graz Centre for Electron Microscopy (ZFE).

IGDK 1754: Optimization and Numerical Analysis for Partial Differential Equations with Nonsmooth Structures



Kristian Bredies   Martin Holler   Georg Haberfehlner

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[2] Graptor (**Gra**z **Ap**plication for **To**mographic **R**econstruction)