INSTITUT FÜR MATHEMATIK UND WISSENSCHAFTLICHES RECHNEN

Optimierung I

Problem Sheet 7

Due date: May 27, 2014

Problem 7.1: [Minima]

i) Compute the gradient and Hessian of the Rosenbruck function

$$f(x,y) = 100(x^2 - y)^2 + (x-1)^2.$$

Investigate the point $(x,y) = (1,1)$.

ii) Investigate the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = 8x + 12y + x^2 - 2y^2$ with respect to maxima, minima and critical points.

iii) Determine all local minima of $f : \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x,y) = \frac{1}{2}x^2 + x\sin(y)$$

Problem 7.2: [Optimality Conditions]
Let $P_n(0,1)$ be the set of polynomials of degree n on $(0,1)$. Find a necessary and sufficient condition on the coefficient vector $a = (a_0, \ldots, a_n)$ of the polynomial $p(x) = \sum_{i=0}^n a_i x^i$ which minimizes

$$f(a) = \int_0^1 |p(x) - g(x)|^2 \, \mathrm{d}x$$

where $g \in C([0,1], \mathbb{R})$ is given.

Problem 7.3: [Directional derivative]
Let $f \in C^1(\mathbb{R}^n, \mathbb{R})$, and let x^* be such that $\alpha = 0$ is a local minimizer of each function of the form $g(\alpha) = f(x^* + \alpha d)$ with $d \in \mathbb{R}^n$ arbitrary.

- Verify that $\nabla f(x^*) = 0$.
- Show that the above condition does not imply that x^* is a local minimizer of f by considering the function $f(y,z) = (z - py^2)(z - qy^2)$ with $0 < p < q$.