INSTITUT FÜR MATHEMATIK UND WISSENSCHAFTLICHES RECHNEN

Optimierung I

Problem Sheet 5

Due date: May 13, 2014

Problem 5.1: [Cheese production]
A cheese factory has two sources of milk: hay milk that costs 0.35€ per litre and regular milk that costs 0.30€ per litre. The factory produces three sorts of cheese for which one litre of milk is giving simultaneously a fraction of three cheese wheels according to the following table:

	Mozzarella	Gouda	Alpine cheese
Hay milk	0.0003	0.0002	0.0003
Regular milk	0.0003	0.0004	0.0002

The factory has contracted to supply 900 wheels of Mozzarella, 800 wheels of Gouda and 500 wheels of Alpine cheese within two years and wishes to find the amounts of hay milk and regular milk in order to minimize its cost.
Formulate this problem as a linear program and solve it with the simplex method.

Problem 5.2: [Two-phase simplex method] Programming Exercise
Extend the implementation of Problem 4.5 to find a basic feasible solution by introducing artificial variables and applying the simplex method (phase I) in prior to solving the original problem (phase II). As a result, you should have a function that takes the problem data A, b, c and returns the solution of the optimization problem. An example would be

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[x, e] = hollerm(A,b,c)
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Again, x should be a solution to the problem (if possible) and e indicate the success by $e = 0$ if the problem is solved, $e = 1$ if a solution became degenerate and $e = 2$ if the problem does not admit a solution. Test your code with the linear program of from Problem 5.1.

Problem 5.3: [Revised simplex method] Programming Exercise
Modify your implementation for Problem 5.2 to realize the revised simplex method. Test the code with the linear programs from Problems 4.1, 4.3 and 5.1.

Remark to the programming exercises: Send the resulting .m files to your group leader by mail at least two hours before class. You should send TWO DIFFERENT MAILS following the rules for programming exercises of the last sheet. In particular, both mails should contain a .m file with your surname and first letter of first name (e.g. hollerm.m) and the subject should read *Optimierung 1, Abgabe 5.2* and *Optimierung 1, Abgabe 5.3*, respectively. If your are using Octave, please indicate this by additionally writing Octave in the subject.

Problem 5.4: [Dual geometric interpretation]
Consider the linear program

$$\min_{x_1,x_2,x_3,x_4} \; 36x_1 + 42x_2 + 6x_3 + x_4 \qquad \text{subject to} \quad \left\{ \begin{array}{rcl} 3x_1 + 6x_2 + x_3 - 2x_4 & = & 5 \\ 6x_1 + 4x_2 - 3x_3 & = & 1 \\ x_1, x_2, x_3, x_4 & \geq & 0. \end{array} \right.$$

Sketch the corresponding column vectors $a_1, \ldots, a_4$, b as well as the dual feasible solution set graphically. Obtain a dual optimal solution from the sketch and use it to compute a primal optimal solution.

Problem 5.5: [Saddle point problem]
For $n, m \in \mathbb{N}$, $K \subset \mathbb{R}^n$, $L \subset \mathbb{R}^m$ and $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ a bilinear function, i.e.

$$F(x, \lambda) = c^T x + \lambda^T b - \lambda^T A x$$

with $c \in \mathbb{R}^n$, $b \in \mathbb{R}^m$, $A \in \mathbb{R}^{m \times n}$, consider the saddle-point problems

$$\inf_{x \in K} \sup_{\lambda \in L} F(x, \lambda) := N \in \mathbb{R} \cup \{\pm\infty\}, \tag{S1}$$

$$\sup_{\lambda \in L} \inf_{x \in K} F(x, \lambda) := M \in \mathbb{R} \cup \{\pm\infty\}. \tag{S2}$$

i) Show that, in general, $N \neq M$.

ii) Set $K = \{x \in \mathbb{R}^n : x \geq 0\}$, $L = \mathbb{R}^m$. Show that, if there exists a solution $\overline{x}$ to

$$\min_{x \in \mathbb{R}^n} c^T x \text{ subject to } \begin{cases} Ax = b \\ x \geq 0 \end{cases}, \tag{P}$$

then $M = N$, there exists a solution $\overline{\lambda}$ to the dual problem of (P), $(\overline{x}, \overline{\lambda})$ solves (S1) and $M = N$ coincides with the optimal value of (P) (and hence of the dual problem).