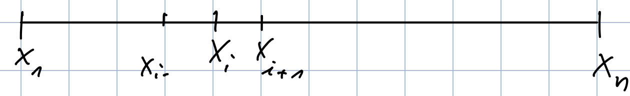
Element matrix computation

1D:



element matrix and rhs [JL, p. 89]

1D assembling [JL §3.4, p 93ff]

- Mapping global to local numbering

  Adjacency matrix $G^{(i)}$ [JL, p. 98]

  1D: ia [nelem][2] dynamically stored as 1D vector

  #vertices in element

Remark on integral calculation:

- Gauß quadrature rules are used [JL, p. 141]
- Gauß-Lobatto integration for tensor-product-like elements in higher dimensions.

Element calculations via the referenz element [JL, § 4.5.2, p. 251 ff]

idea: instead of defining shape functions and calculation of integrals especially for each element $T^{(r)}$

↳ map $T^{(r)}$ ($\xi(x)$) to a reference element $\hat{T}$, calculate the integral on $\hat{T}$ and remap that value regarding $T^{(r)}$ ($x_{T^{(r)}}(\xi)$)

[JL, p. 252]

• Let us consider

$J^{(r)} := \text{Jacobian}(X(\xi))$; $\left(J^{(r)}\right)^{-1} = \text{Jacobian}(\xi(x))$

$$K_{ij}^{(r)} = \int_{T^{(r)}} (\nabla \varphi_i(x))^T \Lambda(x) \nabla \varphi_j(x) \, dx$$

[JL, p. 286]

$$= \int_{\hat{T}} \left[\left(\left(J^{(r)}\right)^{-T} \underline{\nabla \hat{\varphi}_\alpha(\xi)} \right)^T \Lambda(x(\xi)) \left(\left(J^{(r)}\right)^{-T} \underline{\nabla \varphi_\beta(\xi)} \right) \right] \left| \det J^{(r)} \right| d\xi$$

[JL, p. 267 f]

$\left(\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \right)$, triangular element

Structure

$$= \frac{1}{\left|\det J^{(r)}\right|} \left[\text{term}_1(x^{(r)}) \underbrace{\int_{\hat{T}} \lambda_1(x(\xi))\, d\xi}_{\frac{1}{2}\lambda_1} + \text{term}_2(x^{(r)}) \underbrace{\int_{\hat{T}} \lambda_2(x(\xi))\, d\xi}_{\frac{1}{2}\lambda_2} \right]$$

λ_1, λ_2 const

$$= \frac{1}{\left|\det J^{(r)}\right|} \left[k_1 \, \text{term}_1(x^{(r)}) + k_2 \, \text{term}_2(x^{(r)}) \right]$$

Right hand side

$$\int_{T^{(r)}} f(x)\, \varphi_i(x)\, dx \stackrel{\dots}{=} \int_{\hat{T}} f(x(\xi))\, \hat{\varphi}_\alpha(\xi)\, |\det J^{(r)}|\, d\xi$$

From mesh to matrix and rhs

(0) Describe your domain Ω

(A) Discretize your domain

- allowed discretizations
- "good" meshes : aspect ratio $= \frac{\text{outer circ}}{\text{inner circ.}}$ (one def.)
  [mesh quality]

  - Δ : avoid "flat" elements, avoid two $\underbrace{\text{very acute}}_{\text{small}}$ angles

- anisotropic operators: use metrix with $\Lambda(x)$ for aspect ratio

Δ data

Element connectivity $ia[nelem][nloc]$

$\uparrow$ #vertices

1,2,3

xKoord [nvert][ndim]

- That is enough for linear elements
- Higher order elements might distinguish between
  fem connectivity (ia) and geometric description / connectivity
- also different in isoparametric elements
- the number of discretization knots is also called degrees of freedom (dofs)

linear element: ndofs $\equiv$ nverts

(B) • Choose your ansatz space and test space,
e.g. type/degree of shape functions

• implement the functions to calculate
element matrices $K^{(r)}$
element load vectors $f^{(r)}$
via the reference element

• functions for calculating integrals w.r.t. boundary conditions

(C) • Implement assembling routines for matrix, rhs

$$K = \sum_r C^{(r)^T} K^{(r)} C^{(r)}$$

$$f = \sum_r C^{(r)^T} f^{(r)}$$

• Apply b.c. to K, f

Adaptivity

[BL, § 4.10, p. 97ff]