SciComp and FEM in WS25

Exercise 6: Adaptivity for PDE in 1D.

Deadline: Jan 13, 2026, 20:00

Status: 10. Dezember 2025, 17:29

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The goal of this exercise consists in extending your own code from exercise 4 with respect to adaptivity.
You should try two out of the three adaptivity opportunities

- h-adaptivity,
- p-adaptivity
- r-adaptivity [HR, §2.1-2.2]

Use the jump of the flux on an edge (1D: vertex) between elements as error criteria, i.e.,

$$\left\lfloor \lambda(x_j)\frac{\partial u(x_j)}{\partial \vec{n}} \right\rfloor := \lambda(x_j+0)\frac{\partial u(x_j+0)}{\partial \vec{n}} - \lambda(x_j-0)\frac{\partial u(x_j-0)}{\partial \vec{n}}$$

See [Bra, Chapter III./§8] with an excellent introduction to adaptivity and [LB, §2.5 and §4.10].
Try your adaptivity schemes with the following examples:

(A) Consider the PDE [6 pts]

$$\begin{aligned} -u''(x) &= \frac{2p^3x}{(p^2x^2+1)^2} \qquad x \in (-1,1) =: \Omega \\ u(-1) &= -\arctan(p) \\ \frac{\partial u(1)}{\partial \vec{n}} &= \frac{p}{p^2+1} \end{aligned}$$

for various $p \in \{5, 10, 20, 100\}$.

- Exact solution: $u(x) = \arctan(px)$.
- Use a mesh with a vertex $x_j = 0$, later on also an initial mesh without that property.

(B) Consider the PDE [6 pts]

$$\begin{aligned} -\left(\lambda(x)\cdot u'(x)\right)' &= 0 \qquad x \in (0,1) =: \Omega \\ u(0) &= 0 \\ u(1) &= 1 \end{aligned}$$

with $\lambda(x) = \begin{cases} 1 & x \in \left(0, \frac{1}{\sqrt{2}}\right) \\ 10 & x \in \left(\frac{1}{\sqrt{2}}, 1\right) \end{cases}$.

- Start with a coarse mesh that doesn't contain $x_m = \frac{1}{\sqrt{2}}$.

(C) Solve the Péclet[1] problem [6 pts]

$$
\begin{aligned}
-u''(x) + pu'(x) &= 0 \qquad x \in (0,1) \\
u(0) &= 0 \\
u(1) &= 1
\end{aligned}
$$

with FEM for a constant $p \in \mathbb{R}$, see also [JL, §3.10 (example 2)]

- Solve the system of equations with $p = 70$ and with $p = -70$.

Literatur

[LB] Larson/Bengzon: "The Finite Element Method", Springer, TSCE 10, 2013 (e-book Uni Graz)

[DHL] Douglas/Haase/Langer: "A Tutorial on Elliptic PDE Solvers and their Parallelization", SIAM, 2003 (e-book)

[JL] Jung/Langer: "Methode der finiten Elemente für Ingenieure", Springer, 2013 (e-Book Uni Graz)

[Bra] Dietrich Braess: "Finite Elemente", Springer, 2013, 5. Auflage (e-Book Uni Graz, also in English [2007])

[HR] Huang/Russel: "Adaptive Moving Mesh Methods", Springer, 2011, AMS Vol. 174, doi: 10.1007/978-1-4419-7916-2 (Uni Graz Download[2])

[1] https://en.wikipedia.org/wiki/Jean_Claude_Eug%C3%A8ne_P%C3%A9clet

[2] https://link.springer.com/book/10.1007/978-1-4419-7916-2