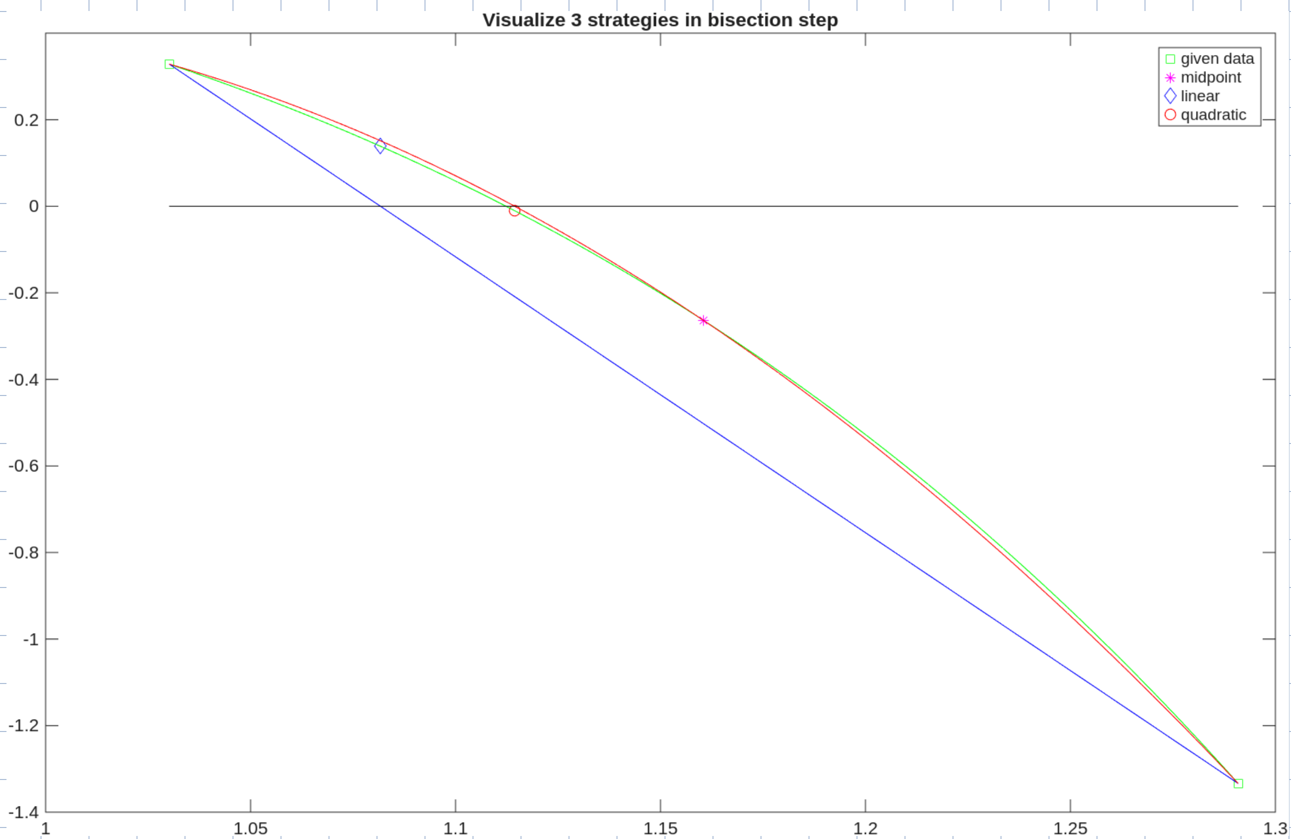
Find root of $f(x)$ in $[a,b]$:

Three ways of subdividing $[a,b]$, i.e. determine $c \in [a,b]$



(A) midpoint

$$c_M := \frac{a+b}{2}$$

(B) *linear*:

Find the root x^* of $\ell(x)$, the linear function from $(a, f(a))$ to $(b, f(b))$.

$$\ell(x) := f(a) + \frac{f(b) - f(a)}{b - a}(x - a) \overset{!}{\equiv} 0$$

$$\Rightarrow \quad c_L := x^* = a - (b-a)\frac{f(a)}{f(b) - f(a)}$$

(C) *quadratisch*:

Find the root x^* of $q(x) = d_0 + d_1 x + d_2 x^2$ mit $q(a) = f(a)$, $q(b) = f(b)$, $q(c_M) = f(c_M)$

$$\Rightarrow \quad \begin{array}{l} d_0 + d_1 a + d_2 a^2 = f(a) \\ d_0 + d_1 b + d_2 b^2 = f(b) \\ d_0 + d_1 c_M + d_2 c_M^2 = f(c_M) \end{array}$$

also c_L possible

$$\Rightarrow \text{Solve} \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c_M & c_M^2 \end{pmatrix} * \begin{pmatrix} d_0 \\ d_1 \\ d_2 \end{pmatrix} = \begin{pmatrix} f(a) \\ f(b) \\ f(c_M) \end{pmatrix}$$

and solve $q(c_q) \equiv 0$ with $c_q \in [a, b]$.