

$$c) \quad -v'' + p v' = 0 \quad / \int \cdot v$$

$$\int_0^1 (-v'' + p v') v dx = 0$$

$$\cancel{(-v' + p v) v} \Big|_0^1 - \int_0^1 \cancel{(-v' + p v)} \cdot \cancel{v'} dx = 0$$

$$= \int_0^1 -v'' v dx + \int_0^1 p v' v dx = 0$$

$$-v' v \Big|_0^1 + \int_0^1 v' v' dx + \int_0^1 p v' v dx = 0$$

$$a(u, v) = \int_0^1 u' v' dx + \int_0^1 p u' v dx \quad u_n = \sum_j \varphi_j U_j \quad v_n = \sum_j \varphi_j V_j$$

$$a(v_n, v_n) = \int_0^1 \sum_j v_j \varphi_j' \sum_i v_i \varphi_i' dx + \int_0^1 p \sum_j v_j \varphi_j' \sum_i v_i \varphi_i dx$$

$$= \sum_j \sum_i v_j v_i \left(\int_0^1 \varphi_j' \varphi_i' dx + \int_0^1 p \varphi_j' \varphi_i dx \right)$$

$$=: K_{ij}$$

case 1) $|i - j| > 1 \quad K_{ij} = 0$

case 2) $i = j$

2a) $i = j = 0$

$$K_{00} = \int_0^{\frac{1}{n}} n^2 dx + \int_0^{\frac{1}{n}} p (-n) \left(\frac{1}{n} - x \right) n dx =$$

$$= n + p \int_0^{\frac{1}{n}} n x - 1 dx = n + p \left(\frac{1}{2n} - \frac{1}{n} \right)$$

$$= n + p \int_0^{\frac{1}{n}} n^2 x - n dx = n + p \left(\frac{1}{2} - 1 \right) = n - \frac{p}{2}$$

b) $i, j \in \{1, \dots, n-1\}$

$$K_{ij} = \int_{x_{i-1}}^{x_{i+1}} n^2 dx + p \int_{x_{i-1}}^{x_i} n^2 (x - x_{i-1}) dx + p \int_{x_i}^{x_{i+1}} -n^2 (x_{i+1} - x) dx$$

$$= 2n$$

c) $i=j=n$

$$K_{nn} = \int_{\frac{n-1}{n}}^n n^2 + p \int_{\frac{n-1}{n}}^n n^2 \left(x - \frac{n-1}{n}\right) dx$$

$u = x - \frac{n-1}{n}$

$$= n + p \int_0^1 n^2 u du = n + \frac{p}{2}$$

case 3) $dj = i+1$

~~$$K_{i,i+1} = \int_{x_i}^{x_{i+1}} -n^2 dx + p \int_{x_i}^{x_{i+1}} n^2 (x - x_i) dx$$~~

$$K_{i,i+1} = \int_{x_i}^{x_{i+1}} -n^2 dx + p \int_{x_i}^{x_{i+1}} n^2 (x_{i+1} - x) dx$$

$u = x_{i+1} - x$

$$= -n + p \int_0^1 n^2 u du$$

$$= -n + p \int_0^1 n^2 u du = -n + \frac{p}{2}$$

3b) $i=j+1$

$$K_{j+1,j} = \int_{x_j}^{x_{j+1}} -n^2 dx + p \int_{x_j}^{x_{j+1}} -n^2 (x - x_j) dx$$

$$= -n - \frac{p}{2}$$

$$K = \begin{pmatrix} n - \frac{p}{2} & -n + \frac{p}{2} & & & \\ -n - \frac{p}{2} & 2n & -n + \frac{p}{2} & & \\ & 0 & -n - \frac{p}{2} & \ddots & \\ & & & 2n & -n + \frac{p}{2} \\ & & & -n - \frac{p}{2} & n + \frac{p}{2} \end{pmatrix}$$

enforce BC

$$K = \begin{pmatrix} 1 & 0 & \dots & 0 & 0 \\ -n - \frac{p}{2} & 2n & \dots & -n + \frac{p}{2} & 0 \\ 0 & -n - \frac{p}{2} & \dots & & \\ & 0 & \dots & 2n & -n + \frac{p}{2} \\ 0 & 0 & \dots & 0 & 1 \end{pmatrix}$$

$$f = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

see code