c) $-u'' + pu' = 0 \quad / \int \cdot v$

$$\int_0^1 (-u'' + pu')v\,dx = 0$$

$$\cancel{(-u' + pu)v]_0^1 - \int_0^1 (-u' + pu) v'\,dx = 0}$$

$$= \int_0^1 -u'' v\,dx + \int_0^1 pu' v\,dx = 0$$

$$-u'v]_0^1 + \int_0^1 u'v'\,dx + \int_0^1 pu'v\,dx = 0$$

$$a(u,v) = \int_0^1 u'v'\,dx + \int_0^1 pu'v\,dx \qquad U_n = \sum_j \varphi_j U_j \quad V_n = \sum_j \varphi_j V_j$$

$$a(u_n, v_n) = \int_0^1 \sum_j u_j \varphi_j' \sum_i v_i \varphi_i'\,dx + \int_0^1 p \sum_j u_j \varphi_j' \sum_i v_i \varphi_i\,dx$$

$$= \sum_j \sum_i u_j v_i \Big(\underbrace{\int_0^1 \varphi_j' \varphi_i'\,dx + \int_0^1 p \varphi_j' \varphi_i\,dx}_{:= K_{ij}} \Big)$$

case 1) $|i-j| > 1 \qquad K_{ij} = 0$

case 2) $i = j$

2) a) $i = j = 0$

$$K_{00} = \int_0^{\frac{1}{n}} n^2\,dx + \int_0^{\frac{1}{n}} p(-n)\left(\tfrac{1}{n} - x\right)n\,dx =$$

$$\cancel{= n + p\int_0^{\frac{1}{n}} nx - 1\,dx = n + p\left(\tfrac{1}{2n} - \right.}$$

$$= n + p\int_0^{\frac{1}{n}} n^2 x - n\,dx = n + p\left(\tfrac{1}{2} - 1\right) = n - \tfrac{p}{2}$$

b) $i, j \in \{1, \dots, n-1\}$

$$K_{ii} = \int_{x_{i-1}}^{x_{i+1}} n^2\,dx + p\int_{x_{i-1}}^{x_i} n^2(x - x_{i-1})\,dx + p\int_{x_{i-1}}^{x_i} -n^2(x_{i+1} - x)\,dx$$

$$= 2n$$

c) $i = j = n$

$$K_{nn} = \int_{\frac{n-1}{n}}^{n} n^2 + p\int_{\frac{n-1}{n}}^{n} n^2\left(x - \frac{n-1}{n}\right) du$$

$u = x - \frac{n-1}{n}$

$$= n + p\int_0^{\frac{1}{n}} n^2 u \, du = n + \frac{p}{2}$$

case 3) a) $j = i+1$

$$\cancel{K_{i,i+1} = \int_{x_i}^{x_{i+1}} n^2 dx + p\int_{x_i}^{x_{i+1}} n^2 (\ldots) x_i}$$

$$K_{i,i+1} = \int_{x_i}^{x_{i+1}} -n^2 dx + p\int_{x_i}^{x_{i+1}} n^2 (x_{i+1} - x) dx$$

$u = x_{i+1} - x$

$$= -n + p\int_{\frac{1}{n}}^{0} n^2(-u) du$$

$$= -n + p\int_0^{\frac{1}{n}} n^2 u \, du = -n + \frac{p}{2}$$

3b) $i = j+1$

$$K_{j+1,j} = \int_{x_j}^{x_{j+1}} -n^2 dx + p\int_{x_j}^{x_{j+1}} -n^2(x - x_j) dx$$

$$= -n - \frac{p}{2}$$

$$K = \begin{pmatrix} n - \frac{p}{2} & -n + \frac{p}{2} & & & \\ -n - \frac{p}{2} & 2n & -n + \frac{p}{2} & & \\ & -n - \frac{p}{2} & \ddots & \ddots & \\ & & \ddots & 2n & -n + \frac{p}{2} \\ & & & -n - \frac{p}{2} & n + \frac{p}{2} \end{pmatrix}$$

enforce BC

$$K = \begin{pmatrix} 1 & 0 & \cdots & & 0 \\ -n - \frac{p}{2} & 2n & -n + \frac{p}{2} & 0 \cdots & 0 \\ 0 & -n - \frac{p}{2} & \ddots & & \\ \vdots & 0 & & \ddots & \\ & \vdots & & 2n & -n + \frac{p}{2} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \qquad f = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

See code