B) $-(\lambda u')' = 0 \quad / \int_0^1 \cdot v \qquad v \in H_0^1(\Omega)$

$$\int_0^1 -(\lambda u')' v \, dx = 0$$

$$\lambda u' v \Big]_0^1 + \int_0^1 \lambda u' v' = 0$$

$$\underbrace{\int_0^1 \lambda u' v' \, dx}_{= a(u,v)} = \underbrace{0}_{= \langle F, v \rangle}$$

$u_h = \sum_i u_i \varphi_i \qquad v_h = \sum_j v_j \varphi_j$

$$a(u_h, v_h) = \int_0^1 \lambda \sum_i \varphi_i' \sum_j \varphi_j' \, dx$$

$$= \sum_i \sum_j u_i v_j \underbrace{\int_0^1 \lambda \varphi_i' \varphi_j'}_{:= K_{ij}}$$

case 1) $|i-j| > 1 \qquad K_{ij} = 0$

case 2 $i=j$

2a) $i=j=0$

$K_{00}=\int_0^1 \lambda (\varphi_1')^2 dx = \int_0^{\frac{1}{n}} \lambda n^2 dx = n$ $\quad \lambda = 1$ at start

b) $= i=j \quad i,j \in \{1 \dots n-1\} \quad \frac{1}{\sqrt{2}} \notin \left(\frac{i-1}{n}, \frac{i+1}{n}\right)$

$K_{ii} = \int_0^1 \lambda (\varphi_i')^2 = \int_{x_{i-1}}^{x_{i+1}} \lambda (\varphi_i')^2 dx = 2n\lambda$

~~+Cath 1~~
2 case

c) $i=j \quad i,j \in \{1 \dots n-1\} \quad \frac{1}{\sqrt{2}} \in \left(\frac{i-1}{n}, \frac{i+1}{n}\right)$

$K_{ii} = n^2 \cdot \int_{x_{i-1}}^{x_{i+1}} \lambda dx = n^2 \left(\frac{1}{\sqrt{2}} - \frac{i-1}{n} + 10\left(\frac{i+1}{n} - \frac{1}{\sqrt{2}}\right)\right)$

d $i=j=n$

$K_{nn} = 10n$ $\quad \lambda = 10$ at end

case 3 $j = i+1$ (symmetric)

3a) $\frac{1}{\sqrt{2}} \notin \left[\frac{i}{n}, \frac{i+1}{n}\right]$

$K_{i\,i+1} = -n\lambda$

3b) $\frac{1}{\sqrt{2}} \in \left[\frac{i}{n}, \frac{i+1}{n}\right]$

$K_{i\,i+1} = \int_{x_i}^{x_{i+1}} -n^2 \lambda dx = -n^2 \left(\frac{1}{\sqrt{2}} - \frac{i}{n} + 10\left(\frac{i+1}{n} - \frac{1}{\sqrt{2}}\right)\right)$

Enforce BC $\quad K_{00} = 1 \quad K_{0j} = 0 \quad j > 0$

$K_{nn} = 1 \quad K_{nj} = 0 \quad j < 0$ $\qquad F = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

See code