

$$B) -(\lambda v')' = 0 \quad / \int_0^1 \cdot v \quad v \in H_0^1(\Omega)$$

$$\int_0^1 -(\lambda v')' v dx = 0$$

$$\lambda v' v \Big|_0^1 + \int_0^1 \lambda v' v' = 0$$

$$\int_0^1 \lambda v' v' dx = 0$$

$$\begin{matrix} \parallel & \parallel \\ a(v, v) & \langle F, v \rangle \end{matrix}$$

$$v_h = \sum_i v_i \varphi_i \quad v_L = \sum_j v_j \varphi_j$$

$$a(v_h, v_h) = \int_0^1 \lambda \sum_i \varphi_i' \sum_j \varphi_j' dx$$

$$= \sum_i \sum_j v_i v_j \underbrace{\int_0^1 \lambda \varphi_i' \varphi_j' dx}_{:= K_{ij}}$$

$$\text{case 1) } |i - j| > 1 \quad K_{ij} = 0$$

case 2 i=j

2a) $i=j=0$

$$K_{00} = \int_0^1 \lambda (\varphi_0)^2 dx = \int_0^1 \lambda n^2 dx = n \quad \lambda = 1 \text{ at start}$$

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b) $i=j \quad i,j \in \{1 \dots n-1\} \quad \frac{1}{\sqrt{2}} \notin \left(\frac{i-1}{n}, \frac{i+1}{n}\right)$

$$K_{ii} = \int_0^1 \lambda (\varphi_i)^2 dx = \int_{x_{i-1}}^{x_{i+1}} \lambda (\varphi_i)^2 dx = 2n\lambda$$

reflected

same

c) $i=j \quad i,j \in \{1 \dots n-1\} \quad \frac{1}{\sqrt{2}} \in \left(\frac{i-1}{n}, \frac{i+1}{n}\right)$

$$K_{ii} = n^2 \cdot \int_{x_{i-1}}^{x_{i+1}} \lambda dx = n^2 \left(\frac{1}{\sqrt{2}} - \frac{i-1}{n} + 10 \left(\frac{i+1}{n} - \frac{1}{\sqrt{2}} \right) \right)$$

d) $i=j=n$

$$K_{nn} = 10n$$

$\lambda = 10$ at end

case 3 $j=i+1$ (symmetric)

3a) $\frac{1}{\sqrt{2}} \notin \left[\frac{i-1}{n}, \frac{i+2}{n}\right] \quad \left[\frac{i}{n}, \frac{i+1}{n}\right]$

$$K_{i,i+1} = -n\lambda$$

3b) $\frac{1}{\sqrt{2}} \in \left[\frac{i}{n}, \frac{i+1}{n}\right]$

$$K_{i,i+1} = \int_{x_i}^{x_{i+1}} -n^2 \lambda dx = -n^2 \left(\frac{1}{\sqrt{2}} - \frac{i}{n} + 10 \left(\frac{i+1}{n} - \frac{1}{\sqrt{2}} \right) \right)$$

Enforce BC $K_{00} = 1 \quad K_{0j} = 0 \quad j > 0$
 $K_{nn} = 1 \quad K_{nj} = 0 \quad j < 0$

$$\Phi = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

See code

