

(c) Poisson problem

$$-u''(x) + pu'(x) = 0, \quad x \in (0,1), \quad p \in \mathbb{R},$$

$$u(0) = 0,$$

$$u(1) = 1.$$

Define $z(x) = u'(x)$. $\Rightarrow -z'(x) + pz(x) = 0$.

Separation of variables: $\frac{z'(x)}{z(x)} = p$ $\left\{ \begin{array}{l} dz = z'(x) dx \\ y = z(x) \end{array} \right.$

Integrate both sides: $\int \frac{z'(x)}{z(x)} dx = \int \frac{1}{y} dy = \ln|y| + C$ $\left. \begin{array}{l} \int p dx = px + C \end{array} \right\} \ln|y| = px + C$

$$\Rightarrow z(x) = Ce^{px} \Rightarrow u'(x) = Ce^{px} \Rightarrow u(x) = \frac{C}{p}e^{px} + D$$

$$\text{BC: } u(0) = \frac{C}{p} + D \stackrel{!}{=} 0 \Rightarrow D = -\frac{C}{p}$$

$$u(1) = \frac{C}{p}e^p + D \stackrel{!}{=} 1 \Rightarrow \frac{C}{p}(e^p - 1) = 1 \Rightarrow C = \frac{p}{e^p - 1}.$$

$$\text{So: } \underline{\underline{u(x) = \frac{1}{e^p - 1} (e^{px} - 1)}}$$