(c) Péclet problem

$$-u''(x) + p\,u'(x) = 0, \quad x \in (0,1), \quad p \in \mathbb{R},$$
$$u(0) = 0,$$
$$u(1) = 1.$$

Define $z(x) = u'(x)$. $\Rightarrow -z'(x) + p\,z(x) = 0$.

Seperation of variables: $\frac{z'(x)}{z(x)} = p$ $\quad dy = z'(x)\,dx$, $y = z(x)$

Integrate both sides: $\int \frac{z'(x)}{z(x)}\,dx = \int \frac{1}{y}\,dy = \ln|y| + C$
$\int p\,dx = px + C$
$\Big\}\ \ln|y| = px + C$

$\Rightarrow z(x) = Ce^{px} \Rightarrow u'(x) = Ce^{px} \Rightarrow u(x) = \frac{C}{p}e^{px} + D$

BC: $u(0) = \frac{C}{p} + D \overset{!}{=} 0 \Rightarrow D = -\frac{C}{p}$

$u(1) = \frac{C}{p}e^{p} + D \overset{!}{=} 1 \Rightarrow \frac{C}{p}(e^{p} - 1) = 1 \Rightarrow C = \frac{p}{e^{p}-1}$.

So: $\underline{\underline{u(x) = \frac{1}{e^{p}-1}\left(e^{px} - 1\right)}}$ ✓