(A) Stationary heat equation without convection or advection

$$-(\lambda(x)u'(x))' = f(x) \quad \text{in } \Omega = (0,1)$$

$$u(0) = 0, \quad u(1) = 1.$$

[1 / ½]

We have $\lambda(x) = \lambda$, $f(x) = c$. $\Longrightarrow -\lambda u''(x) = c$.

Function spaces: $u \in C^2(0,1)$, $\lambda \in L^\infty(0,1)$, $f \in L^\infty(0,1)$.

Add: $u = u_{hom} + u_1$

$u(0)=0, u(1)=1$

Hom.: $-\lambda u''_{hom}(x) = 0 \Longrightarrow u''_{hom}(x) = 0$, $u'_{hom}(x) = A$, $u_{hom}(x) = A_h x + B_h$

Part.: $-\lambda u''_p(x) = c \Rightarrow u''_p(x) = -\frac{c}{\lambda}$, $u'_p(x) = -\frac{c}{\lambda}x + A_p$, $u_p(x) = -\frac{c}{2\lambda}x^2 + A_p x + B_p$

$u(0) = u(1) = 0$

Use B.C.: $u(0) = B \overset{!}{=} 0$

$$u(1) = -\frac{c}{2\lambda} + A \overset{!}{=} 1 \Rightarrow A = 1 + \frac{c}{2\lambda}$$

Hence: $\underline{\underline{u(x) = -\frac{c}{2\lambda}x^2 + \left(1 + \frac{c}{2\lambda}\right)x}}$.

Check: $u''(x) = -\frac{c}{2\lambda}$ ✓, $u(0)=0$, $u(1)=1$.